



# MATHS

# **BOOKS - SHARAM PUBLICATION**

# VECTORS



**1.** If  $\overrightarrow{a}$  and  $\overrightarrow{b}$  are unit vectors and  $\overrightarrow{a} + \overrightarrow{b}$  is also a unit vector, then write the measure of the angle between  $\overrightarrow{a}$  and  $\overrightarrow{b}$ 



**2.** Prove that 
$$: \left| \overrightarrow{a} + \overrightarrow{b} \right| \le \left| \overrightarrow{a} \right| + \left| \overrightarrow{b} \right|.$$

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3. Calculate the area of the triangle ABC (by vector method)

where A(1,2,4), B(3,1,-2), C(4,3,1)

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**4.** If  $\overrightarrow{a}$ ,  $\overrightarrow{b}$ ,  $\overrightarrow{c}$  are mutually perpendicular vectors of equal magnitude, show that  $\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c}$  is equally inclined to  $\overrightarrow{a}$ ,  $\overrightarrow{b}$ ,  $\overrightarrow{c}$ .

5. Prove the following by vector method. Measure of the

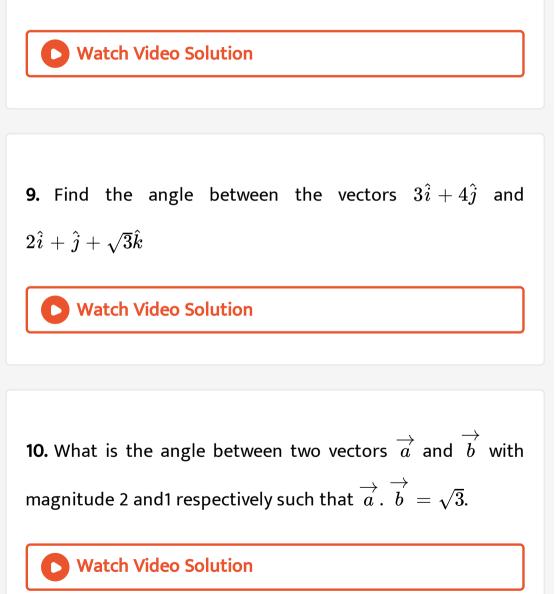
angle between two diagonals of a cube is  $\cos^{-1}\left(rac{1}{3}
ight)$ 



**6.** Prove by vector method that the lines joining the mid points of consecutive sides of a quadrilateral is a parallelogram.

7. if 
$$\overrightarrow{a} = 2\hat{i} - 5\hat{j} + 8\hat{k}, \ \overrightarrow{b} = \hat{i} - 3\hat{j} - 7\hat{k}$$
 and  $\overrightarrow{c} = -3\hat{i} + 2\hat{j} - \hat{k}$  then find  $\left|\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c}\right|$ 

8. If  $\overrightarrow{a} = 3\hat{i} + 3\hat{j} + \hat{k}$  and  $\overrightarrow{b} = -2\hat{i} + \hat{j} - 2\hat{k}$  then what is the unit vector parallel to  $\overrightarrow{a} + \overrightarrow{b}$ 



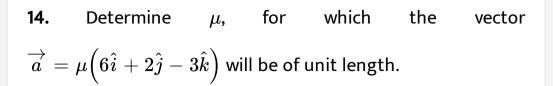
11. If  $\overrightarrow{a} = 3\hat{i} + 3\hat{j} + \hat{k}$  and  $\overrightarrow{b} = -2\hat{i} + \hat{j} - 2\hat{k}$  then what is the unit vector parallel to  $\overrightarrow{a} + \overrightarrow{b}$ 

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12. Find the component of the vector  $\overrightarrow{b} = 8\hat{i} + \hat{j}$  in the direction of the vector  $\overrightarrow{a} = \hat{i} + 2\hat{j} - 2\hat{k}$ .

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13. What is the value of x so that the vector  $6\hat{i} + 2\hat{j} - 3\hat{k}$ and  $\hat{i} - 4\hat{j} + x\hat{k}$  are perpendicular to each other.



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15. How many directions a null vector has ?

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**16.** If  $\overrightarrow{a}$ ,  $\overrightarrow{b}$  and  $\overrightarrow{c}$  are three mutually perpendicular vectors, each of magnitude unity, then what will be the magnitude of  $\left|\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c}\right|$ .

17. What is the volume of the parallelopiped whose sides are

given by the vectors.  $\hat{i}+\hat{j},\,\hat{j}+\hat{k}$  and  $\hat{k}+\,\hat{i}$ 

18. Write the values of m and n for which the vectors  $(m-1)\hat{i}+(n+2)\hat{j}+4\hat{k}$  and  $(m+1)\hat{i}+(n-2)\hat{j}+8\hat{k}$  are parallel

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19. Show that the vectors  $2\hat{i}+3\hat{j},5\hat{i}-5\hat{k}$  and  $6\hat{j}+4\hat{k}$  are

co-planar.

**20.** If A, B, C, D, E are the vertices of a regular pentagon, find the vector sum  $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA}$ .

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**21.** If 
$$\overrightarrow{a}$$
.  $\overrightarrow{b} = 0$  and  $\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{0}$ , then draw the

conclusion.

22. Find a vector in the direction of vector  $2\hat{i} - 3\hat{j} + 6\hat{k}$  which has magnitude 21units.



23. Find the unit vector in the direction of the sum of the vectors  $\overrightarrow{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$  and  $\overrightarrow{b} = 2\hat{i} + 2\hat{j} - 7\hat{k}$ 

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24. Write a unitvector in the direction of the sum of vectors  $\overrightarrow{a}=2\hat{i}-\hat{j}-2\hat{k}$  and  $\overrightarrow{b}=-\hat{i}+\hat{j}+3\hat{k}$ 

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25. If 
$$\overrightarrow{a} = x\hat{i} + 2\hat{j} - z\hat{k}, \ \overrightarrow{b} = 3\hat{i} - y\hat{j} + 2\hat{k}$$
 are two equal

vectors, then write the value of x + y + z.

**26.** If  $\hat{a}$ .  $\hat{b} = \frac{1}{2}$  then what is the angle between  $\hat{a}$  and  $\hat{b}$ ?

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27. Write the value of the cosine of the angle which the vector  $\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}$  makes with y-axis

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**28.** Write a vector in the direction of the vector  $\hat{i} - 2\hat{j} + 3\hat{k}$ 

that has magnitude 9 units.



**29.** Find the value of 'a' such that the vector  $6\hat{i} + 2\hat{j} - 3\hat{k}$  is perpendicular to  $\hat{i} + 6\hat{j} + a\hat{k}$ 



**30.** What is the unit vector perpendicular to the vectors  $\hat{i} - \hat{j}$  and  $2\hat{i} - 3\hat{j}$ ?

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**31.** What is the magnitude of  $2\overrightarrow{a} \times 3\overrightarrow{a}$ ?

32. If 
$$\left(\overrightarrow{a} \times \overrightarrow{b}\right)^2 + \left(\overrightarrow{a} \cdot \overrightarrow{b}\right)^2 = 144$$
, write the value of  $ab$   
.  
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33. Write a vector normal to  $\hat{i} + \hat{k}$  and  $\hat{i} + \hat{j}$ .

**34.** What is the angle between  $\hat{i} + \hat{j}$  and  $\hat{i} - \hat{j}$  ?

**35.** If  $\overrightarrow{a}$  and  $\overrightarrow{b}$  are unit vectors such that  $\overrightarrow{a} \times \overrightarrow{b}$  is a unit vector, then the angle between  $\overrightarrow{a}$  and  $\overrightarrow{b}$  is \_\_\_\_

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**36.** If two vectors 
$$\overrightarrow{a}$$
 and  $\overrightarrow{b}$  are such that  $\left|\overrightarrow{a} + \overrightarrow{b}\right| = \left|\overrightarrow{a} - \overrightarrow{b}\right|$ , then what is the angle between  $\overrightarrow{a}$  and  $\overrightarrow{b}$ ?

**37.** What is the direction of 
$$(\hat{i} + \hat{j} + \hat{k}) + (\hat{i} + \hat{k} - 3\hat{j}) + (\hat{k} + \hat{i} - 3\hat{j}) + (\hat{i} + \hat{j} - 3\hat{k})$$
?



**38.** What is the projection of  $\hat{i}+\hat{j}-\hat{k}$  upon the vector  $\hat{i}$  ?

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39. Prove that 
$$\begin{bmatrix} \overrightarrow{a} \times \overrightarrow{b} \overrightarrow{b} \times \overrightarrow{c} \overrightarrow{c} \times \overrightarrow{a} \end{bmatrix} = \begin{bmatrix} \overrightarrow{a} \overrightarrow{b} \overrightarrow{c} \end{bmatrix}^2$$
  
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40. Let  $\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}, \ \overrightarrow{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$  and  
 $\overrightarrow{c} = \hat{i} - 2\hat{j} + \hat{k}$  and find a vector of magnitude 6 units

which is parallel to the vector  $2\overrightarrow{a} - \overrightarrow{b} + 3\overrightarrow{c}$ .

**41.** Show that 
$$\left[\overrightarrow{a} + \overrightarrow{b}\overrightarrow{b} + \overrightarrow{c}\overrightarrow{c} + \overrightarrow{a}\right] = 2\left[\overrightarrow{a}\overrightarrow{b}\overrightarrow{c}\right]$$

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**42.** If 
$$\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c} = \overrightarrow{0}$$
,  $|\overrightarrow{a}| = 3$ ,  $|\overrightarrow{b}| = 5$  and  $|\overrightarrow{c}| = 7$ , find the angle between  $\overrightarrow{a}$  and  $\overrightarrow{b}$ .

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**43.** The position vectors of the points A, B, C and D are  $4\hat{i} + 3\hat{j} - \hat{k}$ ,  $5\hat{i} + 2\hat{j} + 2\hat{k}$ ,  $2\hat{i} - 2\hat{j} - 3\hat{k}$  and  $4\hat{i} - 4\hat{j} + 3\hat{k}$  respectively. Show that AB and CD are parallel.



44. If the sum of two unit vectors is a unit vectors find the

magnitude of their difference.

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**45.** If 
$$\overrightarrow{a} = (2,-2,1)$$
,  $\overrightarrow{b} = (2,3,6)$  and  $\overrightarrow{c} = (-1,0,2)$ , Find the magnitude and direction of  $\overrightarrow{a} - \overrightarrow{b} + 2\overrightarrow{c}$ .

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46. Prove the following by vector method. An angle inscribed

in a semi-circle is a right angle.

**47.** If  $\overrightarrow{a}, \overrightarrow{b}$  and  $\overrightarrow{c}$  mutually perpendiculars, show that  $\left[\overrightarrow{a}, \left(\overrightarrow{b} \times \overrightarrow{c}\right)\right]^2 = a^2 b^2 c^2$ 



**48.** If 
$$\hat{i} + \hat{j} + \hat{k}$$
 and  $2\hat{i} - \alpha\hat{j} + 3\hat{k}$  are orthogonal to each other then find  $\alpha$ 

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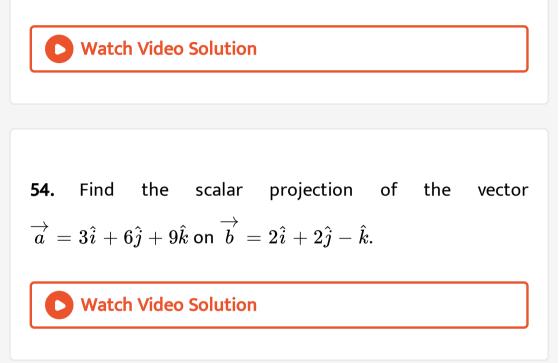
**49.** Find 
$$\begin{bmatrix} \overrightarrow{a} \ \overrightarrow{b} \ \overrightarrow{c} \end{bmatrix}$$
 when  $\overrightarrow{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \overrightarrow{b} = 2\hat{i} + \hat{j} - \hat{k}, \overrightarrow{c} = \hat{j} + \hat{k}$ 

**50.** Find a vector  $\overrightarrow{b}$  such that  $\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{c}$  and  $\overrightarrow{a} \cdot \overrightarrow{b} = 3$ where  $\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}, \overrightarrow{c} = \hat{j} - \hat{k}$ Watch Video Solution Prove by vector method that in 51. а  $\Delta ABC, c^2 = a^2 + b^2 - 2ab\cos C.$ Watch Video Solution

**52.** Determine the area of the parallelogram whose sides are

the vectors  $2\hat{i}+2\hat{j}$  and  $\hat{i}-\hat{k}.$ 

**53.** If the position vectors of the points A, B, C are  $2\hat{i} + \hat{j} - \hat{k}$ ,  $3\hat{i} - 2\hat{j} + \hat{k}$  and  $\hat{i} + 4\hat{j} - 3\hat{k}$  respectively, then prove that A, B, C are collinear.



**55.** Find the value of  $\lambda$  such that the following vectors are coplanar:  $-\hat{i} + \lambda \hat{j} - \lambda \hat{k}, 2\hat{i} + 4\hat{j} + 5\hat{k}, -2\hat{i} + 4\hat{j} - 4\hat{k}$ 

56. Prove that four points with position vectors  $\hat{i}+\hat{j}-3\hat{k},2\hat{i}-\hat{j}-\hat{k},\ -\hat{i}+2\hat{j}+2\hat{k}$ and $2\hat{i}+2\hat{k}$ 

arecoplanar

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57. Using vector method find the area of the triangle with

vertices (1, 0, 0) (0, 1, 0) and (0, 0, 1)

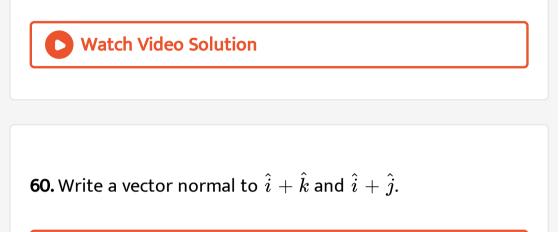


58. Write the volume of the parallelopiped whose sides are

given by $-\hat{j},\hat{k},\ -\hat{i}$ 

59. If the magnitude of the difference of two unit vectors is

 $\sqrt{3}$  then find the magnitude of their sum.



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61. If 
$$\overrightarrow{a} = (2, -2, 1)$$
,  $\overrightarrow{b} = (2, 3, 6)$  and  $\overrightarrow{c} = (-1, 0, 2)$ 

,Find the magnitude and direction of  $\overrightarrow{a} + \overrightarrow{b} - \overrightarrow{c}$ .

62. Find the angle between the vectors  $3\hat{i}+4\hat{j}$  and  $2\hat{i}+\hat{j}+\hat{k}.$ 

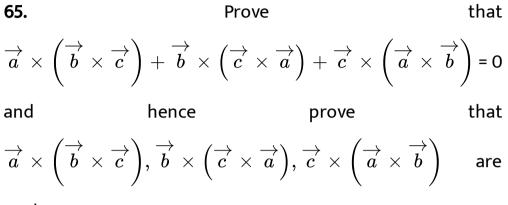
**63.** Find the unit vector in the direction of  $\vec{a} - \vec{b}$  where  $\vec{a} = 4\hat{i} + 4\hat{j} + \hat{k}$  and  $\vec{b} = 3\hat{i} - 11\hat{k}$ .

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**64.** If a and b are perpendicular vectors show that  $\left(\overrightarrow{a} + \overrightarrow{b}\right)^2 = \left(\overrightarrow{a} - \overrightarrow{b}\right)^2$ .  $\left[\left(\overrightarrow{a} + \overrightarrow{b}\right)^2 means(veca+vecb).(veca+vecb), sodoes(veca-vecb))$ 

vecb)^2`.]

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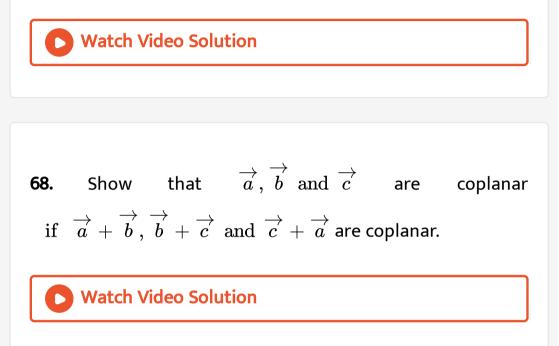


#### coplanar.



**66.** If 
$$\overrightarrow{a} = 3\hat{i} + \hat{j} - 2\hat{k}$$
,  $\overrightarrow{b} = 2\hat{i} - 3\hat{j} + 4\hat{k}$  then verify that  $\overrightarrow{a} \times \overrightarrow{b}$  is perpendicular to both  $\overrightarrow{a}$  and  $\overrightarrow{b}$ .

**67.** Prove that the four points with position vectors  $2\overrightarrow{a} + 3\overrightarrow{b} - \overrightarrow{c}, \overrightarrow{a} - 2\overrightarrow{b} + 3\overrightarrow{c}, 3\overrightarrow{a} + 4\overrightarrow{b} - 2\overrightarrow{c}$  and  $\overrightarrow{a} - 6\overrightarrow{b} + 6\overrightarrow{c}$  are coplanar.



**69.** Find the vector 
$$\overrightarrow{p}$$
 which is perpendicular to both  
 $\overrightarrow{\alpha} = 4\hat{i} + 5\hat{j} - \hat{k}, \overrightarrow{\beta} = \hat{i} - 4\hat{j} + 5\hat{k}$  and  $\overrightarrow{p} \cdot \overrightarrow{q} = 21$  where  
 $q = 3\hat{i} + \hat{j} - \hat{k}$ .

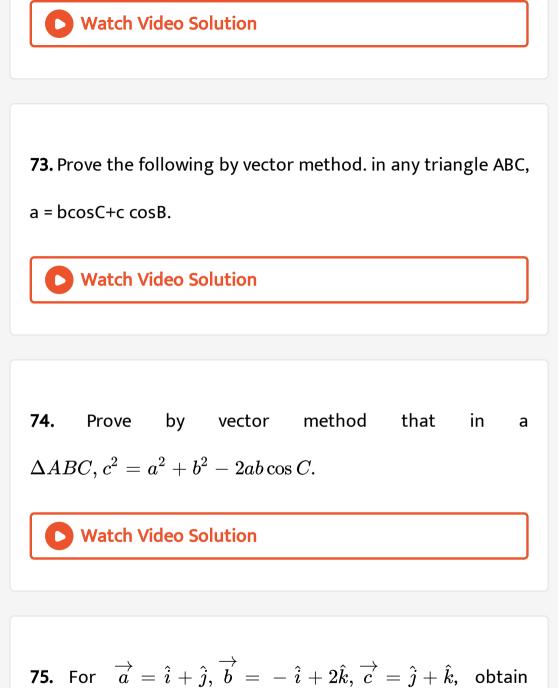
70. If  $\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}$  and  $\overrightarrow{b} = \hat{i} - \hat{k}$ . Find a vector  $\overrightarrow{c}$  such that  $\overrightarrow{a} \times \overrightarrow{c} = \overrightarrow{b}$  and  $\overrightarrow{a} \cdot \overrightarrow{c} = 3$ .

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**71.** If 
$$\overrightarrow{a}, \overrightarrow{b}$$
 and  $\overrightarrow{c}$  are three vectors such that  $\left|\overrightarrow{a}\right| = 5, \left|\overrightarrow{b}\right| = 12, \left|\overrightarrow{c}\right| = 13$  and  $\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c} = 0$  then find the value of  $\overrightarrow{a}, \overrightarrow{b} + \overrightarrow{b}, \overrightarrow{c} + \overrightarrow{c}, \overrightarrow{a}$ .

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72. If  $a = 2\hat{i} + \hat{k}$ ,  $b = \hat{i} + \hat{j} + \hat{k}$  and  $c = 4\hat{i} - 3\hat{j} + 7\hat{k}$ , then find the vector  $\overrightarrow{r}$  which satisfies  $r \times b = c \times b$  and r. a = 0.



 $\overrightarrow{a} imes \left(\overrightarrow{b} imes \overrightarrow{c}
ight)$  and also verify the formula

$$\overrightarrow{a} imes \left( \overrightarrow{b} imes \overrightarrow{c} 
ight) = \left( \overrightarrow{a} . \ \overrightarrow{c} 
ight) \overrightarrow{b} - \left( \overrightarrow{a} . \ \overrightarrow{b} 
ight) \overrightarrow{c}.$$

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76. Prove the following by vector method. The diagonals of a

rhombus are at right angles.



77. Determine the sine of the angle between the vectors

 $\widehat{-3}\hat{j}+\hat{k},\,\hat{i}+\hat{j}+\hat{k}$ 



78. Find the volume of the parallelopiped whose sides are given by the vectors.  $\hat{i} + \hat{j} + \hat{k}, \hat{k}, 3\hat{i} - \hat{j} + 2\hat{k}.$ 

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**79.** Find the value of 
$$\lambda$$
 for which the three points with  
position vectors  $2\hat{i} + 3\hat{j} - 4\hat{k}, -\hat{i} + \hat{j} + 2\hat{k}$  and

 $4\hat{i}+5\hat{j}+\lambda\hat{k}$  are coplanar.

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80. Find a unit vector perpendicular to the following two vectors  $\overrightarrow{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$  and  $\overrightarrow{b} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ 

81. Find the value of 
$$\lambda$$
 so that three vectors  
 $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$  and  
 $\vec{c} = 3\hat{i} + \lambda \vec{j} + 5\hat{k}$  are coplanar.

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82. Show that 
$$\left[\overrightarrow{a} + \overrightarrow{b}\overrightarrow{b} + \overrightarrow{c}\overrightarrow{c} + \overrightarrow{a}\right] = 2\left[\overrightarrow{a}\overrightarrow{b}\overrightarrow{c}\right]$$

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83. If the vertices A,B,C of a triangle ABC are at (1,1,2),(2,2,3),

(3,-1,-1) respectively, then using vector method find the area

of the triangle.

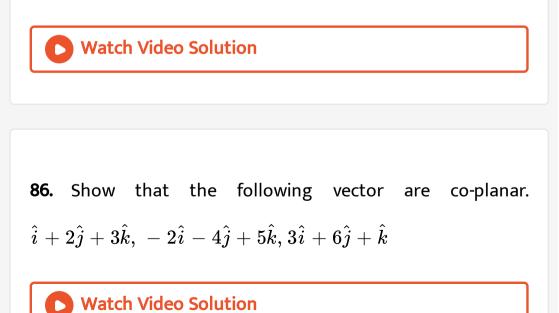


84. If  $\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{b} \times \overrightarrow{c} \neq \overrightarrow{0}$ , prove that  $\overrightarrow{a} + \overrightarrow{c} = m\overrightarrow{b}$ ,

where m is a scalar.

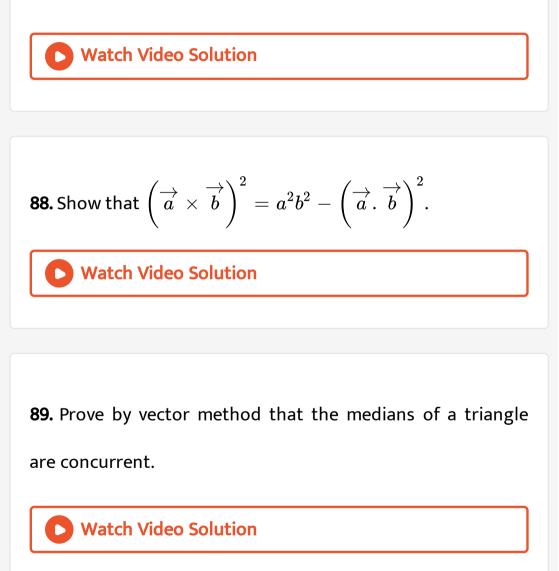
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**85.** Prove analytically : The perpendicular bisector of the sides of a triangle are concurrent.



87. Prove by vector method that the medians of a triangle are

concurrent.



90. Show that the following vector are co-planar. $\hat{i}-2\hat{j}+2\hat{k},3\hat{i}+4\hat{j}+5\hat{k},\ -2\hat{i}+4\hat{j}-4\hat{k}.$