



MATHS

BOOKS - SHARAM PUBLICATION

MODEL QUESTION PAPER 20

Exercise

1. If $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ a symmetric relation on the set $\{1, 2, 3, 4\}$. The relation R is

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2. What is the principal value of $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$?

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3. Show that feasible region for the following constraints in a graph

$$2x + y \leq 4, x \geq 0, y \geq 0.$$

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4. If ω is a complex cube root of 1, then for what value of λ the

determinant
$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \lambda & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0?$$

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5. If $y = \tan^{-1} \sqrt{1 + x^2}$ then find $\frac{dy}{dx}$.

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6. Find the open interval in which $f(x) = x^{\frac{1}{x}}, x > 0$ is decreasing.

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7. Evaluate $\int \cot\left(\frac{x}{3}\right) dx$.

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8. What is the general solution of the differential equation $\frac{dy}{dx} = \frac{y^2}{x^2}$.

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9. Find the scalar projection of the vector $2\hat{i} + \hat{j} + 5\hat{k}$ on the vector $2\hat{i} - \hat{j} + 3\hat{k}$.

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10. Find the direction cosines of the line $\frac{x-4}{3} = \frac{y-2}{6} = \frac{z-1}{3}$.

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11. Evaluate $\int \tan 3x dx$.

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12. Prove that the following.
$$\begin{bmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{bmatrix}$$

 $= abc(1+1/a+1/b+1/c)$

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13. Find the value of $\begin{vmatrix} 17 & 58 & 97 \\ 19 & 60 & 99 \\ 18 & 59 & 98 \end{vmatrix}$ without expanding.

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14. If $A = \begin{bmatrix} 0 & -\tan\left(\frac{\alpha}{2}\right) \\ \tan\left(\frac{\alpha}{2}\right) & 0 \end{bmatrix}$ show that

$(I + A) = (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$ where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

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15. Find the inverse of the matrix $\begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$.

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16. $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$ at $x = 0$

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17. Find $\frac{dy}{dx}$ when $y = e^{x-y}$.

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18. If $y = 2^{x^2} + \tan^{-1} 2x$ then find $\frac{dy}{dx}$.

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19. Find $\frac{dy}{dx}$ when $\sin y = \cos(x + y)$.

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20. Find the intervals in which the function $y = \frac{\ln x}{x}$ is increasing and decreasing.

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21. Integrate $\int e^x \tan e^x dx$.

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22. Evaluate $\int \frac{e^x - 1}{e^x + 1} dx$

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23. Evaluate $\int_0^{\pi/4} \log(1 + \tan x) dx$.

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24. Find the area of the region bounded by the curve $y = 6x - x^2$, the X-axis and the two ordinates $x = 0$ and $x = 9$.

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25. Solve $(x \log x) \frac{dy}{dx} + y = 2 \log x$.

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26. Show that the relation R defined on the set Z of all integers defined as $R = \{(x, y) : x - y \text{ is an integer}\}$ is reflexive, symmetric and transitive.

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27. If $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. Consider the function $f: A \rightarrow B$ defined by

$f(x) = \frac{x - 2}{x - 3}$, for all $x \in A$. Then, show that f is bijective. Find

$f^{-1}(x)$.

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28. Prove that $2 \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{31}{17}\right)$.

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29. Find the feasible region of the system.

$$2y - x \geq 0, 6y - 3x \leq 21, x \geq 0, y \geq 0$$



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30. Prove by vector method that in a ΔABC , $c^2 = a^2 + b^2 - 2ab \cos C$.



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31. The diagonals of a parallelogram are given by $\vec{a} = 2\hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{b} = -2\hat{i} + 2\hat{j} + 2\hat{k}$, Determine the area of the parallelogram .



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32. Find the image of the point (2, 3, 4) with respect to the plane $x - y + 2z = 4$. Obtain the foot of the perpendicular from P on the plane and the corresponding perpendicular distance.



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33. Find the equation of planes passing through the points $(1, 2, 3)$, $(1, -4, 3)$ and $(-1, 3, 2)$

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34. Prove that the lines $x = ay + b, z = cy + d$ and $x = a'y + b', z = c'y + d'$ are perpendicular, if $aa' + cc' + 1 = 0$

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35. Show that $f: N \rightarrow N$, given by

$$f(x) = \begin{cases} x + 1, & \text{if } x \text{ is odd} \\ x - 1, & \text{if } x \text{ is even} \end{cases}$$

is bijective (both one-one and onto).

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36.

If

$$\cos^{-1}\left(\frac{x}{a}\right) = \cos^{-1}\left(\frac{y}{b}\right) = \theta, \text{ prove that } \frac{x^2}{a^2} - \frac{2xy}{ab}\cos\theta + \frac{y^2}{b^2} = \sin^2\theta.$$

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37. Solve the LPP Maximize $z = 5x + 3y$

Subject to $3x + 5y \leq 15$

$5x + 2y \leq 10, x \geq 0, y \geq 0$

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38. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ then find the value of $A^2 - 3A + 2I$

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39. If a, b and c are all positive real, then prove that minimum value of determinant

$$\begin{vmatrix} a^2 + 1 & ab & ac \\ ab & b^2 + 1 & bc \\ ac & bc & c^2 + 1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

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40. Find the value of k so that the function f defined by $f(x) = \left\{ \left(\frac{k \cos x}{\pi - 2x} \right), \text{ when } x \neq \frac{\pi}{2}, \left(0, \text{ at } x = \frac{\pi}{2} \right) : \right\}$ is continuous at $x = \frac{\pi}{2}$.

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41. If $y = x^{\cot x}$ then find $\frac{dy}{dx}$.

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42. Show that the rectangle of maximum area that can be inscribed in a given circle is a square.

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43. Evaluate $\int_0^{\pi} \frac{x}{1 + \sin x} dx$

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44. Find the area of region in the first quadrant enclosed by the X-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

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45. Find the solution of the following differential equations:

$$(2x+y+1)dx+(4x+2y-1)dy=0$$

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46. Three vectors \vec{a} , \vec{b} and \vec{c} satisfy the condition $\vec{a} + \vec{b} + \vec{c} = 0$.

Find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ if

$$|\vec{a}| = 1, |\vec{b}| = 4, |\vec{c}| = 2.$$



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