



MATHS

BOOKS - ARIHANT PRAKASHAN

VERY SIMILAR TEST 8

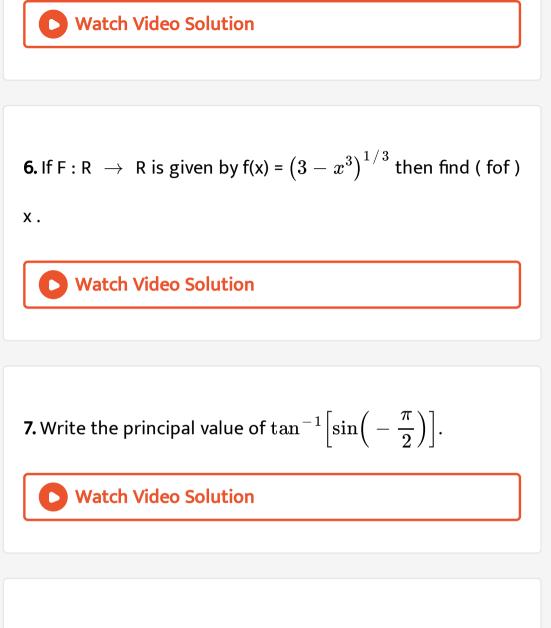


1. Show that $\sin x(1+\cos x), \, x \in [0,\pi]$ is maximum value

at
$$x=rac{\pi}{3}$$

2. Integrate
$$\int \left(2\sqrt{x} + \frac{3}{\sqrt{x}}\right) dx$$
.
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3. Find the integrating factor of $x \frac{dy}{dx} + 2y = x \cos x$.
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4. Find the value of \hat{i} . $(\hat{j} \times \hat{k}) + \hat{j}$. $(\hat{k} \times \hat{i}) + \hat{k}$. $(\hat{i} \times \hat{j})$.
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5. Find the number of points (x, y, z) in space other than the

point (1,-2, 3), such that |x| = 1, |y| = 2 and |z| = 3.



8. If ω is a complex cube root of 1,then for what value of.

lamda the determinant
$$egin{array}{ccc} 1 & \omega & \omega^2 \ \omega & \lambda & 1 \ \omega^2 & 1 & \omega \end{array} = 0 \, ?$$



9. A die is thrown thrice, getting an even number is considered a success. What is the mean and variance of the Binomial distribution?

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10. If
$$x = a \cos \theta$$
, $y = a \sin \theta$, then find $\frac{dy}{dx}$.

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11. Show that $\sin x(1+\cos x), x\in [0,\pi]$ is maximum value

at
$$x=rac{\pi}{3}.$$



12. Integrate
$$\int \left(2\sqrt{x} + \frac{3}{\sqrt{x}} \right) dx.$$

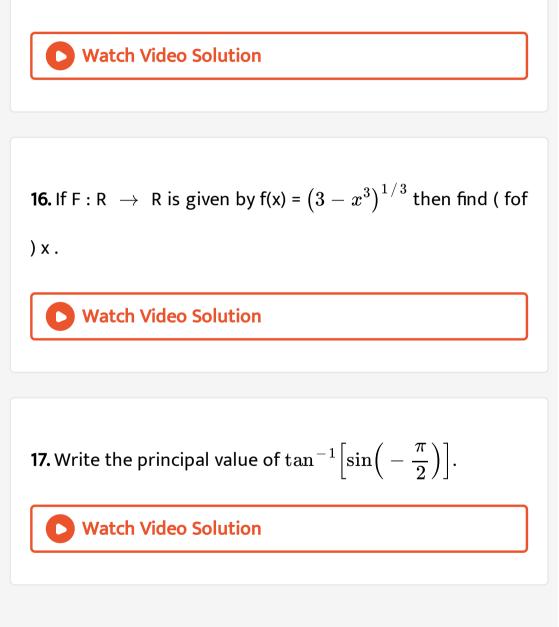
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13. Find the integrating factor of
$$x \frac{dy}{dx} + 2y = x \cos x$$
.

14. Find the value of
$$\hat{i}.$$
 $\left(\hat{j} imes\hat{k}
ight)+\hat{j}.$ $\left(\hat{k} imes\hat{i}
ight)+\hat{k}.$ $\left(\hat{i} imes\hat{j}
ight)$



15. Find the number of points (x, y, z) in space other than the point (1,-2, 3), such that |x| = 1, |y| = 2 and |z| = 3.



18. If ω is a complex cube root of 1,then for what value of.

lamda the determinant
$$egin{array}{ccc} 1 & \omega & \omega^2 \ \omega & \lambda & 1 \ \omega^2 & 1 & \omega \end{array} = 0 \, ?$$

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19. A die is thrown thrice, getting an even number is considered a success. What is the mean and variance of the Binomial distribution?

20. If
$$x=a\cos heta, y=a\sin heta$$
 , then find $rac{dy}{dx}$.



Section B

1. Solve for x,

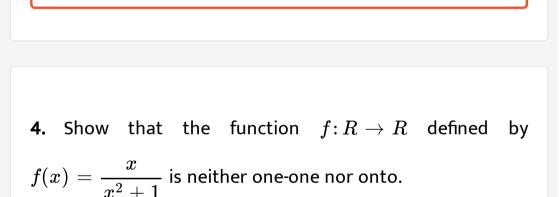
$$an^{-1}igg(rac{2x}{1-x^2}igg) + \cot^{-1}igg(rac{1-x^2}{2x}igg) = rac{\pi}{3}, \ -1 < x < 1$$

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2. Corner points of the feasible region determined by the system of linear constraints are (0,3), (1,1) and (3,0). Let Z = px + qy, where p, q > 0. Find the condition in p and q, so that the minimum of Z

occurs at (3,0) and (1,1).

3. Let R be a relation on the set A of ordered pairs of positive integers defined by (x, y) R (u, v), if and only if xv = yu. Show that R is an equivalence relation.



5. If
$$\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$$
, then find $lpha(eta+\gamma) + eta(\gamma+lpha) + \gamma(lpha+eta).$



6. Sita and Gita throw a dia alternatively till one of them gets a 6 to win the game. Find their respective probability of winning if Sita starts first.



7. If A and B are matrices of the same order and AB=BA, Then

prove that $A^2 - B^2 = (A - B)(A + B)$



8. If
$$A(x_1, y_1), B(x_2, y_2)$$
, and, $C(x_3, y_3)$ are vertices of an

equilateral triangle whose each side is equal to a, then

prove that $egin{array}{c|c} x_1 & y_1 & 2 \\ x_2 & y_2 & 2 \\ x_3 & y_3 & 2 \end{array} egin{array}{c|c} = 3a^4. \end{array}$



9. If you throw a pair of dice n times, find the probability of getting at least one doublet.[When you get identical members you call it a doublet. You can get a double in six ways: (1,1),(2,2),(3,3),(4,4),(5,5) and (6,6) , thus the probability of getting a doublet is $\frac{6}{36} = \frac{1}{6}$, so that the probability of not getting a doublet in one throw is $\frac{5}{6}$].

10. Find the points on the curve $y = x^3 - 3x^2 + 2x$ at which the tangent to the curve is parallel to the line y - 2x + 3 = 0.

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11. Find the interval(s) in which the following functions are

(i) increasing

(ii) decreasing

$$f(x) = (x+2)e^{-x}$$

12. If
$$\sqrt{1-x^4}+\sqrt{1-y^4}=kig(x^2-y^2ig)$$
 then show that $rac{dy}{dx}=rac{x\sqrt{1-y^4}}{y\sqrt{1-x^4}}$

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13. If
$$y = (\sin x)^x + \sin^{-1} \sqrt{x}$$
, then find $\frac{dy}{dx}$.

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14. Examine if Rolle.s theorem is applicable to the function

 $f(x) = \cot x$ on $[0, \pi]$.

15. Find the differential equation for the family of curve $y = a \sin^{-1} x + b \cos^{-1} x$.



16. Obtain the general solution of the following differential

equations.

$$x^2\sqrt{y^2+3}dx+y\sqrt{x^3+1}dy=0$$

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17. Evaluate the following integrals :

$$\int\!\!{(3\sin\phi-2)\cos\phi\over 5-\cos^2\phi-4\sin\phi}d\phi$$

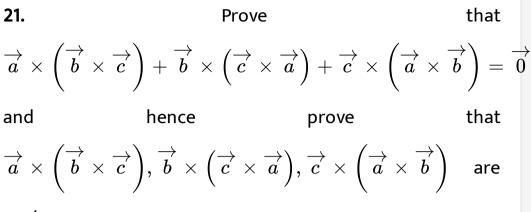
18. Prove that
$$\int_0^{\pi/4} 2 an^3 x dx = 1 - \log 2.$$

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19. Find the area of the region included between the parabola $y^2 = 2x$ and the straight line x - y = 4.

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20. Find the vector equation of the plane through the points (2,1, -1) and (-1, 3, 4) and .. perpendicular to the plane x - 2y + 4z = 10.



coplanar.

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22. If
$$\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{b} \times \overrightarrow{c} \neq \overrightarrow{0}$$
, prove that $\overrightarrow{a} + \overrightarrow{c} = m\overrightarrow{b}$,

where m is a scalar.



23. Find the shortest distance between the lines

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \text{ and } \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}.$$
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$$an^{-1}igg(rac{2x}{1-x^2}igg) + \cot^{-1}igg(rac{1-x^2}{2x}igg) = rac{\pi}{3}, \ -1 < x < 1$$

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25. Corner points of the feasible region determined by the system of linear constraints are (0,3), (1,1) and (3,0). Let Z = px + qy, where p, q > 0.

Find the condition in \boldsymbol{p} and $\boldsymbol{q},$ so that the minimum of \boldsymbol{Z}

occurs at (3,0) and (1,1).



26. Let R be a relation on the set A of ordered pairs of positive integers defined by (x, y) R (u, v), if and only if xv =

yu. Show that R is an equivalence relation.

27. Show that the function
$$f\!:\!R o R$$
 defined by $f(x)=rac{x}{x^2+1}$ is neither one-one nor onto.

28. If $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$, then find $\alpha(\beta + \gamma) + \beta(\gamma + \alpha) + \gamma(\alpha + \beta)$.



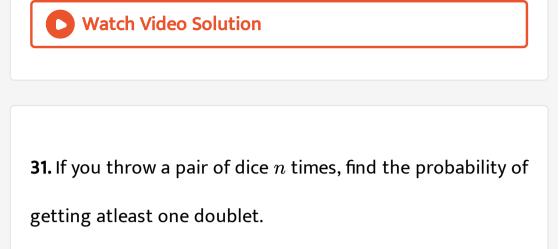
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30. If $A(x_1, y_1), B(x_2, y_2)$, and, $C(x_3, y_3)$ are vertices of an

equilateral triangle whose each side is equal to a, then

prove that
$$egin{array}{c|c} x_1 & y_1 & 2 \ x_2 & y_2 & 2 \ x_3 & y_3 & 2 \ \end{array} = 3a^4.$$



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32. Find the points on the curve $y = x^3 - 3x^2 + 2x$ at which

the tangents to the curve is parallel to the line y-2x+3 = 0.



33. Find the interval(s) in which the following functions are

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34. If
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 then show that $rac{dy}{dx}=rac{x\sqrt{1-y^4}}{y\sqrt{1-x^4}}$

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$$\int \! rac{(3\sin\phi-2)\cos\phi}{5-\cos^2\phi-4\sin\phi} d\phi$$

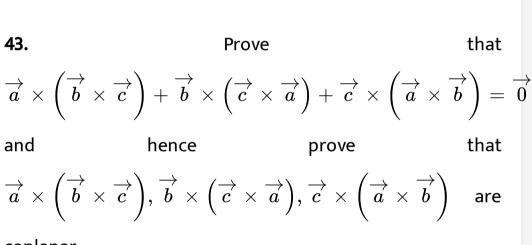
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41. Find the area of the region included between the parabola $y^2 = 2x$ and the straight line x - y = 4.

42. Find the vector equation of the plane through the points (2,1, -1) and (-1, 3, 4) and .. perpendicular to the plane x - 2y + 4z = 10.



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44. If
$$\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{b} \times \overrightarrow{c} \neq \overrightarrow{0}$$
, prove that $\overrightarrow{a} + \overrightarrow{c} = m\overrightarrow{b}$

, where m is a scalar.

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45. Find the shortest distance between the lines

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \text{ and } \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}.$$
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Section C

1. If
$$y = \left[x + \sqrt{x^2 + a^2}
ight]^n$$
, then prove that $rac{dy}{dx} = rac{ny}{\sqrt{x^2 + a^2}}$



2. Show that the right circular cone of least curved surface area and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

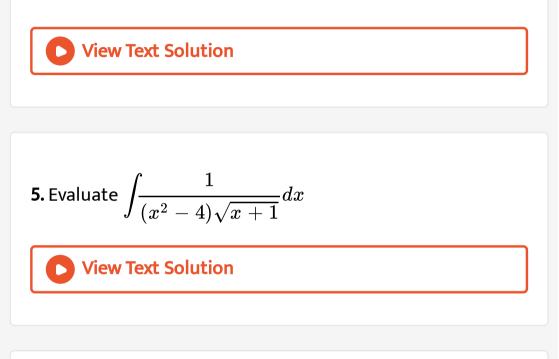


3. Find the area bounded by the curve y = sin x between x =

0 and $x = 2\pi$.



4. Find the particular solution of the differential equation $ig(1-y^2ig)(1+\log|x|)dx+2 imes dy=0$, given y=0, when x=1.



6. Answer any three questions

Find the coordinates of the point, where the line through

the points A(3, 4, 1) and B(5, 1, 6) crosses the XY-plane.



7. Find the equation of the two lines through the origin which intersect the line $\frac{x-3}{2} - \frac{y-3}{1} = \frac{z}{1}$ at angle of $\frac{\pi}{3}$ each.

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8. If a, b, c > 0 such that a + b + c = abc find the value of

 $\tan^{-1}a + \tan^{-1}b + \tan^{-1}c.$

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9. Solve the following LPP graphically.

Maximize Z = 10x + 6y

Subjectto $3x+y \leq 12$

 $2x + 5y \leq 34$

x, y > 0

10. Show the
$$f: R - \{-1\} \rightarrow R - \{1\}$$
 given by $f(x) = \frac{x}{x-1}$ is invertible. Also find f^{-1} .

11. Show the following system of equations is consistent 2x - y + 3z = 5, 3x + 2y - z = 7, 4x + 5y - 5z = 9, Also, find the solution.

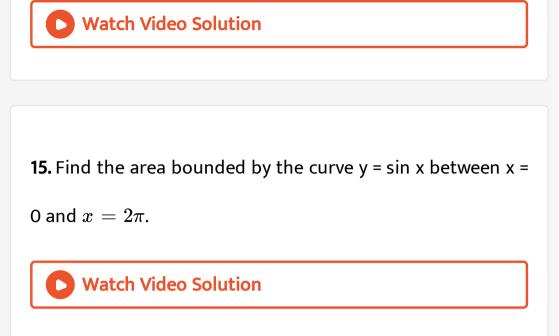
12. If A and B are two independent events such that $P(\overline{A} \cap B) = rac{2}{15}$ and $P(A \cap \overline{B}) = rac{1}{6}$ then find P(A) and P(B)



13. Find the inverse of the matrix $A=egin{bmatrix} 1&2&-2\-1&3&0\0&-2&1 \end{bmatrix}$ by

using row transformations.

14. If
$$y = \left[x + \sqrt{x^2 + a^2}\right]^n$$
, then prove that $\frac{dy}{dx} = \frac{ny}{\sqrt{x^2 + a^2}}$



16. Find the particular solution of the differential equation $ig(1-y^2ig)(1+\log \lvert x
vert) dx+2 imes dy=0$, given y=0, when x=1.

17. Evaluate
$$\int rac{1}{(x^2-4)\sqrt{x+1}} dx$$



18. Answer any three questions

Find the coordinates of the point, where the line through

the points A(3, 4, 1) and B(5, 1, 6) crosses the XY-plane.



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20. If a, b, c > 0 such that a + b + c = abc find the value of $\tan^{-1}a + \tan^{-1}b + \tan^{-1}c$.



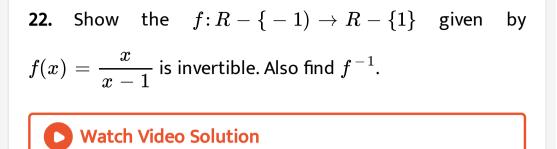
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Maximize Z = 10x + 6y

Subjectto $3x + y \leq 12$

 $2x + 5y \le 34$

x, y > 0



23. Show the following system of equations is consistent

$$2x - y + 3z = 5, 3x + 2y - z = 7, 4x + 5y - 5z = 9,$$

Also, find the solution.

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24. If A and B are two independent events such that $P(\overline{A} \cap B) = \frac{2}{15}$ and $P(A \cap \overline{B}) = \frac{1}{6}$ then find P(A) and P(B)





25. Find the inverse of the matrix $A = egin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ by

using row transformations.