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## MATHS

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## SAMPLE PAPER 2016

## Exercise

1. Write that condition of Rolle's theorem which is violated by the function $f(x)=|x-1|$ in $[0,2]$.

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$$
\begin{aligned}
& \text { 2. Write } \\
& \int_{0}^{\frac{\pi}{2}} \frac{\text { the }}{\sin x} d x-\int_{0}^{\frac{\pi}{2}} \frac{\cos x}{\sin x+\cos x} d x
\end{aligned}
$$

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3. If $p$ and $q$ are the order and degree of the differential equation
$y\left(\frac{d y}{d x}\right)^{2}+x^{2} \frac{d^{2} y}{d x^{2}}+x y=\sin x$, then choose the correct statement out of (i) $p>q$, (ii) $p=q$, (iii)
$p<q$.
4. If $|\vec{a}|=3,|\vec{b}|=2$ and $\vec{a} \cdot \vec{b}=0$, then write the value of $|\vec{a} \times \vec{b}|$.

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5. Write the distance between parallel planes $2 x-y+3 z=4$ and $2 x-y+3 z=18$.
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6. Write the equation of the sphere concentric with the
$x^{2}+y^{2}+z^{2}-4 x-2 x-2 y+2 z-30=0 \quad$ and passing through the origin.

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7. If $A$ is a $4 \times 5$ matrix and $B$ is a matrix such that $A^{T} B$ and $B A^{T}$ both are defined, then write the order of $B$.

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8. If ${ }^{n} C_{r}={ }^{n} P_{r}, r \neq 1$, then write the value of $r$.
9. A binomial distribution has mean 4 and variance 3 .

Write the number of trials.

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10. Find $\frac{d y}{d t}$, when $y=\sin ^{-1}\left(2 \frac{\sqrt{t^{2}-1}}{t^{2}}\right)$

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11. Find $d y / d x$ if
$x^{m} y^{n}=\left(\frac{x}{y}\right)^{m+n}$
12. If $x=a \sec \theta, y=b \tan \theta$, then prove that $\frac{d^{2} y}{d x^{2}}=-\frac{b^{4}}{a^{2} y^{3}}$

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13. if $u=x^{3}-3 x y^{2}$, show that


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14. Find the interval where the following function is increasing:
$y=\sin x+\cos x, x \varepsilon[0,2 \pi]$.

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## 15. Find the following limits: $\lim _{x \rightarrow 0+} \frac{\ln \tan x}{\ln \sin 2 x}$

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16. The radius of a spherical soap bubble is increasing
at the rate of $0.2 \mathrm{~cm} / \mathrm{sec}$. Find the rate of increase of its surface area, when the radius is $7 \mathrm{~cm} .(\pi=3.141$ approx)

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17. If $f^{\prime}(x)=e^{x}+\frac{1}{1+x^{2}}$ and $f(0)=1$, then find $f(x)$.

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18. Evaluate : $\int(\log x)^{2} d x$
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19. Evaluate: $\int \frac{2 x+9}{(x+3)^{2}} d x$
20. $\int_{0}^{1} \frac{x^{5}\left(4-x^{2}\right)}{\sqrt{1-x^{2}}} d x$

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21. Evaluate $\int \frac{\sin x \cos x}{\sin ^{2} x-2 \sin x+3} d x$

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22. Solve $d y+e^{-y} \sin x d x=0$.
23. Solve: $\left(x^{2}-1\right) \frac{d y}{d x}+2 x y=1$

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24. Prove that
$|a+b| \leq|a|+|b|$
State when equality will hold,

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25. Find the area of the triangle $A B C$ with vertices
$A(1,2,4), B(3,1,-2)$ and $C(4,3,1)$ by vector method.
26. For any three vectors veca,vecb,vecc show that $[(\vec{a}-\vec{b})(\vec{c}-\vec{a})(\vec{b}-\vec{c})]=0$

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27. The projection of a line segment $\overline{O P}$, through origin 0 , on the co-ordinate axes are $6,2,3$. Find the length of the line segment OP and its direction cosines.
28. Find the equation of the plane passing through the point ( $-1,3,2$ ) and perpendicular to the planes $x+2 y+2 z=5$ and $3 x+3 y+2 z=8$.

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29. Prove that the lines $\frac{x+4}{3}=\frac{y+6}{5}=\frac{z-1}{-2}$ and $3 x-2 y+z+5=0=2 x+3 y+4 z-4$ are co-planar.

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30. Solve the following LPP graphically

Maximize, $Z=20 x+30 y$

Subject to $3 x+5 y \leq 15$
$x, y \geq 0$.

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31. Find the feasible region of the following system
$2 x+y \geq 6, x-y \leq 3, x \geq 0, y \geq 0$

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32. Show that $(a+1)$ is a factor of
$\left|\begin{array}{ccc}(a+1) & 2 & 3 \\ 1 & a+1 & 3 \\ 3 & -6 & a+1\end{array}\right|$
33. Prove that the following.
$\left[\begin{array}{lll}a & b & c \\ x & y & z \\ p & q & r\end{array}\right]=\left[\begin{array}{lll}y & b & q \\ x & a & p \\ z & c & r\end{array}\right]=\left[\begin{array}{lll}x & y & z \\ p & q & r \\ a & b & c\end{array}\right]$

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34. If $A=\left[\begin{array}{cc}\alpha & 0 \\ 1 & 1\end{array}\right]$ and $B=\left[\begin{array}{ll}1 & 0 \\ 5 & 1\end{array}\right]$, show that for no values of $\alpha, A^{2}=B$.
35. How many 4 digit numbers each greater than 6000 can be formed with be digits $5,6,7$ and 8 ?

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36. If $m={ }^{n} C_{2}$, prove that ${ }^{n} C_{2}=3(n+1) C_{4}$.

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37. If the ratio of the 3 rd term from the beginning to
the 3rd term from the end in the expansion of $(1+\sqrt{2})^{n}$ is $\frac{1}{8}$, then find the value of $n$.
38. Let $A$ and $B$ be events with
$P(A)=\frac{1}{3}, P(A \cup B)=\frac{3}{4}, P(A \cap B)=\frac{1}{4}, \quad$ find $P\left(A \cup B^{C}\right)$.

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39. If $X$ follows a binomial distribution with parameter $n=6$ and p with $4 P(X=4)=P(X=2)$, find p.
40. If $x=\frac{1-\cos ^{2} \theta}{\cos \theta}, y=\frac{1-\cos ^{2 n} \theta}{\cos ^{n} \theta}$ then show
that $\left(\frac{d y}{d x}\right)^{2}=n^{2}\left(\frac{y^{2}+4}{x^{2}+4}\right)$

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41. Shows that the triangle of greatest area that can be inscribed in a circle is equilateral.

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42. Determine the area common to the parabola $y^{2}=x$ and the circle $x^{2}+y^{2}=2 x$.
43. Find the solution of the following differential equations:
$x d y-y d x=\sqrt{x^{2}+y^{2}} d x$

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44. 

Prove
that
$\vec{a} \times(\vec{b} \times \vec{c})+\vec{b} \times(\vec{c} \times \vec{a})+\vec{c} \times(\vec{a} \times \vec{b})$
$=0$ and hence prove that
$\vec{a} \times(\vec{b} \times \vec{c}), \vec{b} \times(\vec{c} \times \vec{a}), \vec{c} \times(\vec{a} \times \vec{b})$
are coplanar.
45. A variable plane meets the coordinate axes at $\mathrm{P}, \mathrm{Q}$,
$R$ points. If the plane passes through a fixed point ( $a$, $\mathrm{b}, \mathrm{c})$, prove that the centre of the shpere passing the origin and $P, Q, R$ will lie on the surface $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$

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46. Solve the following LPP graphically Maximize
$z=5 x_{1}+3 x_{2}$
Subject
$3 x_{1}+5 x_{2} \leq 15,5 x_{1}+2 x_{2} \leq 10, x_{1}, x_{2} \geq 0$.

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47. Solve the following system of equations by the matrix inversion method.
$x+y+z=4$
$2 x-y+3 z=1$
and $3 x+2 y-z=1$

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48. Three persons hit a target with probabilities $\left(\frac{1}{2}\right),\left(\frac{1}{3}\right)$ and $\left(\frac{1}{4}\right)$ respectively. If each one shoots at the target once, (i) find the probaility that
exactly one of them hits the target, (ii) if only one of them hits the target what is the probability that it was the first person?
