MATHS

BOOKS - MODERN PUBLICATION

TEST PAPER 2

Problem

1. Let $f: R \to R$ defined by $f(x) = x + 1$ and $g: R \to R$ defined as $g(x) = \sqrt{x}$ find fog and gof if defined.

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2. Express the value of $\operatorname{cosec}\left(\cos^{-1}\frac{3}{5}+\cos^{-1}\frac{4}{5}\right)$ in simplest form.

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3. Find x and y :

$$\begin{bmatrix} x & -2y \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -8 \\ 0 & -2 \end{bmatrix}$$

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4. If ω is a complex cube root of 1,then for what value of. lamda the determinant $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \lambda & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0$?

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5. What is the derivative of $\cos^{-1}\left(2x^2 - 1\right)$ if x $\in$ (-1,0).

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6. Write the interval in which $y = \ln x, x \text{in} R_+$ is increasing, decreasing.

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7. Evaluate $\int_0^2 |x-2|dx$

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8. What is the differential equation whose solution is $y = 3x + k$?

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9. if $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}-\vec{b}\right|$ the what is the relation between $\vec{a}$ and $\vec{b}$?

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10. Write the equation of the plane perpendicular to z-axis and passing through `(1,-2,4).

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11. let $f: A \to B$ is a bijective function. Do you think $f^{-1}: B \to A$ is also bijective? justify your answer.

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12. Prove that $f: X \to Y$ is injective iff for all subsets A, B of X, $f(A \cap B) = f(A) \cap f(B)$.

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13. Find the number of equivalence, relations on X = {1,2,3),

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14. Find the inverse of the following matrices using elementary transformation

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

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15. Verify that $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ satisfies the equation $A^2 - (a+d)A + (ad-bc)I = 0$ where I is the 2x 2 unit matrix.

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16. Prove that the following.

$$\begin{bmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{bmatrix} = 2\begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix}$$

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17. Prove that the following. $\begin{bmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{bmatrix}$

=4ab

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18. Find the inverse of the following matrix

$$\begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

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19. Examine the continuity of the following functions at indicated points.

$$f(x) = \begin{cases} \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1} & \text{if } x \neq 0 at x = 0 \\ 0 & \text{if } x = 0 \end{cases}$$

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20. if $x = e^t \sin t$ and $y = e^t \cos t$, then prove that $(x+y)^2 \frac{d^2y}{dx^2} = 2\left(x\frac{dy}{dx} - y\right)$.

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21. Differentiate.

$x^{\sin x} + (\tan x)^x$

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22. Find the equation of the normal at a point on the curve $x^2 = 4y$, which passes through the point (1, 2).

Also, find the equation of the corresponding tangent.

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23. etermine the sub-interval of $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, in which f(x)=tan x-4x is increasing.

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24. Integrate the following $\int \frac{x+3}{5-x^2-4x}$

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25. $\int_0^2 [x^2]dx$

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26. Evaluate $\int_0^{\pi/4} \log(1+\tan x)dx.$

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27. Solve the following differential equations

$\frac{dy}{dx} + \frac{y}{x} = x^2, y(1) = 1$

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28. Show that the vector area of the triangle whosse vertices have position vector $\vec{a}, \vec{b}, \vec{c}$ is $\frac{1}{2}\left(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right)$.

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29. If the sum of two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

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30. Find the co-ordinates of the point where the line joining $(3, 4, -5)$ and $(2, -3, 1)$ meets the plane

$2x + y + z - 7 = 0.$

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31. Passing throughthe point $(2, -3, 1)$ and $(-1, 1-7)$ and perpendicular to the plane $x - 2y + 5z + 1 = 0.$

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32. Find the direction cosines of the line which is perpendicular to the lines whose direction ratios are <1,-2,3> and <2,2,1>.

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33. Prove that f:$X \to Y$ is surjective iff for all $A \subseteq X, (f(A))' \subseteq f(A')$, where A' denotes the complement of A in X.

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34. Find the inverse of $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ using elementary operations.

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35. Prove that the following.

$$\begin{vmatrix} (v+w)^2 & u^2 & u^2 \\ v^2 & (w+u)^2 & v^2 \\ w^2 & w^2 & (u+v)^2 \end{vmatrix} = 2uvw(u+v+w)^3$$

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36. Find the points on the curve $y = x^2 + 1$ which are nearest to the point (0,2).

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37. Evaluate $\int_0^{\pi} \frac{x}{1+\sin x} dx$

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38. Find the area of the regions into which the circle $x^2 + y^2 = 4$ is divided by the line $x + \sqrt{3}y = 2$.

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39. Find the solution of the following differential equations:

$$x dy - y dx = \sqrt{x^2 + y^2} dx$$

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40. Show that $\hat{i} \times \left(\vec{a} \times \hat{i}\right) + \hat{j} \times \left(\vec{a} \times \hat{j}\right) + \hat{k} \times \left(\vec{a} \times \hat{k}\right) = 2\vec{a}$.

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41. A variable plane meets the coordinate axes at P, Q, R points. If the plane passes through a fixed point (a, b, c), prove that the centre of the shpere passing the origin and P, Q, R will lie on the surface $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$

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42. Write whether the following statements are true or false.

The equation $\tan^{-1}(\cot x) = 2x$ has exactly two real solutions.

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43. Show that the operation * given by x*y=x+y+ -xy is a binary oeration on Z,Q and R but not on N.

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44. Solve the following : $\begin{bmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{bmatrix}$

=0

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45. If $f'(x) = \sqrt{2x^2 - 1}$ and $y = f(x^2)$ then what is $\frac{dy}{dx}$ at x = 1 ?

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46. Find the extreme points of the function $y = x + \frac{1}{x}$.

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47. What is the differential equation whose solution is $y = c(\sin x + \cos x)$?

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48. For what value of λ, the vectors $\lambda\hat{i} + 3\hat{j} + \lambda\hat{k}$ and $\lambda\hat{i} - 2\hat{j} + \hat{k}$ are perpendicular to each others.

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49. Let A and B be sets.

Show that f : $A \times B \to B \times A$ such that f (a,b) = (b,a) is bijective function .

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50. Maximise $Z = 50x_1 + 60x_2$

Subject to $2x_1 + 3x_2 \leq 6, x_1, x_2 \geq 0.$

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51. Let ~ be defined by (m,n)~(p,q) if mq=np where m, n, p,$q \in Z$-{0}. Show that it is an equivalence

relation.

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52. If $A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 & 2 \\ -2 & 3 & 0 \end{bmatrix}$, verify (A+B)C=AC+BC

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53. Prove that all the diagonal elements of a skew symmetric matrix are zero .

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54. If x,y,z are positive and are the pth, qth and rth terms of a G.P. then prove that

$$\begin{vmatrix} \log x & p & 1 \\ \log y & q & 1 \\ \log z & r & 1 \end{vmatrix} = 0$$

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55. Answer the following:

What is the value of $\begin{bmatrix} o & -h & -g \\ h & o & -f \\ g & f & o \end{bmatrix}$?

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56. Show that no two normals to a parabola are parallel.

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57. Find the intervals where the following functions are (a) increasing and (b) decreasing.

$y = \sin x + \cos x, x \in [0, 2\pi]$

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58. Find $\int \left(\frac{\cos x}{\sin^2 x} + \sin x \right) dx$

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59. Evaluate : $\int_0^{\frac{\pi}{4}} \frac{dx}{\cos x}(\cos x + \sin x).$

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60. Find the differential equation whose general soltution is $c_1 x^2 + c_2 y = 1$ where c_1, c_2 are arbitrary constants.

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61. Find the area of the region bounded by the curve $y = 6x - x^2$, the X-axis and the two ordinates $x = 0$ and $x = 9$.

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62. Prove the following by vector method. The parallelogram whose diagonals are equal is a rectangle.

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63. Prove that the two lines whose direction cosines are connected by the equations $l + 2m + 3n = 0, 3lm - 4\ln + mn = 0$ are perpendicular to each other.

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64. A variable plane meets the coordinate axes at A, B, C and is at a constant distance d from origin. Prove that the locus of the centroid of the triangle ABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{9}{d^2}$

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65. Find the equation of the line through the point $(3, -1, 2)$ and parallel to the planes $x + y + 2z - 4 = 0$ and $2x - 3y + z + 3 = 0$

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66. If N denotes the set of all natural numbers and R be the relation on $N \times N$ defined by (a, b) R (c, d) if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

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67. If a, b and c are all positive real, then prove that minimum value of determinant

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

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68. Solve the following system of equations by the matrix inversion method.

$x + y + z = 4$

$2x - y + 3z = 1$

and $3x + 2y - z = 1$

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69. Find the inverse of $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ using elementary operations.

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70. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

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71. Evaluate : $\int \frac{dx}{\cos x}(5 + 3\cos x)$.

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72. Find the area of the parallelogram whose diagonals are the vectors $3\hat{i}+\hat{j}-2\hat{k}$ and $\hat{i}-3\hat{j}+4\hat{k}$?

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73. For an vector $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, prove that $\vec{r}=\left(\vec{r}.\hat{i}\right)\hat{i}+\left(\vec{r}.\hat{j}\right)\hat{j}+\left(\vec{r}.\hat{k}\right)\hat{k}$

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74. Find the equation of plane through the point $(2, 0, -3)$ and containing the line $3x + y + z - 5 = 0 = x - 2y + 4z + 4$

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