MATHS

BOOKS - MODERN PUBLICATION

TEST PAPER 3

Problem

1. Let $f: R \to R$ defined by $f(x) = x + 1$ and $g: R \to R$ defined as $g(x) = \sqrt{x}$ find fog and gof if defined.

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2. Find the value of

$$\cos^{-1}\left(\frac{-7}{25}\right)+\cos^{-1}\left(\frac{3}{5}\right).$$

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3. If $\begin{vmatrix} a & b & c \\ b & a & b \\ x & b & c \end{vmatrix} = 0$ then `x=____.

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4. If A is a square matrix of order 3 and $|A| = 3$,then write the matrix represented by A.Adj A.

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5. Differentiate $\tan^{-1}$x w.r.t $\cot^{-1}\left(\frac{1}{x}\right)$

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6. Write the set of values of x for which the function f(x) = sinx - x is increasing.

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7. State order and degree of $(Y'')^2 + \cos(Y')$

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8. If $|\vec{a} . \vec{b}| = |\vec{a} \text{ xx } \vec{b}|$` then angle between them is___.

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9. Find the value of λ so that the vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other. $\vec{a}$ = (6,2,-3), $\vec{b}$ = (1,-4,λ)

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10. Show that the relation S defined on set $N \times N$ by $(a, b)S(c, d) \Rightarrow a + d = b + c$ is an equivalence relation.

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11. A trust fund has Rs. 50,000 that is to be invested in two types of bonds .The first and second bonds respectively pay annual interest at the rate of 5% and 6% respectively .Using matrix multiplication ,

determine how to invest the money in these bonds so as to get a total annual interest of Rs. 2780 .

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12. Find the inverse of the following matrices using elementary transformation:

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

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13. Prove that the following.

$$\begin{bmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{bmatrix}$$

= abc(1+1/a+/b+1/c)

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14. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then show that $A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}, k \varepsilon N$

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15. If $y = In\left(x^2 + y^2\right)$,then find $\frac{dy}{dx}$.

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16. If $x=a\cos^3\theta, y=a\sin^3\theta$,then find $\frac{d^2y}{dx^2}$

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17. Test the differentiability and continuity of the following function at x=0: $f(x) = \begin{cases} \frac{1-e^{-x}}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$

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18. Determine the interval in which the function f(x) = $x^3 - 5x^2 + 3x + 97$ is decreasing and that in which it is increasing.

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19. $\int_0^{\pi} |\cos x| dx$

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20. Solve : $\left(1 + y^2\right). dx + x dy = \tan^{-1} y dy$

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21. A plane meets the coordinate axes at A, B and C respectivély. If the centroid of the triangle ABC is (-1, 2, 5) then find the equation of the plane.

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22. Prove by vector method that in any triangle ABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

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23. Prove that the acute angle between the lines whose direction cosines are given by the relation $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$and $\frac{\pi}{3}$

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24. Let f : W $\rightarrow$ W be defined as f (x) = x -1 if x is odd and f(x) = x +1 if x is even then show that f is invertible. Find the inverse of f where W is the set of all whole numbers.

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25. Constract the composition table/multiplication table for the binary operation * defined on {0,1,2,3,4}by $a * b = a \times b(\mathrm{mod} = 5)$. Find the identity element if any. Also find the inverse elements of 2 and 4.

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26. Let A = $\begin{pmatrix} 122 \\ 212 \\ 221 \end{pmatrix}$.Find A^-1 and hence show $A^2 - 4A - 5I = 0.$

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27. Prove that the following.

$$\begin{bmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{bmatrix} = 4a^2b^2c^2$$

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28. Show that the sum of the intercepts on the coordinate axes of any tangent to the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ is constant.

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29. Shows that the triangle of greatest area that can be inscribed in a circle is equilateral.

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30. Find the area of the région on closed between $y = 4x - 1$ and $y^2 = 2x$

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31. Write fog if f:R $\rightarrow$ R and g: $R \rightarrow R$ is given by f(x) = | x| and g(x) = |5x - 2|.

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32. Evaluate :

$$\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$$

.

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33. If $|adjA| = 64$ then find $|A|$

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34. Evaluate the following : $\begin{bmatrix} \sin^2\theta & \cos^2\theta & 1 \\ \cos^2\theta & \sin^2\theta & 1 \\ -10 & 12 & 2 \end{bmatrix}$

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35. A differentiable function f defined for all x > 0 and satisfies $f(x^2) = x^3$ for all x > 0. What is the value f'(16) ?

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36. Write the minimum value of y_2 where $y = \sin^2 x \cos^2 x$

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37. Evaluate $\int_{-1}^{1} e^{|x|} dx$

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38. Write the vector equation ofthe plane whose cartesian equation is $x + y + 2z = 1$.

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39. Let A (1,2,3). Then, find the number of equivalence relations containing (1,2).

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40. Let f ,g: R $\rightarrow R$ be two functions defined as f(x) = |x| +x and g(x) = |x| -x $\forall x \in R$.

Then find fog and gof .

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41. Solve the following : $\begin{bmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{bmatrix}$

=0

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42. If the matrix A is such that $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} A = \begin{bmatrix} -4 & 1 \\ 7 & 7 \end{bmatrix}$, find A.

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43. Find the values of a and b, if the function f defined as f(x) $\begin{cases} x^2 + 3x + a & \text{if } x \leq 1 \\ bx + 2 & \text{if } x > 1 \end{cases}$ is diffentiable at $x = 1$

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44. Differentiate with respect to x:

$$y = 3^{x^2} + \tan^{-1}\left(\frac{\cos x + \sin x}{\cos x - \sin x}\right)$$

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45. Find the dy/dx when

$$\cos x = \sqrt{\frac{1}{1+t^2}}, \sin y = \frac{2t}{1+t^2}$$

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46. Show that the line y = mx + c touches the parabola y^2 = 4ax if c $= \frac{a}{m}$.

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47. Show that 2sinx+tanx $\geq$ 3x for all x $\varepsilon \left(0, \frac{\pi}{2}\right)$.

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48. Evaluate : $\int \frac{xe^x}{(x+1)^2} dx$

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49. Evaluate the following integrals

$\int (x^2) \frac{dx}{(x^2+3)(x^2+2)}$

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50. Solve : $\frac{dy}{dx} + 1 = e^{x+y}$

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51. Solve : $\left(x+2y^3\right)\frac{dy}{dx}=y.$

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52. Find the equation of the plane passing through the points (2,1,3), (3,2,1) and $(1,\ 0,\ -1).$

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53. Find the image of the point $(3,\ -2,\ 1)$ in the plane $x-y+3z=2.$

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54. Determine k such that a vector $\vec{r}$ is at right angles to each of the vectors $\vec{a} = k\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - k\hat{k}$ and $\vec{c} = -2\hat{i} + k\hat{j} + 3\hat{k}$.

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55. Find the foot of the perpendicular drawn from the point (5,7,3) to the line $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$. Find the length of the perpendicular.

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56. Let $f: X \to Y$ and $g: Y \to Z$. Prove that gof is bijective if both f and g are bijective. Also prove that $(gof)^{-1} = f^{-1}og^{-1}$.

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57. Find the product of matrices A and B, where A = $\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ Hence solve the following equations by matrix method,

$x + y + 2z = 1,$ $3x + 2y + z = 7,$

$2x + y + 3z = 2.$

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58. Show that

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

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59. Find the inverse of the following matrices using elementary transformation

$$\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$$

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60. If $x = a\cos\theta + b\sin\theta$ and $y = a\sin\theta - b\cos\theta$, then show that $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

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61. Find the equation of the normal at a point on the curve $x^2 = 4y$, which passes through the point

(1, 2). Also, find the equation of the corresponding tangent.

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62. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the straight line $\frac{x}{a}+\frac{y}{b}=1$

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63. Prove analytically : The perpendicular bisector of the sides of a triangle are concurrent.

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