MATHS

BOOKS - MODERN PUBLICATION

TEST PAPER 8

Exercise

1. Find the value of $\cos \tan^{-1} \cot \left(\cos^{-1} \left(\sqrt{\frac{3}{2}} \right) \right)$.

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2. Constuct a 2 x 3 matrix having elements given by $a_{ij} = ij.$

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3. If $\begin{bmatrix} 3 & 2 \\ 7 & x \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -7 & y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then find the value of x and y.

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4. If $y = at^2$ and $x = 2at$ then find $\frac{d^2y}{dx^2}$ at $x = \frac{1}{2}.$

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5. In what interval $f(x) = \log_5 x$ is decreasing (if any) ?

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6. Write the value of $\int_{-1}^{1} \left(\sin^5 x + 5\right) dx$.

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7. The degree of the differential equation satisfying $\sqrt{1-x^2} + \sqrt{1+y^2} = a(x-y)$ is____.

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8. Write the value of y so that the points (1,y,2),(3,2,-1) and $(-4, 6, 3)$ are collinear.

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9. Write the equation of the plane passing through (1,2,3) and parllel to the plane $x + 2y + 5z = 0$.

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10. Let A = {1, 2, 3). Then, show that the number of relations containing (1, 2) and (2, 3) which are reflexive and transitive but not symmetric is three.

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11. Test whether the relations are reflexive, symmetric or transitive on the sets specified.

R={(m,n):m-n $\geq$ 7) on Z.

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12. Let * be the binary operation on N given by a*b = LCM of a and b. Find

(i) 5*7,20*16

(ii) Is* commutative?

(iii) Is * associative?

(iv) Find the identity of* in N.

(v) Which elements of N are invertible for the operation ?

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13. Show that f:R to R defined as $f(x) = 4x + 3$ is invertible. Find the inverse of 'f' .

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14. Express $A = \begin{bmatrix} x & a & b \\ a & y & c \\ b & c & z \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

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15. Find the inverse of $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ using elementary operations.

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16. Prove that

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3.$$

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17. Find the inverse of the following matrix

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}.$$

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18. Prove that the following.

$$\begin{bmatrix} a & b & c \\ x & y & z \\ p & q & r \end{bmatrix} = \begin{bmatrix} y & b & q \\ x & a & p \\ z & c & r \end{bmatrix} = \begin{bmatrix} x & y & z \\ p & q & r \\ a & b & c \end{bmatrix}$$

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19. Examine the contiunity of the the following functions at indicated points. $f(x)$=

$$\begin{cases} \frac{1}{e^{\frac{1}{x}}-1} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases} \text{ at } x = 0$$

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20. Differentiate : $(\sec x + \tan x)^{\cot x}$.

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21. If $y = e^{ax} \sin bx$ show that $y_2 - 2ay_1 + (a^2 + b^2)y = 0$.

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22. Prove the inequality

$x^2 e^{-x^2} \leq e^{-1}, x \in R.$

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23. Find the interval in which $y = (x-1)^2(x+2)$ is

:

(i) increasing

(ii) decreasing

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24. Evaluate : $\int \frac{dx}{x(x^4+1)}$

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25. Evaluate : $\int \frac{dx}{\sqrt{x^2+8x}}$

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26. Evaluate : $\int_0^{\frac{\pi}{2}} e^x \cos x dx$.

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27. From the differential equation whose general solution is $y = a \sin t + be^t$.

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28. Find the unit vector perpendicular to the plane ABC, where the position vectors of A, B and C are

$2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + \hat{j} + 2\hat{k}$ and $2\hat{i} + 3\hat{k}$, respectively.

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29. Show that $\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right) = 2\left(\vec{a} \times \vec{b}\right)$.

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30. The direction cosines of a straight line in two neighbouring positions are (l,m,n)and< l + deltal,m +deltam,n + deltan>. If dtheta is a small amgle

between them then prove that -

$(\delta\theta)^2 = (\delta l)^2 + (\delta m)^2 + (\delta n)^2$.

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31. Find the equation of the plane Passing through the intersection of the planes $x + 3y - z + 1 = 0$ and $3x - y + 5z + 3 = 0$ and is at a distance $2/3$ units from origin.

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32. Find the equation of the straight line perpendicular to the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-6}{7}$ and lyinng in the plane x - 2y + 4z - 51 = 0.

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33. Exaniming consistency and solvability, solve the following equations by matrix method.

$x + 2y + 3z = 14,$ $2x - y + 5z = 15,$

$2y + 4z - 3x = 13$

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34. Prove

$$\begin{bmatrix} a^3 - x^3 & a^2 & a \\ b^3 - x^3 & b^2 & b \\ c^3 - x^3 & c^2 & c \end{bmatrix} = (a-b)(a-c)(b-c)\left(abc - x^3\right)$$

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35. (i) If $x = \sin t,\ y = \sin 2t$ then prove that

$$\left(1 - x^2\right) d^2 \frac{y}{dx^2} - x\frac{dy}{dx} + 4y = 0.$$

If $y = gof(x)$ then find dy/dx.

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36. Show that the radius of the right cicular cylinder of greatest curved surface that can be inscribed in a given cone is half the redius of the base of the cone.

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37. Find the solution of the differential equation $x \sin\left(\frac{y}{x}\right) dy = \left(y \sin\left(\frac{y}{x}\right) - x\right) dx.$

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38. Find the area of the region included between the parabola $y^2 = 2x$ and the straight line $x - y = 4.$

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39. A variable plane is at a constant distance p from the origin and meets the axes at A,B,C. Through A,B,C plane are drawn parallel to the co-ordinate planes. Show that the locus of their points of intersection is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$.

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40. (i) If the vectors $\vec{a}+\vec{b}+\vec{c}=0$ $\left|\vec{a}\right|=3$, $\left|\vec{b}\right|=5$ and $\left|\vec{c}\right|=7$, find the angle between a and b

(ii) If the vectors 2hat i+3hat j+4hat k and $(a-1)\hat{i}+(b+2)\hat{j}+8\hat{k}$ are parallel then find the values of a and b.

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