MATHS

BOOKS - MODERN PUBLICATION

TEST PAPER 9

Exercise

1. Is the operation * on {0,1,2,3,4,5,6} defined as $a \cdot b = a + b(\mod 7)$ a binary operation ? (Justify)

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2. Evaluate $\tan^{-1} 2 + \tan^{-1} 3$.

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3. Solve the following :

$$\begin{bmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{bmatrix} = 0$$

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4. If $A = \begin{pmatrix} 1 & 0 & 2 \\ 5 & 1 & x \\ 1 & 1 & 1 \end{pmatrix}$ is a singular matrix the, what is the value of x ?

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5. Write the derivative of $y = \sec^{-1}(e^x + 1)$.

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6. Write the set of points, where the function $f(x) = x^3$ has relative (local) extreme.

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7. Write the value of $\int_0^{\pi} \sin^8 x dx$

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8. Write the order and degree of $e^{\frac{dy}{dx}} = x^2$.

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9. In each of the problems given below, find the work done by a force $\vec{F}$ acting on a particle, such that the particle is displaced from a point A to a point B. $\vec{F}$ = $4\hat{i} + 2\hat{j} + 3\hat{k}$ A(1,2,0), B(2,-1,3).

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10. Find the vector equation of the line joining $(2, 1, 3)$ and $(4, -2, 5)$.

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11. Test whether the relations are reflexive, symmetric or transitive on the sets specified.

R={(m,n): $\frac{m}{n}$ is a power of 5} on Z - {0}.

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12. Let A=N $\times$ N and let* be a binary operation on A defined by (a, b) * (c,d) = (ad + bc, bd), $\forall$ (a, b), (c,d) $\in$ N $\times$ N. Show that

(i) * is commutative.

(ii) * is associative.

(iii) A has no identity element.

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13. A factory uses three different respurce for the manufacture of two different products, 20 units of the resource A, 12 units of B and 16 unit of C being available. One unit of the first product requires 2,2 and 4 units of the resources and one unit of the second product requires 4,2 and 0 units of the resources taken

in order. It is known that the first product gives a profit of ₹20 per unit and the second ₹ 30 prt uniy. Formulate the LPP so as to earn maximum profit.

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14. Let A={1,2,3,...,9) and R be the relation on A $\times$ A defined by (a, b) R (c,d), if a+d=b+c for (a, b), (c,d) in A $\times$ A. Prove that R is an equivalence relation and also obtain the equivalence class (2,5).

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15. If

$$A = \begin{bmatrix} 1 & -2 & 2 \\ 3 & 1 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 4 \\ 1 & 2 \\ 3 & -1 \end{bmatrix},$$

then verify that $(AB)^T = B^T A^T$.

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16. Prove that the following. $\begin{bmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{bmatrix}$ =

(b-c)(c-a)(a-b)(bc+ca+ab)

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17. If $A = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$, then find $A^3 - A^2$.

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18. Prove that $\begin{bmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{bmatrix} = 0$

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19. Find "a" and "b" such that the function

$f(x) = \{3ax + b, \quad \text{if} \quad x > 1$

$11 \quad \text{if} \quad x = 1 iscont \in uous.$

$5ax - 2b, \quad \text{if} \quad x < 1$

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20. If $y = \tan^{-1} x$ then prove that $(1 + x^2)y_2 + 2xy_1 = 0.$ Differentiate $\sec^{-1}\left(\frac{1}{2x^2 - 1}\right)$ with respect to sqrt 1-x^2`.

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21. Find the point (S) on the curve

$$x = \frac{3at}{1+t^2}, y = \frac{3at^2}{1+t^2}$$

where the tangent is perependicular to the line 4x+3y+5=0.

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22. Find the intervals in which the function $y = \frac{\ln x}{x}$ is increasing and decreasing.

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23. Evaluate $\int (x+1)\frac{dx}{\sqrt{x^2+2x-3}}\Bigg)$

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24. Evaluate: $\int_0^{2\pi} \frac{dx}{1+e^{\sin x}}$.

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25. Solve: $(1+x^2)\frac{dy}{dx}+2xy-x^3=0$

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26. If the sum of two unit vectors is a unit vectors find the magnitude of their difference.

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27. Prove by vector method that in any ΔABC. $a = b\cos C + c\cos B$.

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28. Find the equation of the plane passing through $(1, 3, 4), (2, 1. \ -1)$ and $(1, \ -4, 3)$.

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29. Obtain the co-ordinates of the foot of the perpendicular drawn from the point $(3, \ -1, 11) \rightarrow thel \in e$x/2=(y-2)/3=(z-3)/4`

Obtain the equation of the perpendicular also.

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30. Let A=R $\times$ R and be the binary operation on A defined by (a, b) *(c,d) = (a + c, b + d). Show that *is commutative and associative. Find the identity element for* on A, if any.

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31. Prove that $f: X \to Y$ is injective iff for all $\subset sA, BofX, f(A \cap B) = f(A) \cap f(B).$

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32. Use matrix product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equation

$x - y + 2z = 1$

$2y - 3z = 1$ and $3x - 2y + 4z = 2$

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33. Prove that the following.

$$\begin{bmatrix} b^2 + c^2 & ab & ac \\ ab & c^2 + a^2 & bc \\ ca & cb & a^2 + b^2 \end{bmatrix} = 4a^2b^2c^2$$

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34. If A+B+C = π, prove that

$$\begin{bmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{bmatrix} = 0$$

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35. Shows that the triangle of greatest area that can be inscribed in a circle is equilateral.

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36. Evaluate the following integrals

$$\int \frac{d\theta}{2 + 3\cos^2\theta - 4\sin^2\theta}$$

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37. Find the equation of the straight line perpendicular to the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-6}{7}$ and lyinng in the plane x - 2y + 4z - 51 = 0.

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