MATHS

BOOKS - MODERN PUBLICATION

TEST PAPER 4

Problem

1. If $f(x) = \left(1 - x^3\right)^{\frac{1}{3}}$ then find $fof(x)$.

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2. Find the value of $\sin^{-1}\left\{\sin 2\frac{\pi}{3}\right\}$.

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3. Let $|A| = n$ and $|B| = m$.How many functions can be there from A to B.

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4. For what k,$x - ky - z = 0$

$kx - y - z = 0$

$x + y - z = 0$

has a non zero solution?

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5. Differentiate $\cos^{-1}\left[x + e^{-x}\right]$ w.r.t. x.

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6. Find the intervals in which the function $y = \frac{\ln x}{x}$ is increasing and decreasing.

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7. what is the value of $\int_1^4 (x)dx$.

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8. What is the order of the differential equation whose solution is $ax^2 + bx + c = 0$.

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9. If $\left|\vec{a} + \vec{b}\right| = \left|\vec{a}\right| + \left|\vec{b}\right|$ then what is the angle between $\vec{a}$ and $\vec{b}$

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10. Write the equation of the plane perpendicular to z-axis and passing through `(1,-2,4).

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11. Constract the composition table/multiplication table for the binary operation * defined on {0,1,2,3,4}by $a * b = a \times b(\text{mod} = 5)$. Find the identity element if any. Also find the inverse elements of 2 and 4.

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12. Suppose a box contains a set of n balls $(n > 4)$ (denoted by B)of four different colours (many have different sizes), viz,red, blue, green and yellow. Show that a relation R defined on B as $R = \{(b_1, b_2) : \text{balls} b_1 \text{and} b_2$ have the same colour} is an equivalence relation on B. How many equivalence classes can you find with respect ot R ?

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13. Let A and B be sets.

Show that f : $A \times B \to B \times A$ such that f (a,b) =

(b,a) is bijective function .

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14. (Allocation Problem.)A farmer has 5 acres of land on which he wishes to grow two crops X and Y. He has to use 4 cart loads and 2cart loads of manure per acre for crops X and Y respectively. But not more than 18 cart loads of manure is available. Other expenses are ₹200 and ₹500 per acre for the crops X and Y respectively . He estimates profit from crops X and Y at the rates ₹1000 and ₹800 per acre respectively. Formulate the LPP as to how

much land he should allocate to each crop for maximum profit.

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15. Without expanding prove that

$$\begin{vmatrix} 1 & a & a^2 & -bc \\ 1 & b & b^2 & -ca \\ 1 & c & c^2 & -ab \end{vmatrix} = 0$$

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16. Prove that

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$$

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17. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ "show that"

$A^3 - 23A - 40I = 0$

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18. If $A = \begin{bmatrix} 1 & -2 & 2 \\ 3 & 1 & -1 \end{bmatrix}$

$B \begin{bmatrix} 2 & 4 \\ 1 & 2 \\ 3 & -1 \end{bmatrix}$ verify

that $(AB)^T = B^T A^T$.

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19. If $f(x) = \begin{cases} ax^2 + b & \text{if} \quad x < 1 \\ 1 \quad \text{if} \quad x = 1 \\ 2ax - b & \text{if} \quad x > 1 \end{cases}$

is continuous at x=1, then find a and b.

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20. Show that `2 sin x + than x ge 3x all x in (0, pi/20).

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21. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles, if $8k^2 = 1$.

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22. Evaluate the following integrals $\int \frac{dx}{\sin x(3 + 2\cos x)}$ put cosx=z

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23. $\int_0^{\pi/2} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$

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24. Determine the area of the region bounded by $y^2 = x^3$ and the double ordinate through (2,0).

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25. Vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 5$ and

$\left|\vec{c}\right| = 7$. Then, find the angle between $\vec{a}$ and $\vec{b}$.

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26. If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines show that the Direction Cosines of the line perpendicular to both of them are $m_1n_2 - n_1m_2, n_1l_2 - l_1n_2, l_1m_2 - m_1l_2$

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27. Find the equation of the plane Which contains the line of intersection of the planes $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ and perpendicular of the plane $5x + 3y + 6z + 8 = 0.$

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28. Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the line $\frac{x-2}{2} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 5.$

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29. Let R be the relation on the set R of real numbers such that aRb iff a-b is and integer. Test whether R is an equivalence relation. If so find the equivalence class of $1 \text{and} \frac{1}{2}$ wrt. This equivalence relation.

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30. Prove that the following.

$$\begin{bmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{bmatrix} = (a+b+c)^3$$

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31. Find the minimum distance of a point on the curve $\frac{2}{x^2}+\frac{1}{y^2}=1$ from the origin.

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32. A variable plane meets the coordinate axes at A, B, C and is at a constant distance d from origin. Prove that the locus of the centroid of the triangle ABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{9}{d^2}$

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33. Find x and y if $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$.

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34. If $\begin{bmatrix} 3 & 5 & 3 \\ 2 & 4 & 2 \\ \lambda & 7 & 8 \end{bmatrix}$ is a singular matrix, write the value of lambda.

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35. Let f={(1,a),(2,b),(3,c),(4,d)} and g={(a,x),(b,x),(c,y),(d,x)} Determine gof and fog if possible. Test whether fog=gof.

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36. Express the value of $\operatorname{cosec}\left(\cos^{-1}\frac{3}{5}+\cos^{-1}\frac{4}{5}\right)$ in simplest form.

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37. Let x $E\psi lon\left(2\frac{\pi}{4}, \pi\right)$, $y = |\cos x| + |\sin x|$ what is $\frac{dy}{dx}$?

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38. Write the least value of a for which the function f defined by $f(x) = x^2 + ax + 1$ increases.

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39. Evaluate:int $x^2 \frac{dx}{x} \cdot 3 + 1.$

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40. Find the differential equation n whcse solution is : $y = ax^2 + bx + c.$

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41. What is the projection of the line segment joining (1,3,-1) and (3,2,4) on z-axis ?

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42. If $\vec{a} = (2, -2, 1)$, $\vec{b} = (2, 3, 6)$ and $\vec{c} = (-1, 0, 2)$ then what is the direction of $\vec{a} - \vec{b} + 2\vec{c}$?

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43. An animal feed company must produce 200kg of a mixture consisting of ingredients A and B. The-ingredient A costs Rs.3 per kg and B costs 5 per kg. No more than 80 kg of A can be uséd and at least 60 kg of B must be used. Formulate the problem to minimise thé cost of mixture.

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44. Check if the relation R on set of real numbers, defined as $R = \{(a, b) : a \leq b^3\}$ is reflexsive, symmetric or transitive.

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45. Verify that $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ satisfies the equation $A^2 - (a + d)A + (ad - bc)I = 0$ where I is the 2x2 unit matrix.

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46. If A+B+C = π, prove that

$$\begin{bmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{bmatrix} = 0$$

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47. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ "show that" $A^3 - 23A - 40I = 0$

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48. Show that $(a+1)$ is a factor of

$$\begin{vmatrix} (a+1) & 2 & 3 \\ 1 & a+1 & 3 \\ 3 & -6 & a+1 \end{vmatrix}$$

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49. Find the dy/dx when

$$x = a[\cos t + \log \tan(t/2)], y = a \sin t$$

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50. Prove that: $y = \dfrac{4 \sin \theta}{2 + \cos \theta} - \theta$ is an increasing function in [0,pi/2]`

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51. $\int \tan^{-1}(\sec x + \tan x)dx$

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52. If the magnitude of the difference of two unit vectors is $\sqrt{3}$ then find the magnitude of their sum.

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53. Find the equation of the plane if the point $(5, -3, 4)$ is the foot of the perpendicular drawn from origin to the plane.

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54. Proved that the line $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-3}{1}$ lies on the plane $7x + 5y + z = 0$

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55. Find the points of intersection of the line $\frac{x-1}{1}=\frac{y+2}{3}=\frac{z-1}{-1}$ and the plane $2x+y+z=9.$

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56. Let A=R $\times$ R and be the binary operation on A defined by (a, b) *(c,d) = (a + c, b + d). Show that *is commutative and associative. Find the identity element for* on A, if any.

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57. If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$ then find A^{-1} Using A^{-1} solve the system of linear equations

$x - 2y = 10$

$2x - y - z = 8$

$-2y + z = 7$

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58. If $A = \begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix}$ then prove that $A^n = ((\cos nx, \sin nx), (-\sin nx, \cos nx)$ for all positive integers n.

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59. If 2s=a+b+c show that

$$\begin{bmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{bmatrix} =$$

$$2s^3(s-a)(s-b)(s-c)$$

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60. If $y\sqrt{x^2+1} = \log\left\{\sqrt{x^2+1} - x\right\}$ then prove that $\left(x^2+1\right)\dfrac{dy}{dx} + xy + 1 = 0$

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61. Show that $\frac{x}{1+x\tan x}, x \in \left(0, \frac{\pi}{2}\right)$ is maximum when x = cos x.

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62. Find the area enclosed by

$y^2 = x^3, x = 0, y = 1$

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63. Solve $y^2 + x^2 \frac{dy}{dx} = xy\frac{dy}{dx}$.

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64. Prove the following by vector method. Altitudes of a triangle are concurrent.

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