

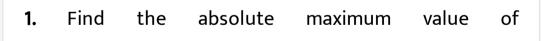


# MATHS

# **BOOKS - ARIHANT PRAKASHAN**

# **VERY SIMILAR TEST 9**

## Section A



 $f(x) = 2x^3 - 24x + 107$  in the interval [1, 3].

**2.** Evaluate 
$$\int \sin^2 x dx$$

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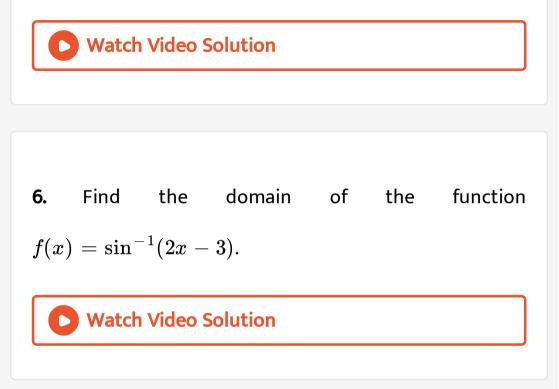
**3.** Show that 
$$y = cx + \frac{a}{c}$$
 is a solution of the differential equation  $y = x \frac{dy}{dx} + \frac{a}{\frac{dy}{dx}}$ .

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**4.** Find 
$$\overrightarrow{a} imes \overrightarrow{b}$$
, if  $\overrightarrow{a} = 2\hat{i} + \hat{k}$  and  $\overrightarrow{b} = \hat{i} + \hat{j} + \hat{k}$ 

5. Find the vector equation of the plane whose Cartesian

from of equation is 3x - 4y + 2z = 5



## 7. Without expanding show that

$$egin{array}{c|c} 1 & a & b+c \ 1 & b & c+a \ 1 & c & a+b \end{array} 
ight| = 0$$

8. Events E and F are independent. Find P(F), if P(E) = 0.35 and  $P(E \cup F) = 0.6$ .

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9. Show that 
$$f(x)=egin{cases} 5x-4 & when & 0< x\leq 1\ 4x^2-3x & when & 1< x<2 \end{cases}$$
 is continuous at  $x=1.$ 

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10. Find the absolute maximum and minimum values of  $f(x) = 2x^3 - 24x + 57$  in the interval [1, 3].



**11.** Evaluate 
$$\int \sin^2 x dx$$

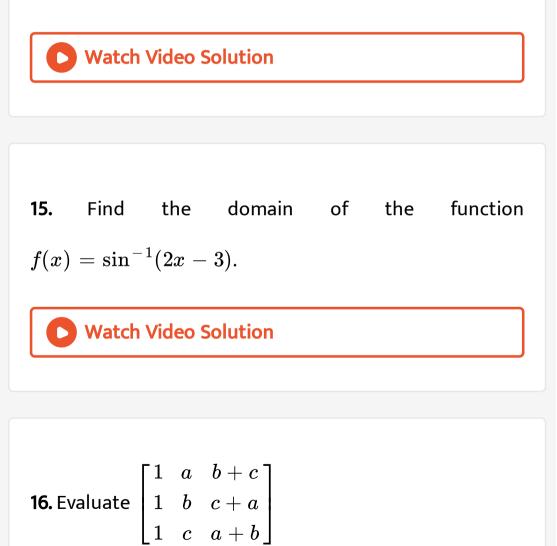
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12. Show that 
$$y = cx + \frac{a}{c}$$
 is a solution of the differential equation  $y = x \frac{dy}{dx} + \frac{a}{\frac{dy}{dx}}$ .

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13. Find  $\overrightarrow{a} imes \overrightarrow{b}$  , if  $\overrightarrow{a} = 2\hat{i} + \hat{k}$  and  $\overrightarrow{b} = \hat{i} + \hat{j} + \hat{k}$ 

14. Find the vector equation of the plane whose Cartesian from of equation is 3x - 4y + 2z = 5



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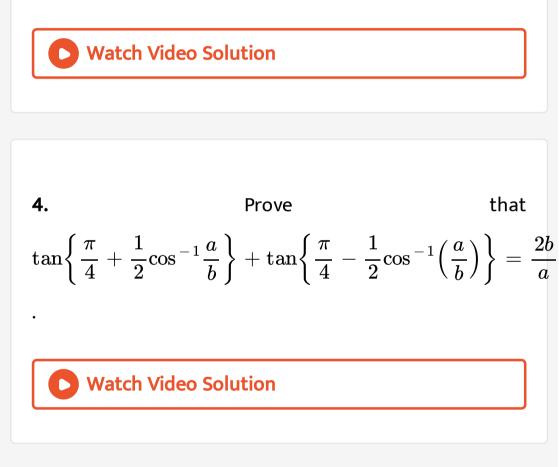
1. If  $\sin\left\{\cot^{-1}(x+1)
ight\} = \cos\left(\tan^{-1}x
ight)$ , then find x.

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**2.** One kind of cake requires 200 g of flour and 25 g of fat and another kind of cake requires 100 g of flour and 50 g of fat. The maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes, formulate the problem as LPP.



**3.** Let R be a relation on the set A of ordered pairs of positive integers defined by (x, y) R (u, v), if and only if xv = yu. Show that R is an equivalence relation.



5. Two persons A and B throw a die alternately till one of them gets a three and wins the game, Find their respective probabilities of winning, if A begins.



6. 
$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$
 and  $C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ , then show that  $A^2 = B^2 = C^2 = I^2$ 

7. Prove that
$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$



**8.** An urn contains 4 white and 6 red balls. Four balls are drawn at random from the urn. Find the probability distribution of the number of white balls.



**9.** Using differentials, find the approximate value of  $(3.68)^{3/2}$ .

10. Prove that:  $y = rac{4\sin heta}{2+\cos heta} - heta$  is an increasing function in [0,pi/2]`

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11. If 
$$xy\log(x+y)=1$$
, then prove that

$$rac{dy}{dx}= \ - \ rac{yig(x^2y+x+yig)}{x(xy^2+x+y)} \, ,$$

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12. If 
$$\sin 2x = rac{2t}{1+t^2}, an y = rac{2t}{1-t^2}$$
 then find  $rac{dy}{dx}$ .

13. Solve 
$$rac{dy}{dx} + y = \cos x - \sin x.$$



14. Find the equation of the curve passing through the point (1,1) whose differential equation is  $xdy = (2x^2 + 1)dx, x \neq 0.$ 

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15. Evaluate 
$$\int rac{x^2}{x^4-x^2+12} dx.$$

**16.** Evaluate the following integrals :

Evaluate 
$$\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}$$

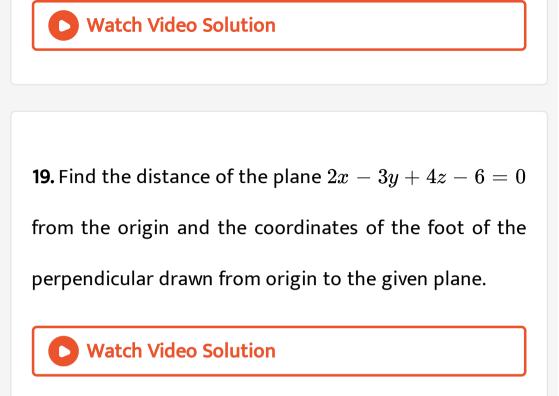
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17. Find the area bounded by the curve y=xert xert, X-axis

and ordinates x = -3 and x = 3.

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18. The plane lx + my = 0 is rotated about its line of intersection with the plane z=0 through angle measure alpha. Prove that the equation of the plane in new position is  $lx + my \pm z\sqrt{l^2 + m^2} \tan \alpha = 0$ 



**20.** Prove that 
$$\left|\overrightarrow{a}\times\overrightarrow{b}\right|^2 = \left|\overrightarrow{a}\right|^2 \left|\overrightarrow{b}\right|^2 - \left(\overrightarrow{a}\cdot\overrightarrow{b}\right)^2$$

**21.** Find the vector equation of a plane which is at a distance of 6 units from the origin and has 2, -1, 2 as the direction ratios of a normal to it. Also, find the coordinates of the foot of the normal drawn from the origin.

22. If 
$$\sin\left\{\cot^{-1}(x+1)\right\} = \cos\left(\tan^{-1}x\right)$$
, then find  $x$ .

**23.** One kind of cake requires 200 g of flour and 25 g of fat and another kind of cake requires 100 g of flour and 50 g of fat. The maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes, formulate the problem as LPP.

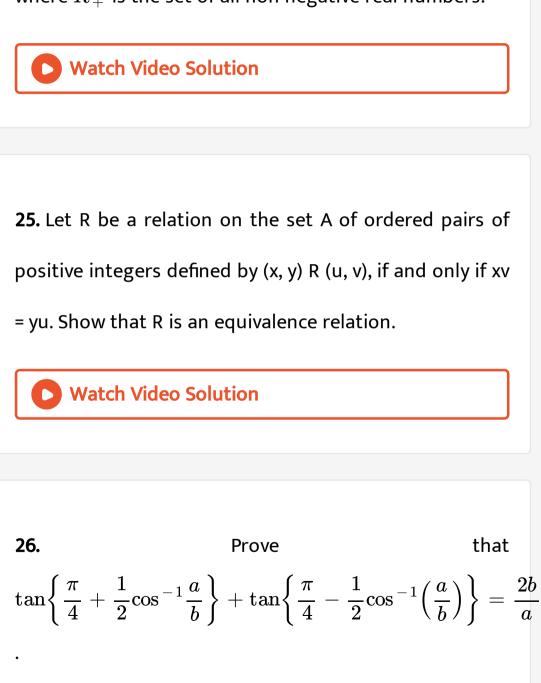
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**24.** Consider  $f\!:\!R_+
ightarrow [\,-5,\infty)$  given by

 $f(x)=9x^2+6x-5.$  Show that f is invertible with  $f^{-1}(y)=igg(rac{\sqrt{y+6}-1}{3}igg).$  Hence. Find

$$(i)f^{-1}(10) \qquad (ii)y \;\; ext{if}\;\; f^{-1}(y) = rac{4}{3}$$

where  $R_+$  is the set of all non-negative real numbers.



**27.** Two persons A and B throw a die alternately till one of them gets a three and wins the game, Find their respective probabilities of winning, if A begins.



**28.** 
$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$
 and  $C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ , then show that  $A^2 = B^2 = C^2 = I^2$ 

**29.** Prove that 
$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$



**30.** An urn contains 4 white and 6 red balls. Four balls are drawn at random from the urn. Find the probability distribution of the number of white balls.

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31. Using differentials, find approximate value

 $(3.968)^{3/2}$ 

**32.** Prove that: 
$$y = \frac{4\sin\theta}{2+\cos\theta} - \theta$$
 is an increasing function in [0 pi/2])

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33. If 
$$xy\log(x+y)=1$$
, then prove that

$$rac{dy}{dx}=\,-\,rac{yig(x^2y+x+yig)}{x(xy^2+x+y)}.$$

**34.** If 
$$\sin 2x = rac{2t}{1+t^2}$$
,  $an y = rac{2t}{1-t^2}$  then find  $rac{dy}{dx}$ .

**35.** Using mean value theorem, prove that sin $x < x, \in (0. \pi/2).$ 

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36. Solve 
$$rac{dy}{dx} + y = \cos x - \sin x$$

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**37.** Find the equation of the curve passing through the point (1,1) whose differential equation is  $xdy = ig(2x^2+1ig)dx, x 
eq 0.$ 





**38.** Evaluate 
$$\int rac{x^2}{x^4-x^2+12} dx.$$

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**39.** Evaluate 
$$\int_0^{\pi/2} \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx.$$

**40.** Find the area bounded by the curve y = x |x|, X-axis

and ordinates x = -3 and x = 3.

**41.** The plane lx + my = 0 is rotated about its line of intersection with the plane z=0 through angle measure alpha. Prove that the equation of the plane in new position is  $lx + my \pm z\sqrt{l^2 + m^2} \tan \alpha = 0$ 



42. Find the distance of the plane 2x - 3y + 4z - 6 = 0 from the origin and the coordinates of the foot of the perpendicular drawn from origin to the given plane.



**43.** Prove that 
$$\left|\overrightarrow{a} \times \overrightarrow{b}\right|^2 = \left|\overrightarrow{a}\right|^2 \left|\overrightarrow{b}\right|^2 - \left(\overrightarrow{a} \cdot \overrightarrow{b}\right)^2$$



**44.** Find the cartesian equation of a plane which is at a distance of 6 units from the origin and which has a normal with direction ratios (2, -1, -2).





1. If 
$$e^x + e^y = e^{x+y}$$
, then prove that  
 $\frac{dy}{dx} = \frac{e^x(e^y - 1)}{e^y(e^x - 1)}$  or  $\frac{dy}{dx} + e^{y-x} = 0.$   
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2. Find a point on the curve  $f(x)=(x-3)^2$ , where the tangent is parallel to the chord joining the points (3, 0) and (4, 1).



**3.** Find the area of the region included between the parabolas  $y^2 = 4ax$  and  $x^2 = 4ay$ , where a > 0.





**4.** Solve 
$$rac{dy}{dx} = y \sin 2x : y(0) = 1.$$

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5. Evaluate 
$$\int \frac{1}{\sin x (2\cos^2 x - 1)} dx$$
.

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6. Prove analytically : The perpendicular bisector of the

sides of a triangle are concurrent.



7. Find the foot of the perpendicular drawn from the

point 
$$2\hat{i}-\hat{j}+5\hat{k}$$
 to the line

$$\overrightarrow{r}=\Big(11\hat{i}-2\hat{j}-8\hat{k}\Big)+\lambda\Big(10\hat{i}-4\hat{j}-11\hat{k}\Big).$$
 Also,

find the length of the perpendicular.



8. Solve the following LPP graphically.

Maximise Z = 5x + 3y

Subject to 3x + 5y = 15

5x + 2y le 10

and x, y ge 0



9. Let f:N o R be a function defined as  $f(x)=4x^2+12x+15$ . Show that f:N o S, where S is the range of f, is invertible. Also, find the inverse of f.

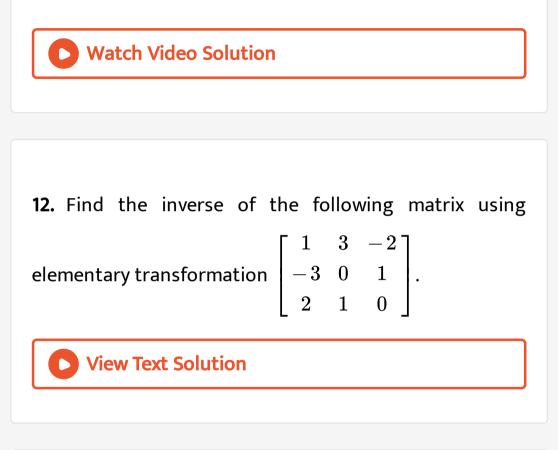
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**10.** If 
$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$$
, find  $A^{-1}$  and hence solve  
the system of linear equations  $x + 2y + z = 4$ ,  
 $-x + y + z = 0, x - 3y + z = 2$ .

## **View Text Solution**

11. A dice is thrown thrice. Find the probability of getting

an odd number atleast once.



13. If 
$$e^x + e^y = e^{x+y}$$
, then prove that $rac{dy}{dx} = rac{e^x(e^y-1)}{e^y(e^x-1)}$  or  $rac{dy}{dx} + e^{y-x} = 0.$ 

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**14.** Find a point on the curve  $f(x)=(x-3)^2$ , where the tangent is parallel to the chord joining the points (3, 0) and (4, 1).



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17. Evaluate 
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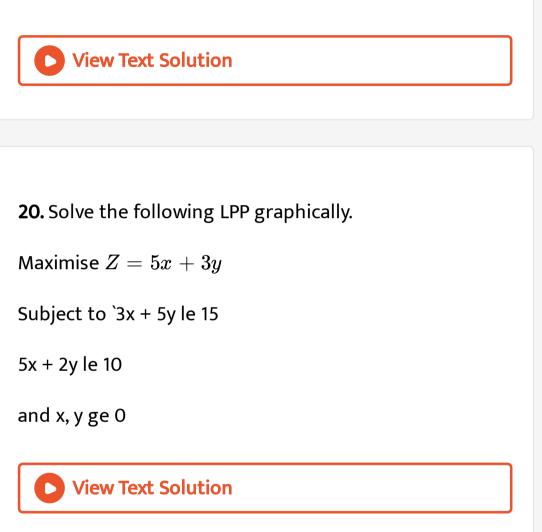


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### **View Text Solution**

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an even number atleast once.

