MATHS

BOOKS - ARIHANT PRAKASHAN

VERY SIMILAR TEST 9

Section A

1. Find the absolute maximum value of $f(x) = 2x^3 - 24x + 107$ in the interval [1, 3].

2. Evaluate $\int \sin^2 x dx$

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3. Show that $y = cx + \frac{a}{c}$ is a solution of the differential equation $y = x\frac{dy}{dx} + \frac{a}{\frac{dy}{dx}}$.

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4. Find $\vec{a} \times \vec{b}$, if $\vec{a} = 2\hat{i} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$

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5. Find the vector equation of the plane whose Cartesian from of equation is $3x - 4y + 2z = 5$

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6. Find the domain of the function $f(x) = \sin^{-1}(2x - 3)$.

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7. Without expanding show that

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$$

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8. Events E and F are independent. Find $P(F)$, if $P(E) = 0.35$ and $P(E \cup F) = 0.6$.

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9. Show that $f(x) = \begin{cases} 5x - 4 & when \quad 0 < x \leq 1 \\ 4x^2 - 3x & when \quad 1 < x < 2 \end{cases}$ is continuous at $x = 1$.

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10. Find the absolute maximum and minimum values of $f(x) = 2x^3 - 24x + 57$ in the interval [1, 3].

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11. Evaluate $\int \sin^2 x dx$

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12. Show that $y = cx + \frac{a}{c}$ is a solution of the differential equation $y = x\frac{dy}{dx} + \frac{a}{\frac{dy}{dx}}$.

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13. Find $\vec{a} \times \vec{b}$, if $\vec{a} = 2\hat{i} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$

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14. Find the vector equation of the plane whose Cartesian from of equation is $3x - 4y + 2z = 5$

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15. Find the domain of the function $f(x) = \sin^{-1}(2x - 3)$.

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16. Evaluate $\begin{bmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{bmatrix}$

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17. Events E and F are independent. Find $P(F)$, if $P(E) = 0.35$ and $P(E \cup F) = 0.6$.

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18. Show that $f(x) = \begin{cases} 5x - 4 & when \quad 0 < x \leq 1 \\ 4x^2 - 3x & when \quad 1 < x < 2 \end{cases}$ is continuous at $x = 1$.

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Section B

1. If $\sin\left\{\cot^{-1}(x+1)\right\} = \cos\left(\tan^{-1}x\right)$, then find x.

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2. One kind of cake requires 200 g of flour and 25 g of fat and another kind of cake requires 100 g of flour and 50 g of fat. The maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes, formulate the problem as LPP.

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3. Let R be a relation on the set A of ordered pairs of positive integers defined by (x, y) R (u, v), if and only if xv = yu. Show that R is an equivalence relation.

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4. Prove that

$$\tan\left\{\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right\}=\frac{2b}{a}$$

.

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5. Two persons A and B throw a die alternately till one of them gets a three and wins the game, Find their respective probabilities of winning, if A begins.

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6. $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then show that $A^2 = B^2 = C^2 = I^2$

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7. Prove that $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

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8. An urn contains 4 white and 6 red balls. Four balls are drawn at random from the urn. Find the probability distribution of the number of white balls.

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9. Using differentials, find the approximate value of $(3.68)^{3/2}$.

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10. Prove that: $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function in [0,pi/2]`

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11. If $xy\log(x+y) = 1$, then prove that

$$\frac{dy}{dx} = -\frac{y\left(x^2y + x + y\right)}{x\left(xy^2 + x + y\right)}.$$

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12. If $\sin 2x = \frac{2t}{1+t^2}$, $\tan y = \frac{2t}{1-t^2}$ then find $\frac{dy}{dx}$.

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13. Solve $\frac{dy}{dx} + y = \cos x - \sin x.$

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14. Find the equation of the curve passing through the point $(1, 1)$ whose differential equation is $xdy = (2x^2 + 1)dx, x \neq 0.$

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15. Evaluate $\int \frac{x^2}{x^4 - x^2 + 12} dx.$

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16. Evaluate the following integrals :

Evaluate $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}$

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17. Find the area bounded by the curve $y = x|x|$, X-axis and ordinates $x = -3$ and $x = 3$.

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18. The plane $lx + my = 0$ is rotated about its line of intersection with the plane z=0 through angle measure alpha. Prove that the equation of the plane in new position is $lx + my \pm z\sqrt{l^2 + m^2} \tan \alpha = 0$

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19. Find the distance of the plane $2x - 3y + 4z - 6 = 0$ from the origin and the coordinates of the foot of the perpendicular drawn from origin to the given plane.

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20. Prove that $\left|\vec{a} \times \vec{b}\right|^2 = \left|\vec{a}\right|^2 \left|\vec{b}\right|^2 - \left(\vec{a} \cdot \vec{b}\right)^2$

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21. Find the vector equation of a plane which is at a distance of 6 units from the origin and has $2, -1, 2$ as the direction ratios of a normal to it. Also, find the coordinates of the foot of the normal drawn from the origin.

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22. If $\sin\left\{\cot^{-1}(x+1)\right\} = \cos\left(\tan^{-1} x\right)$, then find x.

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23. One kind of cake requires 200 g of flour and 25 g of fat and another kind of cake requires 100 g of flour and 50 g of fat. The maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes, formulate the problem as LPP.

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24. Consider $f : R_{+} \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{y+6} - 1}{3}\right)$. Hence. Find

$(i) f^{-1}(10)$ $\quad (ii) y$ if $f^{-1}(y) = \dfrac{4}{3}$

where R_+ is the set of all non-negative real numbers.

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25. Let R be a relation on the set A of ordered pairs of positive integers defined by (x, y) R (u, v), if and only if xv = yu. Show that R is an equivalence relation.

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26. Prove that

$$\tan\left\{\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} + \tan\left\{\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right\} = \frac{2b}{a}$$

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28. $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then show that $A^2 = B^2 = C^2 = I^2$

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29. Prove that $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

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30. An urn contains 4 white and 6 red balls. Four balls are drawn at random from the urn. Find the probability distribution of the number of white balls.

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31. Using differentials, find approximate value

$(3.968)^{3/2}$

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32. Prove that: $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function in [0,pi/2]`

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33. If $xy\log(x+y) = 1$, then prove that

$$\frac{dy}{dx} = -\frac{y\left(x^2y+x+y\right)}{x\left(xy^2+x+y\right)}.$$

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34. If $\sin 2x = \frac{2t}{1+t^2}$, $\tan y = \frac{2t}{1-t^2}$ then find $\frac{dy}{dx}$.

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35. Using mean value theorem, prove that sin $x < x, \in (0. \pi / 2)$.

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36. Solve $\frac{dy}{dx} + y = \cos x - \sin x$

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37. Find the equation of the curve passing through the point $(1, 1)$ whose differential equation is $xdy = (2x^2 + 1)dx, x \neq 0$.

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38. Evaluate $\int \frac{x^2}{x^4 - x^2 + 12} dx.$

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39. Evaluate $\int_0^{\pi/2} \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx.$

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40. Find the area bounded by the curve $y = x|x|$, X-axis and ordinates $x = -3$ and $x = 3.$

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41. The plane $lx + my = 0$ is rotated about its line of intersection with the plane z=0 through angle measure alpha. Prove that the equation of the plane in new position is $lx + my \pm z\sqrt{l^2 + m^2}\tan\alpha = 0$

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43. Prove that $\left|\vec{a} \times \vec{b}\right|^2 = \left|\vec{a}\right|^2\left|\vec{b}\right|^2 - \left(\vec{a} \cdot \vec{b}\right)^2$

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44. Find the cartesian equation of a plane which is at a distance of 6 units from the origin and which has a normal with direction ratios (2, -1, -2).

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Section C

1. If $e^x + e^y = e^{x+y}$, then prove that

$$\frac{dy}{dx} = \frac{e^x(e^y - 1)}{e^y(e^x - 1)} \text{ or } \frac{dy}{dx} + e^{y-x} = 0.$$

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2. Find a point on the curve f(x)=$(x-3)^2$, where the tangent is parallel to the chord joining the points (3, 0) and (4, 1).

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3. Find the area of the region included between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, where $a > 0$.

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4. Solve $\frac{dy}{dx} = y \sin 2x : y(0) = 1.$

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5. Evaluate $\int \frac{1}{\sin x (2 \cos^2 x - 1)} dx.$

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6. Prove analytically : The perpendicular bisector of the sides of a triangle are concurrent.

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7. Find the foot of the perpendicular drawn from the point $2\hat{i} - \hat{j} + 5\hat{k}$ to the line $\vec{r} = \left(11\hat{i} - 2\hat{j} - 8\hat{k}\right) + \lambda\left(10\hat{i} - 4\hat{j} - 11\hat{k}\right)$. Also, find the length of the perpendicular.

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8. Solve the following LPP graphically.

Maximise $Z = 5x + 3y$

Subject to `3x + 5y le 15

5x + 2y le 10

and x, y ge 0

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9. Let $f: N \rightarrow R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f: N \rightarrow S$, where S is the range of f, is invertible. Also, find the inverse of f.

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10. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the system of linear equations $x + 2y + z = 4$, $-x + y + z = 0$, $x - 3y + z = 2$.

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11. A dice is thrown thrice. Find the probability of getting an odd number atleast once.

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12. Find the inverse of the following matrix using elementary transformation $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$.

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13. If $e^x + e^y = e^{x+y}$, then prove that

$$\frac{dy}{dx} = \frac{e^x(e^y - 1)}{e^y(e^x - 1)} \text{ or } \frac{dy}{dx} + e^{y-x} = 0.$$

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23. A die is tossed thrice. Find the probability of getting an even number atleast once.

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24. Find the inverse of the following matrix using elementary transformation $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$.

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