



## MATHS

### BOOKS - SHARAM PUBLICATION

### MODEL QUESTION PAPER-4

#### Exercise

1. Write the largest and smallest relations on a set

$$A = \{a, b, c\}.$$



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2. Write the value of  $\tan^{-1} \left[ 2 \sin \left( 2 \cos^{-1} \left( \frac{\sqrt{3}}{2} \right) \right) \right]$

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3. Is the matrix  $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & -1 & 1 \end{bmatrix}$  invertible?

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4. Differentiate  $\tan^{-1} \left( \frac{\sqrt{1-x^2}}{x} \right)$  w.r.t  $\sin^{-1} \left( \frac{2x}{1+x^2} \right)$ .

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5. An edge of a variable cube is increasing at the rate of  $10\text{cm} / \text{sec}$ . Find the rate of increase of the volume of the cube when the edge is 5 cm long.

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6. Evaluate

$$\int_{-1}^3 \left[ \tan^{-1} \left( \frac{x}{x^2 + 1} \right) + \tan^{-1} \left( \frac{x^2 + 1}{x} \right) \right] dx.$$

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7. If  $p$  and  $q$  are the order and degree of the differential equation

$y\left(\frac{dy}{dx}\right)^2 + x^2\frac{d^2y}{dx^2} + xy = \sin x$ , then choose the correct statement out of

$$p < q$$

A.  $p > q$

B.  $p = q$

C.  $p < q$

D.

**Answer:**



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8. Find the projection of the vector  $\hat{i} + 3\hat{j} + 7\hat{k}$  on the vector  $2\hat{i} - 3\hat{j} + 6\hat{k}$ .



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9. If the cartesian equation of a line is  $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$  then write the corresponding vector equation of the line.



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10. Show that the relation  $S$  defined on set  $N \times N$  by  $(a, b)S(c, d) \Rightarrow a + d = b + c$  is an equivalence

relation.



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11. If  $A = \mathbb{R} - \{3\}$  and  $B = \mathbb{R} - \{1\}$ . Consider the function  $f: A \rightarrow B$  defined by  $f(x) = \frac{x - 2}{x - 3}$ , for all  $x \in A$ . Then, show that  $f$  is bijective. Find  $f^{-1}(x)$ .



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12. Prove that

$$2 \tan^{-1} \left( \frac{1}{2} \right) + \tan^{-1} \left( \frac{1}{7} \right) = \tan^{-1} \left( \frac{31}{17} \right).$$



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13. Find the feasible region of the following system.

$$2y - x \geq 0, 6y - 3x \leq 0, x \geq 0, y \geq 0.$$

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$$14. f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases} \text{ at } x = 0$$

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15. Find  $\frac{dy}{dx}$  when  $y^x = e^{x-y}$ .

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16. Differentiate with respect to  $x$  :

$$Y = 2^{x^2} + \tan^{-1} \left( \frac{\cos x - \sin x}{\cos x + \sin x} \right)$$

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17. Find the equation of the tangent to the curve  $x = y^2 - 1$  at the point where the slope of the normal to the curve is 2.

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18. Find the intervals in which the function  $y = \frac{\ln x}{x}$  is increasing and decreasing.



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19. Prove by vector method that in a  $\Delta ABC$ ,  $c^2 = a^2 + b^2 - 2ab \cos C$ .

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20. The diagonals of a parallelogram are given by  $\vec{a} = 2\hat{i} - 3\hat{j} + 5\hat{k}$  and  $\vec{b} = -2\hat{i} + 2\hat{j} + 2\hat{k}$ ,

Determine the area of the parallelogram .

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**21.** Find the image of the point  $(2, 3, 4)$  with respect to the plane  $x - y + 2z = 4$ . Obtain the foot of the perpendicular from  $P$  on the plane and the corresponding perpendicular distance.



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**22.** Find the equation of planes passing through the points  $(1, 2, 3)$ ,  $(1, -4, 3)$  and  $(-1, 3, 2)$



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**23.** If  $A$  and  $B$  are square matrices of same order, then show by means of an example that  $AB \neq BA$  in general.

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**24.** Prove that the following.

$$\begin{bmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{bmatrix}$$

$= abc(1+1/a+1/b+1/c)$

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25. Find the value of  $\begin{vmatrix} 17 & 58 & 97 \\ 19 & 60 & 99 \\ 18 & 59 & 98 \end{vmatrix}$  without expanding.

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26. Find the inverse of  $\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ .

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27. Evaluate  $\int \tan^{-1} \sqrt{\frac{1-x}{1+x}} dx$

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28. Evaluate  $\int \frac{e^x - 1}{e^x + 1} dx$



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29. Evaluate  $\int_0^{\pi/4} \log(1 + \tan x) dx$ .



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30. Find the area of the region bounded by the curve  $y = 6x - x^2$ , the X-axis and the two ordinates  $x = 0$  and  $x = 9$ .



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**31.** Solve the following differential equations

$$x \log x \frac{dy}{dx} + y = 2 \log x$$



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**32.** Find the value of  $k$  so that the function  $f$  defined

by

$$f(x) = \left\{ \left( \frac{k \cos x}{\pi - 2x} \right), \text{ when } x \neq \frac{\pi}{2}, \left( 0, \text{ at } x = \frac{\pi}{2} \right) : \right\}$$

is continuous at  $x = \frac{\pi}{2}$ .



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**33.** Find  $\frac{dy}{dx}$  if  $x^y = y^x$ .



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**34.** Show that the rectangle of maximum area that can be inscribed in a given circle is a square.



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**35.** Evaluate  $\int_0^{\pi} \frac{x}{1 + \sin x} dx$



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**36.** Find the area of region in the first quadrant enclosed by the X-axis, the line  $y = x$  and the circle

$$x^2 + y^2 = 32.$$



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**37.** Solve the following differential equation

$$(2x + y + 1)dx + (4x + 2y - 1)dy = 0.$$



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**38.** Show that  $f: N \rightarrow N$ , given by

$$f(x) = \begin{cases} x + 1, & \text{if } x \text{ is odd} \\ x - 1, & \text{if } x \text{ is even} \end{cases}$$

is bijective (both one-one and onto).



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39. If  $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$  then find the value of  $A^2 - 3A + 2I$

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40. If  $a, b$  and  $c$  are all positive real, then prove that minimum value of determinant

$$\begin{vmatrix} a^2 + 1 & ab & ac \\ ab & b^2 + 1 & bc \\ ac & bc & c^2 + 1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

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41. Three vectors  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  satisfy the condition

$\vec{a} + \vec{b} + \vec{c} = 0$ . Find the value of

$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$  if

$$|\vec{a}| = 1, |\vec{b}| = 4, |\vec{c}| = 2.$$



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