MATHS

BOOKS - MODERN PUBLICATION

TEST PAPER 5

Problem

1. Find the number of equivalence relations on $x = \{1, 2, 3\}$.

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2. The sum of two non integral roots of $\begin{bmatrix} x & 2 & 5 \\ 3 & x & 3 \\ 5 & 4 & x \end{bmatrix} = 0 i s \_ \_ \_ \_ .$

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3. If $f(x) = x^3 + e^{\frac{x}{2}}$ and $g(x) = f(x)'$ then what is the value of `g'(1)' ?

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4. Write the interval in which the function $f(x) = \sin^{-1}(2 - x)$ is differentiable.

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5. Evaluate : $\int_1^2 xInx'dx$

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6. Name of curve which is represented by the solution of differential equation

$2x\frac{dy}{dx} - y = 3$

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7. Find the distance between the parallel planes $2x - 2y + z + 1 = 0$ and $4x - 4y + 2z + 3 = 0.$

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8. In each of the problems given below, find the work done by a force $\vec{F}$ acting on a

particle, such that the particle is displaced from a point A to a point B. $\overrightarrow{F} = 4\hat{i} - 3\hat{k}$ A(1,2,0), B(0,2,3).

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9. Show that the relation R is in the set $A = \{1, 2, 3, 4, 5\}$ given by R={(a, b): |a-b| is divisible by 2}, is an equivalence relation. Write all the equivalence classes of R.

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10. A binary operation * is defined on the set $X = R - \{-1\}$ by $x * y = x + y + xy,\ \forall x, y \in X.$

Check whether * is commutative and associative. Find its identity element and also find the inverse of each element of X.

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11. Let f, $g: R \to R$ defined by $f(x) = |x| + x$, $g(x) = |x| - x$ for all x in R then find fog and

gof .Hence find $(fog)(-3)$, $(fog)(5)$ and $(gof)(-2)$.

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12. Solve $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

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13. An animal feed company must produce 200 kg of a mixture consisting of ingredients A and B. The-ingredient A costs Rs.3 per kg and

B costs 5 per kg. No more than 80 kg of A can be uséd and at least 60 kg of B must be used. Formulate the problem to minimise thé cost of mixture.

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14. Without expanding prove that

$$\begin{vmatrix} 1 & a & a^2 & -bc \\ 1 & b & b^2 & -ca \\ 1 & c & c^2 & -ab \end{vmatrix} = 0$$

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15. Prove that the following. $\begin{bmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{bmatrix}$ =

(b-c)(c-a)(a-b)(bc+ca+ab)

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16. Find the inverse of the following matrices using elementary transformation:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 1 & 0 & 2 \end{bmatrix}$$

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17. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ "show that" $A^3 - 23A - 40I = 0$

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18. Express as a sum of a symmetric and a skew symmetric matrix:

$\begin{bmatrix} 2 & -1 & 3 \\ 5 & 7 & -2 \\ 1 & 4 & 6 \end{bmatrix}$

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19. If $y = In\left(x^2 + y^2\right)$,then find $\frac{dy}{dx}$.

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20. If $x=a\cos^3\theta$,$y=a\sin^3\theta$,then find $\frac{d^2y}{dx^2}$

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21. Find the equation of the normal to the curve $y = (\log x)^2$ at $x = \frac{1}{e}$.

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22. Determine the interval in which the function f(x) = $x^3 - 5x^2 + 3x + 97$ is decreasing and that in which it is increasing.

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23. Find the tangent to the curve $y = \cos(x + y), 0 \leq x \leq 2\pi$ which is parallel to the line x + 2y = 0

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24. Evaluate $\int_0^2 \left|x^2 + 2x - 3\right| dx$

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25. solve:$ydx - xdy = xydx$

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26. $\int \frac{xe^x}{1+x^2} dx$

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27. $\int_0^{\pi} \frac{x\,dx}{1+\sin x}$

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28. Solve : $\left(x+2y^3\right)\frac{dy}{dx} = y.$

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29. If the position vectors of the points A, B, C are $2\hat{i}+\hat{j}-\hat{k}, 3\hat{i}-2\hat{j}+\hat{k}$ and $\hat{i}+4\hat{j}-3\hat{k}$

respectively, then prove that A, B, C are collinear.

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30. Find the foot of the perpendicular drawn from the point (5,7,3) to the line $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$. Find the length of the perpendicular.

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31. Passing throughthe point $(2, -3, 1)$ and $(-1, 1-7)$ and perpendicular to the plane $x - 2y + 5z + 1 = 0$.

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32. Find the direction cosines of the line which is perpendicular to the lines whose direction ratios are <1,-2,3> and <2,2,1>.

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33. Show that the shrtest distance of the point (0, 8a) from the curve $ax^2 = y^3$ is $2a\sqrt{11}$.

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34. If $\begin{bmatrix} x & x^2 & x^3 - 1 \\ y & y^2 & y^3 - 1 \\ z & z^2 & z^3 - 1 \end{bmatrix} = 0$

then prove that xyz=1 when x,y,z are non zero and unequal.

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35. Exaniming consistency and solvability, solve the following equations by matrix method.

$x + 2y + 3z = 14,$ $2x - y + 5z = 15,$

$2y + 4z - 3x = 13$

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36. Prove the following:

$$\begin{vmatrix} ax - by - cz & ay + bx & az + cx \\ bx + ay & by - cz - ax & bz + cy \\ cx + az & ay + bz & cz - ax - by \end{vmatrix}$$

=

$$\left(a^2 + b^2 + c^2\right)(ax + by + cz)\left(x^2 + y^2 + z^2\right)$$

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37. If x =sin t,y = sin 2t then prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$

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38. Show that the semi-vertical angle of a right circular cone of minimum volume that circumscribes a given sphere is $\sin^{-1}\left(\frac{1}{3}\right)$.

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39. $\int_0^{\pi} \left(\frac{\tan x}{\sin x + \tan x} \right) dx$

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40. Find the area of the regions into which the circle $x^2 + y^2 = 4$ is divided by the line $x + \sqrt{3}y = 2.$

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41. Find the distance of the point $(1, -1, -10)$ from the line $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ measured parallelto the line $\frac{x+2}{2} = \frac{y-3}{-3} = \frac{z-4}{8}$

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