MATHS

BOOKS - SHARAM PUBLICATION

RELATIONS AND FUNCTIONS

Example

1. Write down all the partitions of the set {a,b,c}.

Watch Video Solution

2. Write the domain of the function defined by

f(x)=$\sin^{-1}x+\cos x$

Watch Video Solution

3. A R is a relation on set A such that $R=R^{-1}$,

then write the type of the relation R.

Watch Video Solution

4. Write relations in tabular form and determine their type for $R = \{(x, y) : 2x - y = 0\} on A = \{1, 2, 3, \ldots, 13\}$

Watch Video Solution

5. Sets A and B have respectively m and n elements. The total number of relations from A to B is 64. If $m < n$ and $m \neq 1$, write the values of m and n respectively.

Watch Video Solution

6. Show that the two sets {1, 2, 3,.......} and {3, 4, 5, ...} are equivalent.

Watch Video Solution

7. Test whether the relation : $R = \{(m, n) : 2 \mid (m+n)\}$ on $\mathbb{Z}$ is reflexive, symmetric or transitive.

Watch Video Solution

8. Show that if R is an equivalence relation on X, then Dom R = Rng R = X.

Watch Video Solution

9. If $R = \{(a, a^3)$: a is prime number less than 5} be a relation. Find the range of R.

Watch Video Solution

10. Find the least positive integer r such that $185 \in [r]_7$

Watch Video Solution

11. Let R is the equivalence in the set A = {0, 1, 2, 3, 4, 5} given by R = {(a, b) : 2 divides (a - b)}. Write the equivalence class [0].

Watch Video Solution

12. Is φ an equivalence relaiton on any set?

Watch Video Solution

13. If A={1, 2, 3, 4, 5} and $R: A \rightarrow A$ is {(1,2), (2,3), (4,5), (3,3)} then write $R^{-1}: A \rightarrow A$.

Watch Video Solution

14. What is the range of the function

$$f(x) = \frac{|x-1|}{x-1}, x \neq 1?$$

Watch Video Solution

15. State the reason for the relation R in the set {1, 2, 3} given by R ={(1, 2), (2, 1)} not to be transitive.

Watch Video Solution

16. If $R = \{(x, y) : x + 2y = 8\}$ is a relation on N, then write the range of R.

Watch Video Solution

17. If a set A has n elements and another set B has m elements, what is the number of relations from A to B ?

Watch Video Solution

18. If a set A has n elements and another set B has m elements, what is the number of relations from A to B ?

Watch Video Solution

19. Write the relation $R = \{(x, x^3) : x$ is a prime numeber less than 10} is roaster form.

Watch Video Solution

20. Let A={1,2}, B={1,2,3,4}: Write down the elements of $A \times B$.

Watch Video Solution

21. Let A={1,2}, B={1,2,3,4}:How many relations will be there from A to B.

Watch Video Solution

22. If A={1, 2, 3, 4, 5, 6} and a relation R on A is defined by R={(a, b):$a, b \in A$ and b is exactly divisible by a} then write R is roster form.

Watch Video Solution

23. Let A={1, 2, 3, 5}, B={4, 6, 9}, A relation R form A to B is defined by $R = \{(x, y) : x \in A, y \in B$ and $x - y$ is odd}.write R in roster form.

Watch Video Solution

Watch Video Solution

24. Write the equivalence class $[3]_7$ as a set.

Watch Video Solution

25. Find $[2]_{10} \cap [1]_{13}$

Watch Video Solution

26. If

$f:\{1, 3, 4\} \rightarrow \{1, 2, 5\}$ and $g:\{1, 2, 5\} \rightarrow \{1, 3\}$

given by f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}. Write down gof.

Watch Video Solution

27. Let A={1, 2, 3} and let the relation R={(1, 2), (2, 3)} what is the minimum number of order pairs introduced to R ot make it an equivalence relation.

Watch Video Solution

28. If A = {1, 2, 3}, B = {4, 5, 6, 7} and f = {(1, 4), (2, 5), (3, 6)} is a function from A to B. State whether f is one-one or not.

Watch Video Solution

29. If $f: R \rightarrow R$and $g: R \rightarrow R$ are given by $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$, then write fog.

Watch Video Solution

30. If $f(x) = \left(1 - x^3\right)^{\frac{1}{3}}$ then find $fof(x)$.

Watch Video Solution

31. If $f: R \rightarrow R$ and $g: R \rightarrow R$ is defined by f(x)=sinx and $g(x) = 5x^2$, then (gof)(x).

Watch Video Solution

32. If f: $R \rightarrow R$ and $g: R \rightarrow R$ is given by $f(x) = |x|$ and $g(x) = |5x - 2|$ then write fog(x).

Watch Video Solution

33. If f: R $\rightarrow$ R is defined by f(x) = 3x + 2 define

f (f (x))

Watch Video Solution

34. If the fuction $f: R \rightarrow R$ defined by f(x)=3x-4

is invertible, then find f^{-1}.

Watch Video Solution

35. If $f: R \rightarrow R$ defined by $f(x) = \frac{3x+5}{2}$ is an invertible function, then find $f^{-1}(x)$.

Watch Video Solution

36. State whether the function $f: N \rightarrow N$ defined by f(x)=5x is injective, surjective or both.

Watch Video Solution

37. If $f(x) = x^3$ and $g(x) = x^{\frac{1}{3}}$. Then find gof(x):

Watch Video Solution

38. If R be a relation from the set A to the set B, then-

A. $R = A \cap B$

B. $R = A \cup B$

C. $R \subseteq A \times B$

D. $R \subseteq B \times A$

Answer:

Watch Video Solution

39. If A={1, 2, 3, 4, 5} then the relation R={(1,1), (2, 2), (3, 3), (4, 4), (1, 2), (2,3)} is

A. reflexive

B. symmetric

C. transitive

D. none of these.

Answer:

Watch Video Solution

40. If A be non-empty set of children in a family then the relation "a is a brother of b" on A is-

A. reflexive

B. symmetric

C. transitive

D. none of these.

Answer:

Watch Video Solution

41. If a set A has n elements and another set B has m elements, what is the number of relations from A to B ?

A. 2^{mn}

B. $2^{mn}-1$

C. $2m^n$

D. m^n

Answer:

Watch Video Solution

42. If R be a relation on a finite set A having n elements, then the number of relations on A is-

A. 2^n

B. 2^{n^2}

C. n^2

D. n^n

Answer:

Watch Video Solution

43. If R be the largest equivalence relation on a set A and S is any relation on A then

A. $R \subset S$

B. $S \subset R$

C. R=S

D. none of these.

Answer:

Watch Video Solution

44. If $n(A)=4$ and $n(B)=6$ then the number of one-one function from A to B is-

A. 360

B. 370

C. 380

D. 390

Answer:

Watch Video Solution

45. If $f(x) = \cos \log_e^x$ then $f(x) \cdot f(y) - \frac{1}{2}\left[f(xy) + f\left(\frac{x}{y}\right)\right] =$

A. 0

B. $\frac{1}{2} f(x) f(y)$

C. f(x+y)

D. none of these.

Answer:

Watch Video Solution

46. If the mapping is $f: R \to R$ given by $f(x) = 4x^3 - 12x$ then image of the interval [-1, 3]is -

A. [8,72]

B. [-8,72]

C. [0,8]

D. none of these.

Answer:

Watch Video Solution

47. If $f(x) = (a - x^n)^{\frac{1}{n}}$ where a>0 and $n \in N$ then fof(x) is equal to-

A. x

B. n

C. x^n

D. a^n

Answer:

Watch Video Solution

48. If $f: R \to R$ be a function defined by f(x)=cos(5x+2), then f is

A. injective

B. surjective

C. bijective

D. none of these.

Answer:

Watch Video Solution

49. Sets A and B have respectively m and n elements. The total number of relations from set A to set B is 64. If $m < n$ and $m \neq 1$, write the values of m and n, respectively.

A. $m \leq n$ is-

B. n^m

C. $\dfrac{n!}{(n-m)!}$

D. none of these.

Answer:

Watch Video Solution

50. The total number one-one function from a finite set with m elements to a set with n elements form>n is

A. $\frac{m!}{(m-n)!}$

B. $\frac{n!}{(n-m)!}$

C. n^m

D. none of these.

Answer:

Watch Video Solution

51. The number of bijective function from a set A to itself when A contains n elements is-

A. n^2

B. n

C. n!

D. 2^n

Answer:

Watch Video Solution

52. Show that the two sets {1, 2, 3,.......} and {3, 4, 5, ...} are equivalent.

Watch Video Solution

53. Find the domain of the functions: $f(x) = \log\left(\frac{12}{x^2 - x}\right)$ and $f(x) = \cos^{-1}\left[\log_3\left(\frac{x^2}{3}\right)\right]$

Watch Video Solution

54. Let the binary operation on Q defined as $a \cdot b = 2a + b - ab$, find $3 \cdot 4$.

Watch Video Solution

55. If the binary operation $*$ on set of integers Z is defined as $a * b = a + 3b^2$ then find the vlaue of $2 * 4$.

Watch Video Solution

56. Let * be a binary operation on set of integer. I defined by $a * b = 2a + b - 3$. Find the value of $3 * 4$.

Watch Video Solution

57. Let $* : R \times R \to r$ is defined as $a * b = 2a + bF \in d(2 * 3) * 4.$

Watch Video Solution

58. Let * is a binary operation on set of integers I defined by $a * b = 3a + 4b - 2$, then find the value of $4 * 5.$

Watch Video Solution

59. Prove that for any $f: X \rightarrow Y, foid_x = f = id_Y$ of.

Watch Video Solution

60. Test whether the relation : $R = \{(m, n) : 2 \mid (m+n)\}$ on $\mathbb{Z}$ is reflexive, symmetric or transitive.

Watch Video Solution

61. Let R be the relation on the set R of real numbers such that aRb iff a-b is and integer. Test whether R is an equivalence relation. If so find the equivalence class of $1 \text{and} \frac{1}{2}$ wrt. This equivalence relation.

Watch Video Solution

62. Let ~ be defined by (m,n)~(p,q) if mq=np where m, n, p,$q \in Z$-{0}. Show that it is an equivalence relation.

Watch Video Solution

Watch Video Solution

63. Show that the relation R defined on the set Z of all integers defined as R={(x,y):x-y is an integer} is reflexive, symmertric and transtive.

Watch Video Solution

64. Find least positive integer x, satisfying $276x + 128 = 4$ (mod 7).

Watch Video Solution

65. Test whether the relations are reflexive, symmetric or transitive on the sets specified.

R={(m,n): $\frac{m}{n}$ is a power of 5} on Z - {0}.

Watch Video Solution

66. Test wheter relations are reflexive, symmetric or transitive on the sets specified for

$R = \{(m, n) : 3 \text{divides} m - n\} on \{1, 2, 3. \ldots, 10\}$.

Watch Video Solution

67. If R and S are two equivalence relation on the set then prove that $R \cap S$ is also an equivlaence relation on the set.

Watch Video Solution

68. If A = R -{3} and B = R -{1}. Consider the function $f: A \rightarrow B$ defined by $f(x) = \frac{x-2}{x-3}$, for all $x \in A$. Then, show that f is bijective. Find $f^{-1}(x)$.

Watch Video Solution

69. $\text{Let} f(x) = \sqrt{x}\text{and} g(x) = 1 - x^2.$

Compute fog and gof and find their natural domains.

Watch Video Solution

70. The ralation R on Z is defined by for m, $n \in Z$, $mRn \Rightarrow \frac{m}{n}$ is a power of 2. Examine whether it is an equivalence relation.

Watch Video Solution

71. Show that the relation R on the set A={1, 2, 3, 4, 5} given by R={(a,b):$|a-b|$ is even} in an equivalence ralation.

Watch Video Solution

72. Show that the relation S defined on set $N \times N$ by $(a, b)S(c, d) \Rightarrow a + d = b + c$ is an equivalence relation.

Watch Video Solution

73. Check whether the relation R defined on the set A = {1,2,3,4,5,6} as R = {(x,y):y is divisible by x} is reflexive, symmetric and transitive.

Watch Video Solution

74. If the function'f : R $\rightarrow$ R is given by f(x) = $x^2 + 2$ and g:R $\rightarrow$ R is given by g(x) = $\frac{x}{x-1}, x \neq 1$ then find fog and gof and hence find fog (2) and gof (-3) .

Watch Video Solution

75. Let A = R-{2} and B = R - {1}. If f : A $\rightarrow$ B is a function defined by f(x)= $\frac{x-1}{x-2}$ then show that f is one-one and onto. Hence, find f^{-1}.

Watch Video Solution

76. Show that the fuction f in $A=R-\left\{\frac{2}{3}\right\}$ definde as $f(x)=\frac{4x+3}{6x-4}$ is one-one and onto. Hence find f^(-1)`.

Watch Video Solution

77. Show that $f: N \rightarrow N$, given by

$$f(x) = \begin{cases} x + 1, \text{ if x is odd} \\ x - 1, \text{ if x is even} \end{cases}$$

is bijective (both one-one and onto).

Watch Video Solution

78. Let f : W $\rightarrow$ W be defined as f (x) = x -1 if x is odd and f(x) = x +1 if x is even then show that f is invertible. Find the inverse of f where W is the set of all whole numbers.

Watch Video Solution

79. If $f: R \to R$ is defined as $f(x) = 10x + 7$. Find the function $g: R \to R$, such that $gof = fog = I_R$.

Watch Video Solution

80. If $f: R \to R$ is the function defined by $f(x) = 4x^3 + 7$, then show that f is a bijection.

Watch Video Solution

81. If the function $f: R \rightarrow R$ is given by $f(x) = (x)^2 + 3x + 1$ and $g: R \rightarrow R$ is given by $g(x) = 2x - 3$ than find fog and gof.

Watch Video Solution

82. If S is the set of all rational numbers except 1 and * be defined on S by $a * b = a + b - ab$, for all $a, b \in S$.

Prove that

(i) * is a binary operation on S.

(ii) * is commutative as well as associative.

Watch Video Solution

83. If S is the set of all rational numbers except 1 and * be defined on S by $a * b = a + b - ab$, for all $a, b \in S$.

Prove that

(i) * is a binary operation on S.

(ii) * is commutative as well as associative.

Watch Video Solution

84. Construct the multiplication table $\times 7$ on the set {1, 2, 3, 4, 5, 6}. Also find the converse of 4 if exists.

Watch Video Solution

85. Consider the binary operation $* : R \times R \rightarrow R$ and $o : R \times R \rightarrow R$ defined as $a * b = |a - b|$ and $aob = a$. For all $a, b \in R$. Show that * is commutative but not associative, .o. is associative but not commutative.

Watch Video Solution

86. Consider the binary operation $*$ on the set {1, 2, 3 , 4, 5} defined by $a * b = \min \{amb\}$. Write the operation talbe of operation $*$.

Watch Video Solution

87. If $*$ is a binary operation on set Q of rational numbers such tht $a * b = (2a - b)^2, a, b \in Q$. Find $3 * 5$ and $5 * 3$. Is $3 * 5 = 5 * 3$?

Watch Video Solution

88. if $*$ is the binary operation on N given by $a * b$= L. C. M of a and b. Find $20 * 16$. Is $*$ Commutative.

Watch Video Solution

89. if $*$ is the binary operation on N given by $a * b$= L. C. M of a and b. Find $20 * 16$. Is $*$ Associative.

Watch Video Solution

90. Prove that $f: X \to Y$ is injective iff for all subsets A, B of X, $f(A \cap B) = f(A) \cap f(B)$.

Watch Video Solution

91. Prove that $f: X \to Y$ is injective iff $f^{-1}(f(A)) = \mathrm{A} \text{ for all} A \subseteq X$.

Watch Video Solution

92. Prove that $f: X \to$ Y is surjective iff for all $B \subseteq Y, f\left(f^{-1}(B)\right) = B$.

Watch Video Solution

93. Prove that for any $f: X \to Y, foid_x = f = id_Y$ of.

Watch Video Solution

94. Let f: $X \rightarrow Y$

If there exists a map g:Y $\rightarrow$ X such that gof = id_X and fog = id_y, then show that

f is bijective and (ii) $g = f^{-1}$

Watch Video Solution

95. Let $f: X \rightarrow Y$. If there exists a map $g: Y \rightarrow X$ such that g of=$id_x$and fo g=id_y then show that `g=f^(-1)

Watch Video Solution

96. If f$f(x)=\cos[\pi^2]x+\cos[-\pi^2]x$ where [x] stands for the greatest integer functions, then evaluate $f\left(\frac{\pi}{2}\right), f(\pi), f(-\pi)$, and $f\left(\frac{\pi}{4}\right)$.

Watch Video Solution

97. If f:$R \rightarrow R, g : R \rightarrow R$ and $h : R \rightarrow R$ such that f(x)=x^2, $g(x)$= tan x and h(x)= log x then find [ho (gof)] (x) at$x=\frac{\sqrt{\pi}}{2}$

Watch Video Solution

98. If p is a prime and $ab \equiv 0$ (mod p) then show that either a=0 (mod p) or $b \equiv 0$ (mod p).

Watch Video Solution

99. Prove that the relation R on the set Z of all integers defined by $R = \{(a, b) : a - b$is divisible by n} is an equivalence relation.

Watch Video Solution

100. Let n be positive integer and a function f be defined as $f(n)=\begin{cases}0 & when n=1\\ r\left(\left[\frac{n}{2}\right]\right)+1 & when n>1\end{cases}$ then find f(35).

Watch Video Solution

101. If $f: R \rightarrow R$ defined by $f(x)=5x-8$ for all $x \in R$, then show that f is invertible. Find the corresponding inverse function.

Watch Video Solution

102. Show that the inverse of a bijective function is unique.

Watch Video Solution

103. Show that the inverse of a bijective is also a bijection.

Watch Video Solution

104. Let f={(1,a),(2,b),(3,c),(4,d)} and g={(a,x),(b,x),(c,y),(d,x)} Determine gof and fog if possible. Test whether fog=gof.

Watch Video Solution

105. Prove that the greatest integer function f:R $\rightarrow$ R, given by f(x) = [x] is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x.

Watch Video Solution

106. Let A and B be sets.

Show that f : $A \times B \rightarrow B \times A$ such that f (a,b) = (b,a) is bijective function .

Watch Video Solution

107. Show that the fuction $f: R \rightarrow R$ defined by f (x)=sin x is neither one-one nor onto.

Watch Video Solution

108. If $f: N \to N$ is defined by.

$$f(n) = \begin{cases} \frac{n+1}{2}, \text{if n is odd} \\ \frac{n}{2}, \text{if n is even} \end{cases}$$ for all $n \in N$.

Find whether the function f is bijective.

Watch Video Solution

109. Show that a fuction $f: R \to R$ given by $f(x) = ax + b, a, b \in R$ and $a \neq 0$ is bijective.

Watch Video Solution

110. Let* be a binary operation on Q, defined by a*b=$\frac{3ab}{5}$ .Show that is commutative as well as associative. Also, find its identity, if it exists.

Watch Video Solution

111. If $A = N \times N$ and * is a binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$. Show that * is commutative and associative. Also, find identity element for * on A, if any.

Watch Video Solution

112. A binary operation $\cdot$ on the set {0,1,2,3,4,5} is defined as

$$a \cdot b = \begin{cases} a+b & \text{if} a+b < 6 \\ a+b-6 & \text{if} a+b \geq 6 \end{cases}$$

Find the composition table for $\cdot$ Also, show that zero is the identity for this operation and each non-zero element a of the set is invertible with 6-a, being the inverse of a.

Watch Video Solution

113. Let us consider a binary operation $*$ on the set {1, 2, 3, 4, 5} given in the following table.

Compute$(2 * 3) * 3$ and $2 * (3 * 4)$

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

Watch Video Solution

114. Let us consider a binary operation $*$ on the set {1, 2, 3, 4, 5} given in the following table.

Is $*$ commutative.

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

Watch Video Solution

115. Let us consider a binary operation $*$ on the set {1, 2, 3, 4, 5} given in the following table.

Compute $(2 * 3) * (4 * 5)$.

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

Watch Video Solution