



# MATHS

## BOOKS - SHARAM PUBLICATION

### MODEL QUESTION PAPER 14

#### Exercise

1. If  $f: R \rightarrow R$  and  $g: R \rightarrow R$  is defined by  $f(x) = \sin x$  and  $g(x) = 5x^2$ , then  $(g \circ f)(x)$ .



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2. If  $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$  then what is  $\cos^{-1} x + \cos^{-1} y$



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3. Determine the maximum value of

$$\left| \begin{array}{cc} \cos x & \sin x \\ -\sin x & \cos x - 1 \end{array} \right|.$$



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4. If  $A$  is a  $4 \times 5$  matrix and  $B$  is a matrix such that

$A^T B$  and  $BA^T$  both are defined, then write the

order of B.



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5. What is the derivative of  $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$  w.r.t  $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ ?



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6. What is the value of  $a$  for which the function

$f(x) = a \sin x + \frac{1}{3} \sin 3x$  has an extremum at

$x = \frac{\pi}{3}$ ?



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7. Evaluate  $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

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8. Form the differential equation, whose solution is

$$y = e^{x+a}$$

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9. If  $\left(\vec{a} \times \vec{b}\right)^2 + \left(\vec{a} \cdot \vec{b}\right)^2 = 144$ , write the value of  $ab$ .



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10. What are the direction cosines of the normal to the plane  $3x - 2y - 2z + 1 = 0$ ?



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11. Let  $f : W \rightarrow W$  be defined as  $f(x) = x - 1$  if  $x$  is odd and  $f(x) = x + 1$  if  $x$  is even then show that  $f$  is invertible. Find the inverse of  $f$  where  $W$  is the set of all whole numbers.



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12. Prove the  $\sin^2(\sin^{-1} x + \sin^{-1} y + \sin^{-1} z)$   
 $= \cos^2(\cos^{-1} x + \cos^{-1} y + \cos^{-1} z)$



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13. Solve for x

$$\tan^{-1}(x + 2) + \tan^{-1}(x - 2) = \tan^{-1}\left(\frac{8}{79}\right), x > 0$$



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14. Solve the following LPP graphically

$$\text{Maximize, } Z = 20x + 30y$$

$$\text{Subject to } 3x + 5y \leq 15$$

$$x, y \geq 0.$$



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15. Find  $x$ , so that 
$$\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & 1 \\ 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix} = O$$



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16. If  $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$  then prove that

$$A^2 - 4A - 5I = 0.$$



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17. Prove that:  $\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix}$  is a perfect square.



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18. Solve for  $x$ ,

$$\begin{vmatrix} 15 - 2x & 11 & 10 \\ 11 - 3x & 17 & 16 \\ 7 - x & 14 & 13 \end{vmatrix} = 0$$



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19. If  $\sin y = x \sin (a + y)$  then show that

$$\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$$



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20. If  $x = a(\cos t + t \sin t)$  and

$y = a(\sin t - t \cos t)$ , then find  $\frac{d^2y}{dx^2}$  at  $t = \frac{\pi}{4}$ .



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21. Find the equation of tangent to the curve

$x^2 + 3y = 3$  which is parallel to  $y - 4x + 5 = 0$



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22. Find the extreme values of the function

$$y = X + \frac{1}{x}.$$

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23. 
$$\int_0^1 \frac{x^5(4-x^2)}{\sqrt{1-x^2}} dx$$

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24. Integrate the following 
$$\int \frac{\sec^2 \sqrt{x}}{\sqrt{x}} dx$$

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25. 
$$\int x^2 \tan^{-1} x dx$$

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26. Solve  $\frac{dy}{dx} + y = e^{-x}$



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27. Using vector method find the area of the triangle with vertices  $(1, 0, 0)$   $(0, 1, 0)$  and  $(0, 0, 1)$



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28. Find  $\begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix}$  when

$$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{c} = \hat{j} + \hat{k}$$

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29. Find the image of the point  $(2, -1, 3)$  in the plane  $3x - 2y + z - 9 = 0$

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30. Find the point where the line  $\frac{x - 2}{1} = \frac{y}{-1} = \frac{z - 1}{2}$  meets the plane  $2x + y + z = 2$ .

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**31.** Find the equation of the plane through the points  $(1, 2, -3)$ ,  $(2, 3, -4)$  and perpendicular to the plane  $x + y + z + 1 = 0$ .



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**32.** Show that the relation  $S$  defined on set  $N \times N$  by  $(a, b)S(c, d) \Rightarrow a + d = b + c$  is an equivalence relation.



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33. If  $\sin^{-1}\left(\frac{x}{a}\right) + \sin^{-1}\left(\frac{y}{b}\right) = \sin^{-1}\left(\frac{c^2}{ab}\right)$ ,

then prove that

$$b^2x^2 + 2xy\sqrt{a^2b^2 - c^4} + a^2y^2 = c^4$$



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34. Solve the following LPP graphically Maximize

$$z = 5x_1 + 3x_2$$

Subject to

$$3x_1 + 5x_2 \leq 15, 5x_1 + 2x_2 \leq 10, x_1, x_2 \geq 0.$$



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**35.** Show that

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$



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**36.** Examining consistency and solvability, solve the following equation by matrix method.

$$x+y+z=4$$

$$2x-y+3z=1$$

$$3x+2y-z=1$$



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37. Find the point on the curve  $y^2 - x^2 + 2x - 1 = 0$  where the tangent is parallel to the x-axis.



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38. Find the maximum of the function

$$f(x) = \left(\frac{1}{x}\right)^x$$



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39. Find  $\int x^2 (\sin^4 x + \cos^4 x) dx$



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40. solve:  $(1 + y^2)xdx + (1 - x^2)ydy = 0$



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41. Prove that the two lines whose direction cosines are connected by the equations  $l + 2m + 3n = 0$ ,  $3lm - 4ln + mn = 0$  are perpendicular to each other.



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