MATHS

BOOKS - MODERN PUBLICATION

TEST PAPER 6

Exercise

1. For real numbers x and y, define x R y if and only if $x - y + \sqrt{2}$ is an irrational number. Is R transitive? Explain your answer.

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2. Find the least value of n for which $\tan^{-1}\left(\frac{n}{\pi}\right) > \frac{\pi}{4}$,n varepsilon N is valid.

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3. If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then write the value of k .

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4. IF

$$\begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix} = a + bx + cx^2 + dx^3 + ex^4 + fx^5$$

then write the value of a.

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5. If x $\in \left(\frac{3\pi}{4}, \pi\right)$ what is $\frac{dy}{dx}$ for y = | cos x | + | sin x | ?

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6. Give example of a function which is increasing in $(-\infty, 2)$ and $(3, \infty)$ and decreases in (2,3).

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7. What is the value of $\int e^x dx$.

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8. Write the solution of $\sqrt{3}x + \frac{dy}{dx} = 2$.

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9. Write the equation of the line passing through the points $(3, -2, -5)$ and `(3,-2,6).

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10. Find the volume of parallelopiped with edges $2\vec{i} - 3\vec{j}$, $\vec{i} + \vec{j} - \vec{k}$ and $3\vec{i} - \vec{k}$.

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11. Show that the relation R on the set A = {1,2,3,4,5) given by R = {(a,b): la -bl is even)} is an equivalence relation. Also, show that all elements of (1, 3, 5) are related to each other and all the elements of (2, 4) are related to each other, but no element of (1, 3, 5) is related to any element of (2, 4).

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12. Consider f:R_+ $[4, \infty]$ is given by f(x)= $x^2 + 4$. Show that f is invertible with the inverses f^{-1} of f given by $f^{-1}(y) = \sqrt{y-4}$, where R_+, is the set of all non-negative real numbers.

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13. A merchant sells two models X and Y of TV with cost price ₹ 25000 and ₹ 50000 Per set respectively. He gets a profit of ₹ 1500 on model X and ₹ 2000 on model Y . The sales connot exceed 20 sets in a month. If he cannot invest more than 6 lakh rupees, formulate the problem of determining the number of sets of each type he must keep in stock for maximum profit.

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14. A binary operation * is defined on the set

$X = R - \{-1\}$ by $x * y = x + y + xy$, $\forall x, y \in X$.

Check whether * is commutative and associative. Find its identity element and also find the inverse of each element of X.

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15. Prove that the following. $\begin{bmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{bmatrix}$ =4ab

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16. Using determinants, find the values of a if the area of triangle with vertices (a,0),(4,0),(0,2) is 4sq. units.

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17. If A,B,C are matrices of order 2×2 each and $2A + B + C = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$

$A + B + C = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$

$A + B - C = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$ find A,B and C.

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18. Using properties of determinant prove that

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$$

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19. Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$.

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20. Examine continuity and differentiability $f(x)=$

$$\begin{cases} x\frac{\sin 1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \text{ at } x = 0.$$

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21. Find $\frac{dy}{dx}$ if $y = (\cos x)^x$.

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22. Find the tangent to the curve $y = \cos(x + y), 0 \leq x \leq 2\pi$ which is parallel to the line x + 2y = 0

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23. Find the interval where

$y = \sin 2x - \cos 2x$, $x$in$[0, 2$pi$]$ is

(a) incresing (b) decreasing

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24. Evaluate: $\int \frac{\sin x - x \cos x}{x(x + \sin x)} dx.$

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25. Integrate : $\int \sin^{-1} x dx.$

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26. Find the particular solution of $d^2\frac{y}{dx^2} = 2x$,given that x=0,y=2 and $\frac{dy}{dx} = 3$.

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27. Solve :-$\left(x^2 + y^2\right)dx - 2xydy = 0$

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28. If with reference to the eight handed system of mutually perpendicular unit vectors $\hat{i}, \hat{j}, \hat{k}, \vec{\alpha} = 3\hat{i} - \hat{j}, \beta = 2\hat{i} + \hat{j} - 3\hat{k}$, then express $\vec{\beta}$ in the form $\vec{\beta} = \vec{\beta_1} + \vec{\beta_2}$, where $\vec{\beta_1}$ is parallel to $\vec{\alpha}$ and $\vec{\beta_2}$ is perpendicular to $\vec{\alpha}$.

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29. If $\vec{a} = (2, 3, 6)$, $\vec{b} = (2, -2, 1)$, $\vec{c} = (-1, 0, 2)$, find the direction cosines of $\vec{b} - \vec{a} + 2\vec{c}$ and the unit vector in direction of $\vec{b} - \vec{a} + 2\vec{c}$.

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30. Find the equation of the plane passing through $(1, 3, 4)$, $(2, 1. -1)$ and $(1, -4, 3)$.

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31. Find the equation of the plane passing through the line $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and the point $(1, 2, -4)$.

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32. Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the line $\frac{x-2}{2} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 5$.

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33. Let A=R $\times$ R and be the binary operation on A defined by (a, b) *(c,d) = (a + c, b + d). Show that *is commutative and associative. Find the identity element for* on A, if any.

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34. Let A=R-{3/5} $Brf: A \to A$ defined as $f(x) = \frac{3x+2}{5x-3}$. `Br show that is invertible, hence find f^-1.

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35. Prove the following:

$$\begin{vmatrix} ax-by-cz & ay+bx & az+cx \\ bx+ay & by-cz-ax & bz+cy \\ cx+az & ay+bz & cz-ax-by \end{vmatrix}$$

$$=\left(a^2+b^2+c^2\right)(ax+by+cz)\left(x^2+y^2+z^2\right)$$

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36. Find the inverse of the following matrices using elementary transformation

$$\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$$

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37. Show that the homogenous system of equations

$x - 2y + z = 0 x + y - z = 0$

$3x + 6y - 5z = 0$

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38. (i) if$x = \sin t, y = \sin pt,$ then show that

$$\left(1 - x^2\right) d^2 \frac{y}{dx^2} - x \frac{dy}{dx} + p^2 y = 0.$$

$BrDeter\ \min ethed \Leftrightarrow erentialsofz$=cos 2t -2 cot t`.

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39. A cylindrical open water tank with a circular base is to be made out of 30 sq metres of metal sheet. Find the dimensions so that it can hold maximum water. (Neglect thickness of sheet).

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40. Evaluate $\int \frac{x^2 dx}{(x^2+1)(x^2+4)}$

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41. Determine the area of the region bounded by $y^2 = 4ax$ by the double ordinate through (3,0).

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42. Prove by vector method that in any triangle ABC, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

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43. Show that the lines $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ are coplaner. Find their point of intersection.

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