MATHS

BOOKS - MODERN PUBLICATION

TEST PAPER 1

Problem

1. Let X and Y be the sets containing m and n elements respectively. How many one-one functions are there from X to Y.

2. Find the value of $\cos \, \tan^{-1} \cot$

$$\left(\cos^{-1} \left(\sqrt{\frac{3}{2}} \right) \right).$$

3. Determine the maximum value of

$$\left| \begin{array}{cc} \cos x \, \sin x \\ -\sin x \, \cos x - 1 \end{array} \right|.$$

4. $f(x) = x|x|$ differentiable at x = 0.

5. The tangent to the curve $x = e^t \cos t$, $y = e^t \sin t$ and $t = \dfrac{\pi}{4}$ makes what angle with x-axis.•

6. If $\left|\vec{a}\right| = 8$, $\left|\vec{b}\right| = 3$ and $\left|\vec{a} \times \vec{b}\right| = 12$, then find the angle between $\vec{a}$ and $\vec{b}$.

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7. Write the equation of X-axis in space.

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8. Evaluate $\int e^x (\cos x - \sin x) dx$.

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9. What is the order of the differential equation of all circles of given radius a ?

10. An animal feed company must produce 200 kg of a mixture consisting of ingredients A and B. The-ingredient A costs Rs.3 per kg and B costs 5 per kg. No more than 80 kg of A can be uséd and at least 60 kg of B must be used.

Formulate the problem to minimise thé cost of mixture.

11. Test whether the relations are reflexive, symmetric or transitive on the sets specified.

R={(m,n): $\dfrac{m}{n}$ is a power of 5} on Z - {0}.

12. Show that the operation * given by x*y=x+y+ -xy is a binary oeration on Z,Q and R but not on N.

13. $\text{Let } f(x) = \sqrt{x} \text{and} g(x) = 1 - x^2.$

Compute fog and gof and find their natural domains.

14. Express $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \\ -1 & 5 & -2 \end{bmatrix}$ as a sum of a symmetric and a skew symmeuic matrices.

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15. If ax+hy+g=0, hx+by+f=0 and gx+fy+c=λ, find the value of λ in the form of a determinant.

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16. If

$$\begin{bmatrix} x & y \\ x & \frac{x}{2} + t \end{bmatrix} + \begin{bmatrix} y & x + t \\ x + 2 & \frac{x}{2} \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

find x,y,z and t.

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17. Find x if '||a b c| |b a b| |x b c||` = 0

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18. Test differentiability and continuity of the following functions.

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases} \quad at\, x = 0.$$

19. If $y = \left(\sin^{-1} x\right)^2$, prove that

$$\left(1 - x^2\right)y_2 - xy_1 - 2 = 0$$

20. Find the dy/dx when

$$x = a[\cos t + \log \tan(t/2)], \, y = a \sin t$$

21. Show that `2 sin x + than x ge 3x all x in (0, pi/20).

22. Show that the line y = mx + c touchees

$$y = 4ax \quad \text{if} \quad c = \frac{a}{m}.$$

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23. Evaluate $\int \left(4 \frac{x^2}{(x-3)(x+1)} \right) dx$

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24. $\int_{\frac{\pi}{5}}^{3\frac{\pi}{10}} \frac{\sin x \, dx}{\sin x + \cos x}$

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25. Solve : $\left(\dfrac{dy}{dx} \right) = (x + y)^2$

26. Find the particular solution of the following differential equation: $\left(\dfrac{dy}{dx} \right) = \dfrac{1 + y^2}{1 + x^2}$ given that $y = \sqrt{3}$ when $x = 1$

27. Find the area of the region bounded by the line $y = 3x + 2$, x-axis and the ordinates $x = -1$ and $x = 1$.

28. Vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $|\vec{c}| = 7$. Then, find the angle between $\vec{a}$ and $\vec{b}$.

29. Find the equation of a plane biscting the line segment joining $(-1, 4, 3)$ and $(5, -2, -1)$ at right angle.

30. Find the foot of the perpendicular drawn from the point (5,7,3) to the line $\dfrac{x-15}{3} = \dfrac{y-29}{8} = \dfrac{z-5}{-5}$. Find the length of the perpendicular.

31. Congruence modulo 3 relation partitions the set Z into how many equivalence classes ?

32. Maximise $Z = 5x_1 + 7x_2$

Subject to $x_1 + x_2 \leq 4$,

$5x_1 + 8x_2 \leq 24$

and $10x_1 + 7x_2 \leq 35$, $x_1, x_2 \geq 0$.

33. If $\begin{bmatrix} x & x^2 & x^3 - 1 \\ y & y^2 & y^3 - 1 \\ z & z^2 & z^3 - 1 \end{bmatrix} = 0$

then prove that xyz=1 when x,y,z are non zero

and unequal.

34. Exaniming consistency and solvability,

solve the following equations by matrix

method. $x + 2y + 3z = 14, 2x - y + 5z = 15$

, $2y + 4z - 3x = 13$

35. If $A = \begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix}$ then prove that

$A^n = ((\cos nx, \sin nx), (-\sin nx, \cos nx))$

for all positive integers n.

36. Find dy/dx $x = \dfrac{\cos^3 t}{\sqrt{\cos 2}} t, y = \dfrac{\sin^3 t}{\sqrt{\cos 2}} t$

37. Show that the shrtest distance of the point $(0, 8a)$ from the curve $ax^2 = y^3$ is $2a\sqrt{11}$.

38. Integrate : $\int \left(\ln\left(x + \sqrt{(x^2 + a^2)}\right) \right) dx.$

39. Solve the following differential equations

$$\frac{dy}{dx} + 2y\tan x = \sin x, \ y\left(\frac{\pi}{3}\right) = 0$$

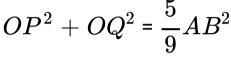**40.** Find the area of the région on closed between $y = 4x - 1$ and $y^2 = 2x$

41. Find the distance of the point (3,-4,5)from the plane 2x+5y-6z-19=0 measured parallel to the line $\dfrac{x - 1}{2} = \dfrac{y}{1} = \dfrac{z + 3}{-2}$.

42. Prove the following by vector method. In a triangle AOB, $m\angle AOB = 90°$. If P and Q are the points of trisection of AB, prove that

$$OP^2 + OQ^2 = \frac{5}{9}AB^2$$

43. Let $f : R \to R$ defined by $f(x) = x + 1$ and $g : R \to R$ defined as $g(x) = \sqrt{x}$ find fog and gof if defined.

44. Solve $\cos\left(2\sin^{-1}x\right) = \dfrac{1}{9}$.

45. Find x and y if $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$.

46. Fill in the blanks with appropriate answer from the brackes. If $\begin{bmatrix} a & b & c \\ b & a & b \\ x & b & c \end{bmatrix} = 0$, then

x=_____

47. What is the slope of the normal to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 20$ at the point $(8, 64)$?

48. What is the value of $\displaystyle\int_0^{\frac{\pi}{2}} \log\tan x\,dx$?

49. Write the general solution of $\dfrac{ydx - xdy}{y} = 0.$

50. Write the distance between the plane $x - 2y + z = 6$ and $2x - 4y + 2z = 8$

51. What is the image of the point (6, 3, -4) with respect to yz- plane ? '

52. Check if the relation R on set of real numbers, defined as $R = \{(a, b) : a \leq b^3\}$ is reflexsive, symmetric or transitive.

53. Examine $f : (-1, 1) \to R$, $f(x) = \dfrac{x}{1 - x^2}$ functions if it is (i) injective (ii) surjective, (iii) bijective and (iv) none of the three.

54. Solve the following LPP graphically

Maximize, $Z = 20x + 30y$

Subject to $3x + 5y \leq 15$

$x, y \geq 0$.

55. Let A and B are symmetric matrices of the same order. Prove that AB is symmetric if and if AB = BA.

56. Express $\begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix}$ in the form of a perfect square.

57. Verify that $[AB]^T = B^T A^T$ where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$$

58. Discuss the continuity and differentiability of function $f(x) = |x| + |x - 1|$ in the interval (-1,2).

59. If sin (x + y) = y cos (x + y) then prove that

$$\frac{dy}{dx} = -\frac{1 + y^2}{y^2}$$

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60. Find $\frac{dy}{dx}$ for $(x + y)^{\cos x} = e^{x+y}$.

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61. Prove that , if $y = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$, then

$$\frac{dy}{dx} = \sec x$$

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62. Evaluate : $\displaystyle\int \frac{\cos px + \cos qx}{\sin px + \sin qx}\,dx.$

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63. Evaluate $\displaystyle\int \frac{4x - 5}{x^2 - x - 2}\,dx$

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64. FInd the area of the region bounded by $y^2 = x$ and $y = x$.

65. Solve : $\dfrac{dy}{dx}\left(x^2 - 1\right) + 2xy = 1.$

66. Form the differential equation for :

$y = a\sin^{-1}x + b\cos^{-1}x.$

67. Find the point of intersection of the line $2x - 4 = 3y = z$ with plane $x + y + z = 13$.

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68. Find the equation of the plane parallel to x - 5y + z + 1 =0 and at a distance of 3 units from origin.

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69. If 2s=a+b+c show that

$$\begin{bmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{bmatrix} =$$

$2s^3(s-a)(s-b)(s-c)$

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70. If $A = \begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix}$ then prove that

$A^n = ((\cos nx, \sin nx), (-\sin nx, \cos nx))$

for all positive integers n.

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71. Exaniming consistency and solvability, solve the following equations by matrix method.

$$x + 2y + 3z = 14, \qquad 2x - y + 5z = 15,$$

$$2y + 4z - 3x = 13$$

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72. Prove that $e^x - 2 = 0$ has a solution between 0 and 1.

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73. Show that the radius of the right cicular cylinder of greatest curved surface that can be inscribed in a given cone is half the redius of the base of the cone.

74. Evaluate $\int_0^{\pi/2} \log \sin x \, dx$.

75. Find the distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$, measured parallel to the line $\dfrac{x}{2} = \dfrac{y}{3} = \dfrac{z}{-6}$

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Exercise

1. Let $f(x) = 2\ln x$ and $g(x) = \ln x^2$ .Do you think $f = g$? Justify

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2. If $\cot^{-1} x + \dfrac{\sin^{-1} 1}{\sqrt{5}} = \dfrac{\pi}{4}$ then what is the value of x.

3. If $A = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ then find A - alpha I, alpha in R.

4. Evaluate $[[-6, 0, 0], [3, -5, 7], [2, 8, 11]]$

5. Give example of a function whose is continuous but not differentiable at x=2.

6. Is there any tangent to the curve $y = |2x - 1|$ at $\left(\frac{1}{2}, 0\right)$?

7. Find the primitive of : $\dfrac{x+1}{x}$

8. Is $y = \dfrac{A}{x+A}$ a solution of the differential equation $x\dfrac{dy}{dx} + y = y^2$.

9. Find the co-ordinates of the foot of the perpendicular drawn from origin to the plane, `x + y + z -1 = 0.

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10. Write the projection of $\hat{i} - \hat{j}$ in the direction of $\hat{i} - \hat{j}$.

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11. Let A = {1, 2, 3). Then, show that the number of relations containing (1, 2) and (2, 3) which are reflexive and transitive but not symmetric is three.

12. A factory uses three different respurce for the manufacture of two different products, 20 units of the resource A, 12 units of B and 16 unit of C being available. One unit of the first

product requires 2,2 and 4 units of the resources and one unit of the second product requires 4,2 and 0 units of the resources taken in order. It is known that the first product gives a profit of ₹20 per unit and the second ₹ 30 prt uniy. Formulate the LPP so as to earn maximum profit.

13. Verify that $[AB]^T = B^T A^T$ where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 6 & 7 & 8 \\ 6 & -3 & 4 \end{bmatrix} B = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 2 \\ 5 & 6 & 1 \end{bmatrix}.$$

14. Solve $x + 2y = 3, \ 3x + y = 4$ by matrix method.

15. Prove the following :

$$\begin{bmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{bmatrix} = \left(1 - x^3\right)^2$$

16. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, show

that for no values of α, $A^2 = B$.

17. Prove that $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = $ (a-b)(b-c)(c-a)

(a+b+c).

18. If $x^7 y^3 = (x + y)^{10}$, then find $\dfrac{d^2 y}{dx^2}$

19. If tangents are drawn from the origin to the curve y = sin x, then show that the locus of the points of contact is $x^2 y^2 = x^2 - y^2$.

20. Integrate the following $\displaystyle\int \dfrac{3x + 4}{x\sqrt{2x^2 - 5}} dx$

21. Evaluate $\displaystyle\int_0^4 \left(x + e^{2x}\right)dx$, as limit of sum.

22. Solve : $\dfrac{dy}{dx} = \sin(x + y) + \cos(x + y)$

23. Solve the following differential equations

$$(1 - x^2)\frac{dy}{dx} + 2xy = x\sqrt{1 - x^2}$$

24. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are mutually perpendicular vectors of equal magnitude show that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$, $\vec{c}$.

25. Prove that the lines joining the midpoints of consecutive sides of a quadrilateral form a parallelogram using vector method.

26. $\displaystyle\int_0^1 \frac{x^5\left(4-x^2\right)}{\sqrt{1-x^2}}\,dx$

27. find the point of intersection of the line through (1,3,-2) and (3,4,1) with the plane $x - 2y + 4z = -1$.

28. Prove that the lines $\dfrac{x+3}{2} = \dfrac{y+5}{3} = \dfrac{z-7}{-3}$ and $\dfrac{x+1}{4} = \dfrac{y+1}{5} = \dfrac{z+1}{-1}$ are coplanar. Find the equation of plane containing them.

29. Let R be the relation on the set R of real numbers such that aRb iff a-b is and integer. Test whether R is an equivalence relation. If so find the equivalence class of $1 \text{and} \dfrac{1}{2}$ wrt. This equivalence relation.

30. Show that f : A to B, where $A = R - \{3\}$, $B = R - \{1\}$ defined as $f(x) = \dfrac{x - 2}{x - 3}$ is bijective. Find f^{-1}.

31. Find the particular solution of the differential equation $\frac{dy}{dx} + y\cot x = 2x + x^2\cot x$,given that $y = 0$ when $x = \frac{\pi}{2}$.

32. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the

square bounded by $x = 0, x = 4, y = 4$ and $y = 0$ into three equal parts.

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33. A line with direction ratios $< 2, 1, 2 >$ meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$. Find the co-ordinates of the points of intersection.

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