

MATHS

BOOKS - SHARAM PUBLICATION

MODEL QUESTION PAPER -16

Exercise

1. If $f: R \rightarrow R$ and $g: R \rightarrow R$ are given by $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$, then write fog.



Watch Video Solution

2. Write the principal value of

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right).$$



Watch Video Solution

3. If $[2x \ 4] \begin{bmatrix} x \\ -8 \end{bmatrix} = 0$ then find the positive value of x .



Watch Video Solution

4. Find $\frac{d}{dx} \ln \sin^{-1} \cos \left(\frac{\pi - 2e^x}{2} \right)$.



[Watch Video Solution](#)

5. Find the open interval in which $f(x) = x^{\frac{1}{x}}$, $x > 0$ is decreasing.



[Watch Video Solution](#)

6. What is the value of $\int \frac{1 + \frac{1}{x^2}}{x - \frac{1}{x} + 4} dx$



[Watch Video Solution](#)

7. Write the particular solution of $\frac{dy}{dx} = 8x$,
given that $y=2$, when $x=1$.



[Watch Video Solution](#)

8. If $\vec{a} = 3\hat{i} + 3\hat{j} + \hat{k}$ and
 $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$ then what is the unit
vector parallel to $\vec{a} + \vec{b}$



[Watch Video Solution](#)

9. Find the value of k for which the line $\frac{x - 2}{3} = \frac{1 - y}{k} = \frac{z - 1}{4}$ is parallel to the plane $2x + 6y + 3z - 4 = 0$.



[Watch Video Solution](#)

10. If $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined as $f(x) = 10x + 7$. Find the function $g: \mathbb{R} \rightarrow \mathbb{R}$, such that $g \circ f = f \circ g = I_{\mathbb{R}}$.



[Watch Video Solution](#)

11. Show that the relation R on the set A of real numbers defined as $R = \{(a,b): a \leq b\}$ is reflexive, and transitive but not symmetric.



[Watch Video Solution](#)

12. If S is a set of all rational numbers except 1 and $*$ be defined on S by $a * b = a + b - ab$ for all $a, b \in S$ then prove that $*$ is a binary operation.



[Watch Video Solution](#)

13. If S is a set of all rational numbers except 1 and $*$ be defined on S by $a * b = a + b - ab$ for all $a, b \in S$ then prove that $*$ is commutative as well as associative.



[Watch Video Solution](#)

14. Prove that

$$\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18 = \cot^{-1} 3$$



[Watch Video Solution](#)

15. Solve the following LPP graphically

$$\text{Maximize, } Z = 20x + 30y$$

$$\text{Subject to } 3x + 5y \leq 15$$

$$x, y \geq 0.$$



Watch Video Solution

16. Verify that $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

satisfies the equation

$$A^2 - (a + d)A + (ad - bc)I = 0 \text{ where } I \text{ is}$$

the 2×2 unit matrix.



Watch Video Solution

17. If the matrix A is such that

$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} A = \begin{bmatrix} -4 & 1 \\ 7 & 7 \end{bmatrix}, \text{ find } A.$$



Watch Video Solution

18. Show that $(a + 1)$ is a factor of

$$\begin{vmatrix} (a + 1) & 2 & 3 \\ 1 & a + 1 & 3 \\ 3 & -6 & a + 1 \end{vmatrix}$$



Watch Video Solution

19. Solve :
$$\begin{vmatrix} x - a & 0 & 0 \\ a & x - b & 0 \\ a & b & x - c \end{vmatrix} = 0$$



[Watch Video Solution](#)

20. For what value of k is the function defined

by
$$\begin{cases} k(x^2 + 2) & \text{when } x \leq 0 \\ 3x + 1 & \text{when } x > 0 \end{cases}$$
 .Continuous at x

$= 0$. Also write whether the function is

continuous at $x = 1$.



[Watch Video Solution](#)

21. If $\sin y = x \sin (a + y)$ then show that

$$\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$$



Watch Video Solution

22. Find $\frac{dy}{dx}$ when $y = x^{\sin y}$



Watch Video Solution

23. Find the interval in which the function

$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is strictly

increasing.



Watch Video Solution

24. Find two numbers whose sum is 24 and product is maximum.



Watch Video Solution

25. Integrate: $\int x \frac{e^x}{(1+x)^2} dx$



Watch Video Solution

26. Integrate : $\int \frac{\tan^{-1} x}{1 + x^2} dx$

 [Watch Video Solution](#)

27. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

 [Watch Video Solution](#)

28. Find the area bounded by the line $y = 2x$, x -axis and the ordinate $x = 3$.

 [Watch Video Solution](#)

29. Solve $2y^3 \frac{dy}{dx} = ax$



Watch Video Solution

30. Find $\left[\begin{matrix} \vec{a} & \vec{b} & \vec{c} \end{matrix} \right]$ when
 $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{c} = \hat{j} + \hat{k}$



Watch Video Solution

31. If the position vectors of the points A, B, C are $2\hat{i} + \hat{j} - \hat{k}$, $3\hat{i} - 2\hat{j} + \hat{k}$ and $\hat{i} + 4\hat{j} - 3\hat{k}$ respectively, then prove that A, B, C are collinear.



Watch Video Solution

32. Write the volume of the parallelepiped whose sides are given by $-\hat{j}, \hat{k}, -\hat{i}$



Watch Video Solution

33. The angle between the plane $3x + 3z - 5 = 0$ and the line $\frac{x - 1}{1} = \frac{y - 2}{-1} = \frac{z - 3}{0}$ is.



Watch Video Solution

34. Find the equation of the plane passing through the intersection of the planes $3x + y - z = 2$ and $x - y + 2z = 1$ and the point $(1, 0, 2)$



Watch Video Solution

35.

Prove

that

$$\left(\vec{a} \cdot \vec{b}\right)^2 = a^2 b^2 - \left(\vec{a} \times \vec{b}\right)^2$$

[Watch Video Solution](#)**36.**

Show

that

the

lines

$$\frac{x - 4}{1} = \frac{y + 3}{-4} = \frac{z + 1}{7}$$

and

$$\frac{x - 1}{2} = \frac{y + 1}{-3} = \frac{z + 10}{8}$$

are coplaner. Find

their point of intersection.

[Watch Video Solution](#)

37. If $x = \frac{1 - \cos^2 \theta}{\cos \theta}$, $y = \frac{1 - \cos^{2n} \theta}{\cos^n \theta}$ then
show that $\left(\frac{dy}{dx}\right)^2 = n^2 \left(\frac{y^2 + 4}{x^2 + 4}\right)$



Watch Video Solution

38. Find the altitude of a right circular cylinder of maximum volume inscribed in a sphere of radius r .



Watch Video Solution

39. Evaluate the following integrals

$$\int \frac{12 \sin x - 2 \cos x + 3}{\sin x + \cos x} dx$$



Watch Video Solution

40. Determine the area common to the parabola $y^2 = x$ and the circle $x^2 + y^2 = 2x$.



Watch Video Solution

41. Solve the following differential equations

$$\frac{dy}{dx} + 2y \tan x = \sin x, y\left(\frac{\pi}{3}\right) = 0$$



Watch Video Solution

42. If $A = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 3 \\ -3 & 1 \end{bmatrix}$ then

show that $(A + B)^2 \neq A^2 + 2AB + B^2$.



Watch Video Solution

43. If $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 5x - 8$ for all $x \in \mathbb{R}$, then show that f is invertible. Find the corresponding inverse function.



[Watch Video Solution](#)

44. Prove statement

$$\sin^{-1} \frac{3}{5} + \cos^{-1} \frac{12}{13} = \cos^{-1} \frac{33}{65}$$



[Watch Video Solution](#)

45. Solve the following LPP graphically:

Maximize: $Z = 4x_1 + 3x_2$ subject to

$$x_1 + x_2 \leq 50, x_1 + 2x_2 \leq 80, 2x_1 + x_2 \geq 20,$$

$$x_1, x_2 \geq 0$$



Watch Video Solution