MATHS

BOOKS - ARIHANT PRAKASHAN

VERY SIMILAR TEST 8

Section A

1. Show that $\sin x(1+\cos x), x \in |0, \pi|$ is maximum value at $x = \frac{\pi}{3}$.

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2. Integrate $\int\left(2\sqrt{x}+\frac{3}{\sqrt{x}}\right)dx.$

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3. Find the integrating factor of $x\frac{dy}{dx}+2y=x\cos x.$

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4. Find the value of $\hat{i}.\left(\hat{j}\times\hat{k}\right)+\hat{j}.\left(\hat{k}\times\hat{i}\right)+\hat{k}.\left(\hat{i}\times\hat{j}\right).$

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5. Find the number of points (x, y, z) in space other than the point (1,-2, 3), such that |x| = 1, |y| = 2 and |z|= 3.

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6. If F : R $\rightarrow$ R is given by f(x) = $\left(3 - x^3\right)^{1/3}$ then find (fof) x .

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7. Write the principal value of $\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$.

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8. If ω is a complex cube root of 1,then for what value of. lamda the determinant $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \lambda & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0$?

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9. A die is thrown thrice, getting an even number is considered a success. What is the mean and variance of the Binomial distribution?

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10. If $x = a\cos\theta, y = a\sin\theta$, then find $\frac{dy}{dx}$.

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11. Show that $\sin x(1+\cos x), x \in |0, \pi|$ is maximum value at $x = \frac{\pi}{3}$.

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Section B

1. Solve for x,

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{\pi}{3},\ -1<x<1$$

.

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2. Corner points of the feasible region determined by the system of linear constraints are (0,3), (1,1) and (3,0). Let $Z=px+qy$, where $p, q>0$.

Find the condition in p and q, so that the minimum of Z occurs at (3,0) and (1,1).

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3. Let R be a relation on the set A of ordered pairs of positive integers defined by (x, y) R (u, v), if and only if xv = yu. Show that R is an equivalence relation.

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4. Show that the function $f: R \to R$ defined by $f(x) = \frac{x}{x^2 + 1}$ is neither one-one nor onto.

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5. If $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$, then find $\alpha(\beta + \gamma) + \beta(\gamma + \alpha) + \gamma(\alpha + \beta)$.

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6. Sita and Gita throw a dia alternatively till one of them gets a 6 to win the game. Find their respective probability of winning if Sita starts first.

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7. If A and B are matrices of the same order and AB=BA, Then prove that $A^2 - B^2 = (A - B)(A + B)$

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8. If $A(x_1, y_1)$, $B(x_2, y_2)$, and, $C(x_3, y_3)$ are vertices of an equilateral triangle whose each side is equal to a, then prove that $\begin{vmatrix} x_1 & y_1 & 2 \\ x_2 & y_2 & 2 \\ x_3 & y_3 & 2 \end{vmatrix}^2 = 3a^4$.

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9. If you throw a pair of dice n times, find the probability of getting at least one doublet.[When you get identical members you call it a doublet. You can get a double in six ways: (1,1),(2,2),(3,3),(4,4),(5,5) and (6,6) , thus the probability of getting a doublet is $\frac{6}{36} = \frac{1}{6}$, so that the probability of not getting a doublet in one throw is $\frac{5}{6}$].

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10. Find the points on the curve $y = x^3 - 3x^2 + 2x$ at which the tangent to the curve is parallel to the line $y - 2x + 3 = 0$.

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11. Find the interval(s) in which the following functions are

(i) increasing

(ii) decreasing

$f(x) = (x + 2)e^{-x}$

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12. If $\sqrt{1-x^4}+\sqrt{1-y^4}=k\left(x^2-y^2\right)$ then show that $\frac{dy}{dx}=\frac{x\sqrt{1-y^4}}{y\sqrt{1-x^4}}$

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13. If $y=(\sin x)^x+\sin^{-1}\sqrt{x}$, then find $\frac{dy}{dx}$.

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14. Examine if Rolle.s theorem is applicable to the function $f(x)=\cot x$ on $[0,\pi]$.

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15. Find the differential equation for the family of curve $y = a\sin^{-1}x + b\cos^{-1}x$.

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16. Obtain the general solution of the following differential equations.

$x^2\sqrt{y^2+3}dx + y\sqrt{x^3+1}dy = 0$

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17. Evaluate the following integrals :

$$\int \frac{(3\sin\phi - 2)\cos\phi}{5 - \cos^2\phi - 4\sin\phi} d\phi$$

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18. Prove that $\int_0^{\pi/4} 2\tan^3 x dx = 1 - \log 2.$

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19. Find the area of the region included between the parabola $y^2 = 2x$ and the straight line $x - y = 4.$

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20. Find the vector equation of the plane through the points (2,1, -1) and (-1, 3, 4) and .. perpendicular to the plane $x - 2y + 4z = 10.$

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21. Prove that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) + \vec{b} \times \left(\vec{c} \times \vec{a}\right) + \vec{c} \times \left(\vec{a} \times \vec{b}\right) = \vec{0}$ and hence prove that $\vec{a} \times \left(\vec{b} \times \vec{c}\right), \vec{b} \times \left(\vec{c} \times \vec{a}\right), \vec{c} \times \left(\vec{a} \times \vec{b}\right)$ are coplanar.

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22. If $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} \neq \vec{0}$, prove that $\vec{a} + \vec{c} = m\vec{b}$, where m is a scalar.

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23. Find the shortest distance between the lines $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ and $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$.

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Section C

1. If $y = \left[x + \sqrt{x^2 + a^2}\right]^n$, then prove that $\frac{dy}{dx} = \frac{ny}{\sqrt{x^2 + a^2}}$

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2. Show that the right circular cone of least curved surface area and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

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3. Find the area bounded by the curve y = sin x between x = 0 and $x = 2\pi$.

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4. Find the particular solution of the differential equation $(1-y^2)(1+\log|x|)dx+2\times dy=0$, given $y=0$, when $x=1$.

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5. Evaluate $\int\frac{1}{(x^2-4)\sqrt{x+1}}dx$

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6. Answer any three questions

Find the coordinates of the point, where the line through the points $A(3,4,1)$ and $B(5,1,6)$ crosses the XY-plane.

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7. Find the equation of the two lines through the origin which intersect the line $\frac{x-3}{2} - \frac{y-3}{1} = \frac{z}{1}$ at angle of $\frac{\pi}{3}$ each.

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8. If $a, b, c > 0$ such that $a + b + c = abc$ find the value of $\tan^{-1} a + \tan^{-1} b + \tan^{-1} c$.

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9. Solve the following LPP graphically.

Maximize $Z = 10x + 6y$

Subjectto $3x + y \leq 12$

$2x + 5y \leq 34$

$x, y > 0$

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10. Show the $f: R - \{-1) \to R - \{1\}$ given by $f(x) = \frac{x}{x-1}$ is invertible. Also find f^{-1}.

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11. Show the following system of equations is consistent

$2x - y + 3z = 5, 3x + 2y - z = 7, 4x + 5y - 5z = 9,$

Also, find the solution.

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12. If A and B are two independent events such that $P(\overline{A} \cap B) = \frac{2}{15}$ and $P(A \cap \overline{B}) = \frac{1}{6}$ then find P(A) and P(B)

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13. Find the inverse of the matrix $A = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix}$ by using row transformations.

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