



# MATHS

## BOOKS - SHARAM PUBLICATION

### APPLICATIONS OF DERIVATIVES

#### Example

1. Write the interval in which the function  $\sin^2 x - x$  is increasing.



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2. Write that condition of Rolle's theorem which is violated by the function  $f(x) = |x - 1|$  in  $[0, 2]$ .



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3. or the curve  $y = 3x^2 + 4x$ , find the slope of the tangent to the curve at a point where x-coordinate is  $-2$



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4. What is the acceleration, at the end of 2 s of the particle that moves with rule  $s = \sqrt{t} + 1$  ?



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5. Write the interval in which the function  $\sin^2 x - x$  is increasing.



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6. Write the maximum value of the function

$y = x^5$  in the interval  $[1, 5]$ .



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7. Mention the values of  $x$  for which the

function  $f(x) = x^3 - 12x$  is decreasing,



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8. Find the point on the curves  $x = a(\theta - \sin \theta)$

and  $y = a(1 - \cos \theta)$ , at which the tangent is

parallel to X-axis.



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**9.** If the tangent at each point of the curve  $y = x^3 - ax^2 + x + 1$  is inclined at an acute angle with the positive direction of x-axis then find a.



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10. Find the open interval in which

$f(x) = x^{\frac{1}{x}}, x > 0$  is decreasing.



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11. Find the intervals in which the function

$y = \frac{\ln x}{x}$  is increasing and decreasing.



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12. For which value of  $x$ , the function  $f(x) = 5-6x$  is increasing.



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13. What is the value of  $a$  for which the function  $f(x) = a \sin x + \frac{1}{3} \sin 3x$  has an extremum at  $x = \frac{\pi}{3}$ ?



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14. If  $f(x) = \sin x + 2$  in the interval  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ , what can you say about the greatest value of  $f(x)$ ?



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15. Find interval (s) in which the function  $f(x) = \sin x + \cos x, x \in (0, \pi/2)$  is increasing or decreasing.



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**16.** Find the extreme points of the function

$$y = x + \frac{1}{x}.$$



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**17.** What is the equation of the normal to the

curve  $y = \sqrt{x}$  at the point  $\left(\frac{1}{4}, \frac{1}{2}\right)$ ?



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**18.** Write the equation of the tangent to the curve  $y = |x|$  at the point  $(-2, 2)$ .



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**19.** If the tangent to the curve  $x = at^2, y = 2at$  is perpendicular to the x-axis then what is its point of contact ?



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20. What is the equation of the normal to the curve  $y = \sin x$  at  $(0, 0)$



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21. If  $f(x) = x^3 + ax^2 + bx + 5\sin^2 x$  be an increasing function on the set  $\mathbb{R}$ , then what is the relation between  $a$  and  $b$  ?



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**22.** If  $f$  and  $g$  are two increasing functions then show that  $f \circ g$  is an increasing function.



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**23.** Mention the values of  $x$  for which the function  $f(x) = x^3 - 12x$  is increasing.



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**24.** Write the maximum value of the function

$$y = x^5 \text{ in the interval } [1, 5].$$



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**25.** What is the slope of the normal to the

$$\text{curve } 2y = 3 - x^2 \text{ at the point } (1,1)?$$



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**26.** Find the equation of the tangent to the curve  $x = y^2 - 1$  at the point where the slope of the normal to the curve is 2.



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**27.** Find approximately the difference between the volumes of two cubes of sides 4 cm and 4.03 cm.



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28. Find the open interval in which

$f(x) = x^{\frac{1}{x}}, x > 0$  is decreasing.



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29. Evaluate  $\lim_{x \rightarrow \frac{\pi}{2}} (\sec x - \tan x)$



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30. Find the intervals in which the function

$y = \frac{\ln x}{x}$  is increasing and decreasing.



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**31.** Write the equation of the tangent to the curve  $y = |x|$  at the point  $(-2, 2)$ .



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**32.** Find the intervals in which the function

$y = \frac{\ln x}{x}$  is increasing and decreasing.



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**33.** Write the set of points, where the function

$f(x) = x^3$  has relative (local) extreme.



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**34.** Determine the interval in which

$g(x) = \frac{x^2 + 3x + 3}{x + 1}$  is decreasing.



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**35.** Write slope of the tangent to the curve

$y = \sqrt{3} \sin x + \cos x$  at  $\left[\frac{\pi}{3}, 2\right]$



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**36.** What is the slope of the normal to the curve  $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 20$  at the point  $(8, 64)$ ?



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**37.** Write the x-coordinate of the extreme point of the function  $y = \cos x + \sin x$ ,  $x \in \left[0, \frac{\pi}{2}\right]$



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**38.** For what value of  $x$ , is the function  $f(x) = 3 - 2x^2$  the maximum?



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**39.** Write the subinterval of  $(0, \pi)$  in which  $\sin\left(x + \frac{\pi}{4}\right)$  is increasing.



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40. The curves  $y = 4x^2 + 2x - 8$  and  $y = x^3 - x + 10$  touch each other at the point (3, 4). TRUE or FALSE



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41. Show that the tangent to the curve  $x = a(t - \sin t)$ ,  $y = at(1 + \cos t)$  at  $t = \frac{\pi}{2}$  has slope  $(1 - \frac{\pi}{2})$



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42. Find the approximate value of  $\sqrt{48.96}$



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43. Find the point on the curve

$$x^2 + y^2 - 4xy + 2 = 0$$

where the normal is parallel to the x-axis.



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**44.** Show that  $\sqrt{2} \sin x + x \geq 3x$  all  $x$  in  $(0, \pi/20)$ .



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**45.** Show that the sum of the intercepts on the coordinate axes of any tangent to the curve  $\sqrt{x} + \sqrt{y} = \sqrt{a}$  is constant.



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**46.** Find the interval in which the function  $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$  is strictly increasing.



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**47.** Find the interval in which the function  $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$  is strictly increasing.



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**48.** Prove that:  $y = \frac{4 \sin \theta}{2 + \cos \theta} - \theta$  is an increasing function in  $[0, \pi/2]$



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**49.** Show that the function  $f(x) = x^3 - 3x^2 + 3x, x \in R$  is increasing on  $R$ .



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50. Find the equations of tangent and normal

to the curves  $x = a \sin^3 \theta$  and  $y = a \cos^3 \theta$  at

$$\theta = \frac{\pi}{4}$$



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51. Find the points on curve  $y = x^3 - 11x + 5$

at which equation of the tangent is  $y = x - 11$



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**52.** Find the points on the curve  $x^2 + y^2 - 2x - 3 = 0$  at which tangent is parallel to x-axis.



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**53.** Find the equation of tangent to the curve  $x = \sin 3t, y = \cos 2t$  at  $t = \frac{\pi}{4}$



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**54.** Find the equation of the tangent to the curve  $y = x^4 - 6x^3 + 13x^2 - 10x + 2$  at the point  $(1, 0)$



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**55.** Find the equation of tangent to the curve  $x^2 + 3y = 3$  which is parallel to  $y - 4x + 5 = 0$



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**56.** Find the equation of the normal to the curve  $y = (\log x)^2$  at  $x = \frac{1}{e}$ .



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**57.** The sum of two numbers is 24. Find the numbers, so that their product is maximum.



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**58.** Find two positive numbers whose product is 256 and whose sum is least.



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**59.** Show that all the rectangles with a given perimeter, the square has the largest area.



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**60.** Find the intervals where function is increasing function  $y = \cos x + \sin x, x \in [0, 2\pi]$



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**61.** Show that  $\sqrt{2} \sin x + \tan x \geq 3x$  all  $x$  in  $(0, \pi/20)$ .



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**62.** Find the approximate value of  $\sqrt[6]{63}$ .



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**63.** Find the equation to the tangent and normal to the parabola  $y^2 = 4ax$  at the point  $(at^2, 2at)$ .



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**64.** Show that no two normals to a parabola are parallel.



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**65.** The slope of the curve  $2y^2 = ax^2 + b$  at (1, -1) is -1. Find a and b.



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**66.** Find the equation of the normal to the curve  $y = 2x^2 + 3 \sin x$  at  $x = 0$



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**67.** Determine the point on the curve  $y = \ln x$ , at which the tangent will be parallel to the chord joining the points  $P(1, 0)$  and  $Q(e, 1)$ .



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**68.** Find the extreme values of the function

$$y = X + \frac{1}{x}.$$



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**69.** Find the maximum and minimum value of

$$x + \frac{1}{x}$$



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**70.** Find the intervals in which the function

$$f(x) = 2x^3 + 9x^2 + 12x + 20 \text{ is increasing}$$

and decreasing.



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71. For which value of  $x$ , the function  $f(x) = 4 - x - x^2$  is maximum or minimum.



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72. Determine the sub-interval of  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ , in which  $f(x) = \tan x - 4x$  is increasing.



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**73.** Find the equation of the normal to the curve

$$5x^2 + 3y^2 = 23 \text{ at } (2,-1)$$



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**74.** Find the slope of the tangent to the curve

$$y = (\log_e x)^2 \text{ at } x = \frac{1}{e}$$



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**75.** Find the equation of normal to the curve

$$3y^2 = 16x \text{ at } (3,4).$$



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**76.** Determine the point on the curve  $y = \ln x$ , at which the tangent will be parallel to the chord joining the points  $P(1, 0)$  and  $Q(e, 1)$ .



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77. Find the equation of the tangent to the curve  $x = y^2 - 1$  at the point where the slope of the normal to the curve is 2.



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78. Find approximately the difference between the volumes of two cubes of sides 4 cm and 4.03 cm.



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**79.** Find the set of value of  $x$  where the function  $f(x) = 2x^3 + 3x^2 - 36x - 7$  is increasing or decreasing.



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**80.** Find the value of  $x$  for which the function  $f(x) = x^4 - 4x^3 + 4x^2 - 1$  is maximum or minimum.



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**81.** Find the extreme point of the function

$$f(x) = \sin x \cos x, x \in \left(\frac{\pi}{8}, \frac{\pi}{2}\right).$$



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**82.** Using differential, find approximately the difference between the volumes of two cubes of sides 2 cm and 2.01 cm.



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**83.** Show that the semivertical angle of a cone of given slant height is  $\tan^{-1} \sqrt{2}$  when its volume is maximum.



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**84.** Show that the radius of the right circular cylinder of greatest curved surface that can be inscribed in a given cone is half the radius of the base of the cone.



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**85.** A cylindrical open water tank with a circular base is to be made out of 30 sq metres of metal sheet. Find the dimensions so that it can hold maximum water. (Neglect thickness of sheet).



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**86.** Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is  $\frac{\cos^{-1}(1)}{\sqrt{3}}$ .



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**87.** Show that the height of a closed right circular cylinder of given surface and maximum volume is equal to diameter of base.



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**88.** Shows that the triangle of greatest area that can be inscribed in a circle is equilateral.



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**89.** Find the tangent to the curve  $y = \cos(x + y)$ ,  $0 \leq x \leq 2\pi$  which is parallel to the line  $x + 2y = 0$



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**90.** Find the minimum distance of a point on the curve  $\frac{2}{x^2} + \frac{1}{y^2} = 1$  from the origin.



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**91.** Find the minimum value of  $64 \sec \theta + 27 \csc \theta$  when  $\theta$  lies in  $\left(0, \frac{\pi}{2}\right)$



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**92.** Show that the minimum distance of a point on the curve  $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 1$  from the origin is  $a + b$ .



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**93.** Show that the length of the portion of the tangent to  $x^{2/3} + y^{2/3} = a^{2/3}$  intercepted between the axes is constant.



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**94.** Show that the sum of the intercepts on the coordinate axes of any tangent to the curve  $\sqrt{x} + \sqrt{y} = \sqrt{a}$  is constant.



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**95.** Find the equation of the normal to the curve given by  $x = \cos^3 \theta$ ,  $y = \sin^3 \theta$  at  $\theta = \frac{\pi}{4}$



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**96.** Find the point on the curve  $y^2 - x^2 + 2x - 1 = 0$  where the tangent is parallel to the x-axis.



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**97.** Show that the tangent to the curve  $y = x^2 + 3x - 2$  at (1,2) is parallel to tangent at (-1,1) to the curve  $y = x^3 + 2x$ .



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**98.** Show that the curves  $y = 2^x$  and  $y = 5^x$

intersect at an angle  $\tan^{-1} \left| \frac{1n\left(\frac{5}{2}\right)}{1 + 1n21n5} \right|$ .

Note Angle between two curves is the angle between their tangents at the point of intersection.





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**99.** Find the altitude of a right circular cylinder of maximum volume inscribed in a sphere of radius  $r$ .



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**100.** Find the coordinates of the point on the curve  $x^2y - x + y = 0$  where the slope of the tangent is maximum.





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**101.** Find two numbers  $x$  and  $y$  whose sum is 15 such that  $xy^2$  is maximum.



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**102.** Find the values of  $x$  for which  $f(x) = x^4 + 2x^3 - 2x^2 - 6x + 5$  is locally maximum and minimum.



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**103.** Find the interval where  $y = \sin x - \cos x$   
 $x \in [0, 2\pi]$  is increasing.



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**104.** Find the maximum value of

$$y = (1 + \cos x) \sin x, x \in \left[0, \frac{3\pi}{4}\right]$$



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**105.** Obtain the extreme point of  $f(x) = e^x(x^2 - 6x + 9)$ . As certain whether they are maximum or minimum points. Find the extreme values at these points.



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**106.** Find the maximum of the function

$$f(x) = \left(\frac{1}{x}\right)^x$$



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**107.** If  $a$  is positive, find the minimum value of

$$\frac{a + x}{\sqrt{ax}}.$$



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**108.** Use the function  $f(x) = x^{1/x}$ ,  $x > 0$  to show that  $e^\pi > \pi^e$ .



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**109.** Mention the values of  $x$  for which the function  $f(x) = x^2 - 12x$  is increasing.



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**110.** Find the interval for which the function  $f(x) = \tan x - 4(x - 2)$  is increasing and decreasing in  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ .



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**111.** What is the value of  $a$  for which the function  $f(x) = a \sin x + \frac{1}{3} \sin 3x$  has an extremum at  $x = \frac{\pi}{3}$ ?



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**112.** What is the maximum value of the function  $f(x) = \sin x(1 + \cos x)$ ?



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**113.** If  $f(x) = a \ln x + bx^2 + x$  has extreme values at  $x = -1$  and  $x = 2$  then find a and b.



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**114.** Find the coordinates of a point on the parabola  $y^2 = 8x$  which is at minimum distance from the circle  $x^2 + (y + 6)^2 = 1$



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**115.** Find the difference between the greatest and least values of the function

$$f(x) = \cos x + \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x$$



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**116.** If  $f(x) = a \ln x + bx^2 + x$  has extreme values at  $x = -1$  and  $x = 2$  then find  $a$  and  $b$ .



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**117.** Find the approximate value of  $(26.9)^{\frac{1}{3}}$



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**118.** Discuss the extreme value of the function

$$y = (x + 2)^4(x - 1)^5$$



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