



MATHS

BOOKS - MODERN PUBLICATION

CONTINUITY AND DIFFERENTIABILITY

Problem

1. Find $\frac{dy}{dx}$ for $\sqrt{x} + \sqrt{y} = \sqrt{c}$.



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2. If $f(x) = \log_x^2(\log x)$, then $f(e)$



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3. If $y = 10^x$ and $z = 100^{\frac{x}{2}}$ then find

$$\frac{d}{dx} \left(\frac{y^2}{z} \right).$$



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4. Find the differential of $y=\sin^2 x$.



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5. If $f(x) = \sqrt{x^2 - 2x + 1}$, $x \in [0, 2]$ then at $x=1 f'(x)=$ _____



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6. Differentiate $\sqrt{\sin \sqrt{x}}$



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7. If $f(x) = [\tan^2 x]$, what is $f'(0)$?



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8. If $x^y = e^{x-y}$ then find $\frac{dy}{dx}$.



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9. If $x=a \cos^3 \theta, y=a \sin^3 \theta$, then find $\frac{d^2y}{dx^2}$



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10. Find $\frac{dy}{dx}$ if $y = \cot^{-1} \left(\frac{(1 + \cos x)^{\frac{1}{2}}}{(1 - \cos x)^{\frac{1}{2}}} \right)$.



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11. Find the derivative of $\sin x^0$ w.r.t. x.



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12. Differentiate $\sin^{-1}(2x\sqrt{1 - x^2})$ w.r.t. x.



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13. Let $f(x) = e^x g(x)$, $g(0) = 2$ and $g'(0) = 1$, then find $f'(0)$.



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14. If $x \in \left(0, \frac{\pi}{2}\right)$ then find $\frac{d}{dx}(\sin x)$.



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15. Find $\frac{d}{dx} (\sin^{-1} x)^{x^2}$



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16. If $f(x) = [x^2]$ then $f(3/2) = \underline{\hspace{2cm}}$.



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17. A differentiable function f defined for all $x > 0$ satisfies $f(x^2) = x^3$ for all $x > 0$. what is $f'(b)$?



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18. What is the derivative of $f'(\ln x)$ w.r.t. x

where $f(x) = \ln x$.



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19. Find the derivative of $\log_x a$.



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20. If $x^y = e^{x-y}$ then find $\frac{dy}{dx}$.



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21. Examine the continuity of the function f

defined by $f(x) = \begin{cases} 2x + 1 & \text{if } x < 1 \\ 0 & \text{if } x = 1 \\ x^2 - 1 & \text{if } x > 1 \end{cases}$.



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22. examine the contiuity of the following

functions at indicated points . $f(x) =$

$$\begin{cases} (1 + 2x)^{\frac{1}{x}} & \text{if } x \neq 0 \\ (e^2) & \text{if } x = 0 \end{cases} \text{ at } x = 0.$$



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23. Examine the continuity of the following functions at the indicated points . $f(x) = \begin{cases} 2x + 1 & \text{if } x \leq 0 \\ x & \text{if } 0 < x < 1 \text{ at } x = 0, 1. \\ 2x - 1 & \text{if } x \geq 1 \end{cases}$



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24. Find the value of a such that the function f defined by

$$f(x) = \begin{cases} \frac{\sin ax}{\sin x} & \text{if } x \neq 0 \\ \frac{1}{a} & \text{if } x = 0 \end{cases}$$

is continuous at $x=0$.



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$$25. \text{ If } f(x) = \begin{cases} ax^2 + b & \text{if } x < 1 \\ 1 & \text{if } x = 1 \\ 2ax - b & \text{if } x > 1 \end{cases}$$

is continuous at $x=1$, then find a and b.



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26. Examine the continuity of the following

functions at indicated points. $f(x) =$

$$\begin{cases} \frac{1}{e^{\frac{1}{x}} - 1} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases} \quad \text{at } x = 0$$



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27. Examine continuity and differentiability

$$f(x) = \begin{cases} x \frac{\sin 1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \text{ at } x = 0.$$



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28. Test differentiability of the following function at the indicated points.

$$f(x) = x + |\cos x| \text{ at } x = \frac{\pi}{2}$$



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29. Differentiate $x^{\sin^{-1} x}$ w.r.t. \sin^{-1} .



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30. if $x = e^t \sin t$ and $y = e^t \cos t$, then prove
that $(x + y)^2 \frac{d^2y}{dx^2} = 2 \left(x \frac{dy}{dx} - y \right)$.



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31. Find $\frac{dy}{dx}$ for $(x + y)^{\cos x} = e^{x+y}$.



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32. If $2^x + 2^y + 2^{x+y}$, then find $\frac{dy}{dx}$ at $x = y = 1$.



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33. If $y=x + e^x$, then find $d^2 \frac{x}{dy^2}$.



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34. If $e^x + e^y = e(x + y)$ find $\frac{dy}{dx}$.



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35. Differentiate the following functions by proper substitution.

$$\left[\left(\frac{1+t^2}{1-t^2} \right)^2 - 1 \right]^{\frac{1}{2}}$$



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36. If $\sin(x + y) = y \cos(x + y)$ then prove that

$$\frac{dy}{dx} = -\frac{1 + y^2}{y^2}$$



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37. Find dy/dx if

$$\log \sqrt{x^2 + y^2} = \tan^{-1}\left(\frac{y}{x}\right)$$



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38. $\sin x = 2\frac{t}{1} + t^2$, $\tan y = 2\frac{t}{1} - t^2$, find $\frac{dy}{dx}$.



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39. Find $\frac{dy}{dx}$,

$$x = 3 \cos t - 2 \cos^3 t,$$

$$y = 3 \sin t - 2 \sin^3 t.$$



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40. If $y = \sin(m \cos^{-1} x)$ then prove that

$$(1 - x^2)y_2 - xy_1 + m^2y = 0.$$



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41. If $y = \cos^{-1} \left(\frac{2 \cos x - 3 \sin x}{\sqrt{13}} \right)$, then find dy/dx .



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42. Examine the continuity of the following functions at indicated points.

$$f(x) = \begin{cases} \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1} & \text{if } x \neq 0 \text{ at } x = 0 \\ 0 & \text{if } x = 0 \end{cases}$$



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43. Show that $f(x) = \begin{cases} x \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

is continuous at $x=0$



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44. Prove that $e^x - 2 = 0$ has a solution

between 0 and 1.



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45. So that $x^5 + x + 1 = 0$ for some value of x between -1 and 0.



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46. Examine continuity and differentiability

$$f(x) = \begin{cases} \frac{1-e^{-x}}{x} & x \neq 0 \\ 1 & x = 0 \end{cases} \text{ at } x=0.$$



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47. If $x=2 \cos \theta - \cos 2\theta$, $y= 2 \sin \theta - \sin 2\theta$
then find $d^2 \frac{y}{dx^2}$ at $\theta = \frac{\pi}{2}$.



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48. If $y = (\sin^{-1} x)^2$, prove that

$$(1 - x^2)y_2 - xy_1 - 2 = 0$$



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49. Find $\frac{dy}{dx}$ for the following . $y = \log_2 \sqrt{\sin \phi}(x)$.



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50. Find dy/dx if

$$\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1$$



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51. Find dy/dx if

$$y = x \cot^{-1} \left(\frac{x}{y} \right)$$



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52. Find dy/dx if $(\cos x)^y = (\sin y)^x$



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53. Find $\frac{dy}{dx}$, if $y = \tan^{-1} \left(\frac{\cos x}{1 + \sin x} \right)$.



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54. Find dy/dx if $e^{3x} \ln(\sin 2x) + e^{\sqrt{\cos x}}$.



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55. Differentiate $\tan^{-1} \left(\frac{1 + \sin x}{1 - \sin x} \right)^{\frac{1}{2}}$ w.r.t.
 $\log \left(\frac{1 + \cos x}{1 - \cos x} \right)$.



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56. Find dy/dx $y = \tan^{-1}(x \sin \alpha - x \cos \alpha)$



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57. Find $\frac{dy}{dx}$

$$y = \cos^{-1} \left(1 - \frac{1}{x} - \frac{1}{x^2} \right)^{\frac{1}{2}}$$



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58. Find $\frac{dy}{dx}$ $x = \frac{\cos^3 t}{\sqrt{\cos 2}} t, y = \frac{\sin^3 t}{\sqrt{\cos 2}} t$



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59. If $x = \sin t$, $y = \sin 2t$ then prove that

$$(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + 4y = 0$$



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60. If $y = e^{m \cos^{-1} x} (1 - x^2) y_2 - xy_1 = m^2 y$



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61.

Differentiate

$$y = \tan^{-1} \cdot \frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}$$



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62. If $\sqrt{1 - x^4} + \sqrt{1 - y^4} = k(x^2 - y^2)$

then show that

$$\frac{dy}{dx} = \frac{x\sqrt{1 - y^4}}{y\sqrt{1 - x^4}}$$



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63. If $x = \frac{1 - \cos^2 \theta}{\cos \theta}, y = \frac{1 - \cos^{2n} \theta}{\cos^n \theta}$ then

show that $\left(\frac{dy}{dx}\right)^2 = n^2 \left(\frac{y^2 + 4}{x^2 + 4}\right)$



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