MATHS

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VECTORS

Example

1. If $\vec{a}$ and $\vec{b}$ are unit vectors and $\vec{a}+\vec{b}$ is also a unit vector, then write the measure of the angle between $\vec{a}$ and $\vec{b}$

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2. Prove that : $\left|\vec{a}+\vec{b}\right| \leq \left|\vec{a}\right| + \left|\vec{b}\right|$.

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3. Calculate the area of the triangle ABC (by vector method) where A(1,2,4), B(3,1,-2), C(4,3,1)

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4. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitude, show that $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}.\ \vec{b}.\ \vec{c}.$

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5. Prove the following by vector method. Measure of the angle between two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$

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6. Prove by vector method that the lines joining the mid points of consecutive sides of a quadrilateral is a parallelogram.

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7. if $\vec{a} = 2\hat{i} - 5\hat{j} + 8\hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} - 7\hat{k}$ and $\vec{c} = -3\hat{i} + 2\hat{j} - \hat{k}$ then find $\left|\vec{a} + \vec{b} + \vec{c}\right|$

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8. If $\vec{a} = 3\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$ then what is the unit vector parallel to $\vec{a} + \vec{b}$

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9. Find the angle between the vectors $3\hat{i} + 4\hat{j}$ and $2\hat{i} + \hat{j} + \sqrt{3}\hat{k}$

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10. What is the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitude 2 and1 respectively such that $\vec{a} . \vec{b} = \sqrt{3}$.

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11. If $\vec{a} = 3\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$ then what is the unit vector parallel to $\vec{a} + \vec{b}$

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12. Find the component of the vector $\vec{b} = 8\hat{i} + \hat{j}$ in the direction of the vector $\vec{a} = \hat{i} + 2\hat{j} - 2\hat{k}$.

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13. What is the value of x so that the vector $6\hat{i} + 2\hat{j} - 3\hat{k}$ and $\hat{i} - 4\hat{j} + x\hat{k}$ are perpendicular to each other.

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14. Determine μ, for which the vector $\vec{a} = \mu\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)$ will be of unit length.

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15. How many directions a null vector has ?

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16. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors, each of magnitude unity, then what will be the magnitude of $\left|\vec{a} + \vec{b} + \vec{c}\right|$.

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17. What is the volume of the parallelopiped whose sides are given by the vectors. $\hat{i}+\hat{j}, \hat{j}+\hat{k}$ and $\hat{k}+\hat{i}$

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18. Write the values of m and n for which the vectors $(m-1)\hat{i}+(n+2)\hat{j}+4\hat{k}$ and $(m+1)\hat{i}+(n-2)\hat{j}+8\hat{k}$ are parallel

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19. Show that the vectors $2\hat{i}+3\hat{j}, 5\hat{i}-5\hat{k}$ and $6\hat{j}+4\hat{k}$ are co-planar.

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20. If A, B, C, D, E are the.vertices of a regular pentagon, find the vector sum $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA}$.

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21. If $\vec{a} . \vec{b} = 0$ and $\vec{a} \times \vec{b} = \vec{0}$, then draw the conclusion.

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22. Find a vector in the direction of vector $2\hat{i} - 3\hat{j} + 6\hat{k}$ which has magnitude 21units.

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23. Find the unit vector in the direction of the sum of the vectors $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{b} = 2\hat{i} + 2\hat{j} - 7\hat{k}$

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24. Write a unitvector in the direction of the sum of vectors $\vec{a} = 2\hat{i} - \hat{j} - 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + 3\hat{k}$

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25. If $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$, $\vec{b} = 3\hat{i} - y\hat{j} + 2\hat{k}$ are two equal vectors, then write the value of x + y + z.

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26. If $\hat{a}.\ \hat{b} = \frac{1}{2}$ then what is the angle between $\hat{a}$ and $\hat{b}$?

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27. Write the value of the cosine of the angle which the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ makes with y-axis

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28. Write a vector in the direction of the vector $\hat{i} - 2\hat{j} + 3\hat{k}$ that has magnitude 9 units.

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29. Find the value of 'a' such that the vector $6\hat{i} + 2\hat{j} - 3\hat{k}$ is perpendicular to $\hat{i} + 6\hat{j} + a\hat{k}$

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30. What is the unit vector perpendicular to the vectors $\hat{i} - \hat{j}$ and $2\hat{i} - 3\hat{j}$?

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31. What is the magnitude of $2\vec{a} \times 3\vec{a}$?

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32. If $\left(\vec{a} \times \vec{b}\right)^2 + \left(\vec{a} \cdot \vec{b}\right)^2 = 144$, write the value of ab

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33. Write a vector normal to $\hat{i} + \hat{k}$ and $\hat{i} + \hat{j}$.

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34. What is the angle between $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$?

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35. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $\vec{a} \times \vec{b}$ is a unit vector, then the angle between $\vec{a}$ and $\vec{b}$ is ____

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36. If two vectors $\vec{a}$ and $\vec{b}$ are such that $\left|\vec{a} + \vec{b}\right| = \left|\vec{a} - \vec{b}\right|$, then what is the angle between $\vec{a}$ and $\vec{b}$?

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37. What is the direction of $\left(\hat{i} + \hat{j} + \hat{k}\right) + \left(\hat{i} + \hat{k} - 3\hat{j}\right) + \left(\hat{k} + \hat{i} - 3\hat{j}\right) + \left(\hat{i} + \hat{j} - 3\hat{k}\right)$?

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38. What is the projection of $\hat{i}+\hat{j}-\hat{k}$ upon the vector $\hat{i}$?

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39. Prove that $\left[\vec{a}\times\vec{b}\,\vec{b}\times\vec{c}\,\vec{c}\times\vec{a}\right]=\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$

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40. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k},\ \vec{b}=4\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$ and find a vector of magnitude 6 units which is parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}$.

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41. Show that $\left[\vec{a}+\vec{b}\,\vec{b}+\vec{c}\,\vec{c}+\vec{a}\right]=2\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

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42. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, $\left|\vec{a}\right|=3$, $\left|\vec{b}\right|=5$ and $\left|\vec{c}\right|=7$, find the angle between $\vec{a}$ and $\vec{b}$.

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43. The position vectors of the points A, B, C and D are $4\hat{i}+3\hat{j}-\hat{k}$, $5\hat{i}+2\hat{j}+2\hat{k}$, $2\hat{i}-2\hat{j}-3\hat{k}$ and $4\hat{i}-4\hat{j}+3\hat{k}$ respectively. Show that AB and CD are parallel.

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44. If the sum of two unit vectors is a unit vectors find the magnitude of their difference.

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45. If $\vec{a}$ = (2,-2,1), $\vec{b}$ = (2,3,6) and $\vec{c}$ = (-1,0,2), Find the magnitude and direction of $\vec{a} - \vec{b} + 2\vec{c}$.

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46. Prove the following by vector method. An angle inscribed in a semi-circle is a right angle.

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47. If $\vec{a}, \vec{b}$ and $\vec{c}$ mutually perpendiculars, show that $\left[\vec{a}.\left(\vec{b} \times \vec{c}\right)\right]^2 = a^2b^2c^2$

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48. If $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} - \alpha\hat{j} + 3\hat{k}$ are orthogonal to each other then find α

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49. Find $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ when $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{c} = \hat{j} + \hat{k}$

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50. Find a vector $\vec{b}$ such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} . \vec{b} = 3$ where $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{c} = \hat{j} - \hat{k}$

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51. Prove by vector method that in a $\Delta ABC, c^2 = a^2 + b^2 - 2ab\cos C.$

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52. Determine the area of the parallelogram whose sides are the vectors $2\hat{i} + 2\hat{j}$ and $\hat{i} - \hat{k}.$

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53. If the position vectors of the points A, B, C are $2\hat{i}+\hat{j}-\hat{k}, 3\hat{i}-2\hat{j}+\hat{k}$ and $\hat{i}+4\hat{j}-3\hat{k}$ respectively, then prove that A, B, C are collinear.

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54. Find the scalar projection of the vector $\vec{a}=3\hat{i}+6\hat{j}+9\hat{k}$ on $\vec{b}=2\hat{i}+2\hat{j}-\hat{k}$.

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55. Find the value of λ such that the following vectors are coplanar: $-\hat{i}+\lambda\hat{j}-\lambda\hat{k}, 2\hat{i}+4\hat{j}+5\hat{k}, -2\hat{i}+4\hat{j}-4\hat{k}$

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56. Prove that four points with position vectors $\hat{i}+\hat{j}-3\hat{k}, 2\hat{i}-\hat{j}-\hat{k}, -\hat{i}+2\hat{j}+2\hat{k}$ and $2\hat{i}+2\hat{k}$ arecoplanar

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57. Using vector method find the area of the triangle with vertices (1, 0, 0) (0, 1, 0) and (0, 0, 1)

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58. Write the volume of the parallelopiped whose sides are given by $-\hat{j}, \hat{k}, -\hat{i}$

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59. If the magnitude of the difference of two unit vectors is $\sqrt{3}$ then find the magnitude of their sum.

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60. Write a vector normal to $\hat{i}+\hat{k}$ and $\hat{i}+\hat{j}$.

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61. If $\vec{a}=(2,\ -2,1)$, $\vec{b}=(2,3,6)$ and $\vec{c}=(-1,0,2)$,Find the magnitude and direction of $\vec{a}+\vec{b}-\vec{c}$.

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62. Find the angle between the vectors $3\hat{i}+4\hat{j}$ and $2\hat{i}+\hat{j}+\hat{k}$.

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63. Find the unit vector in the direction of $\vec{a}-\vec{b}$ where $\vec{a}=4\hat{i}+4\hat{j}+\hat{k}$ and $\vec{b}=3\hat{i}-11\hat{k}$.

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64. If a and b are perpendicular vectors show that $\left(\vec{a}+\vec{b}\right)^2=\left(\vec{a}-\vec{b}\right)^2$.

$\left[\left(\vec{a}+\vec{b}\right)^2 means$(veca+vecb).(veca+vecb), $sodoes$(veca-vecb)^2`.]

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65. Prove that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) + \vec{b} \times \left(\vec{c} \times \vec{a}\right) + \vec{c} \times \left(\vec{a} \times \vec{b}\right) = 0$ and hence prove that $\vec{a} \times \left(\vec{b} \times \vec{c}\right), \vec{b} \times \left(\vec{c} \times \vec{a}\right), \vec{c} \times \left(\vec{a} \times \vec{b}\right)$ are coplanar.

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66. If $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}, \vec{b} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ then verify that $\vec{a} \times \vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{b}$.

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67. Prove that the four points with position vectors $2\vec{a}+3\vec{b}-\vec{c}, \vec{a}-2\vec{b}+3\vec{c}, 3\vec{a}+4\vec{b}-2\vec{c}$ and $\vec{a}-6\vec{b}+6\vec{c}$ are coplanar.

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68. Show that $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar if $\vec{a}+\vec{b}, \vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are coplanar.

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69. Find the vector $\vec{p}$ which is perpendicular to both $\vec{\alpha}=4\hat{i}+5\hat{j}-\hat{k}, \vec{\beta}=\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{p}.\vec{q}$ = 21 where $q=3\hat{i}+\hat{j}-\hat{k}$.

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70. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{k}$. Find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$.

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71. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $\left|\vec{a}\right| = 5, \left|\vec{b}\right| = 12, \left|\vec{c}\right| = 13$ and $\vec{a} + \vec{b} + \vec{c} = 0$ then find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$.

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72. If $a = 2\hat{i} + \hat{k}, b = \hat{i} + \hat{j} + \hat{k}$ and $c = 4\hat{i} - 3\hat{j} + 7\hat{k}$, then find the vector $\vec{r}$ which satisfies $r \times b = c \times b$ and $r.a = 0$.

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73. Prove the following by vector method. in any triangle ABC, a = bcosC+c cosB.

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74. Prove by vector method that in a $\Delta ABC, c^2 = a^2 + b^2 - 2ab\cos C.$

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75. For $\vec{a} = \hat{i} + \hat{j}, \vec{b} = -\hat{i} + 2\hat{k}, \vec{c} = \hat{j} + \hat{k},$ obtain $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$ and also verify the formula

$\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \left(\vec{a} . \vec{c}\right)\vec{b} - \left(\vec{a} . \vec{b}\right)\vec{c}.$

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76. Prove the following by vector method. The diagonals of a rhombus are at right angles.

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77. Determine the sine of the angle between the vectors $\widehat{-3\hat{j} + \hat{k}}, \hat{i} + \hat{j} + \hat{k}$

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78. Find the volume of the parallelopiped whose sides are given by the vectors. $\hat{i}+\hat{j}+\hat{k}, \hat{k}, 3\hat{i}-\hat{j}+2\hat{k}$.

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79. Find the value of λ for which the three points with position vectors $2\hat{i}+3\hat{j}-4\hat{k}, -\hat{i}+\hat{j}+2\hat{k}$ and $4\hat{i}+5\hat{j}+\lambda\hat{k}$ are coplanar.

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80. Find a unit vector perpendicular to the following two vectors $\vec{a}=2\hat{i}+3\hat{j}+6\hat{k}$ and $\vec{b}=3\hat{i}-6\hat{j}+2\hat{k}$

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81. Find the value of λ so that three vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} + \lambda\vec{j} + 5\hat{k}$ are coplanar.

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82. Show that $\left[\vec{a} + \vec{b}\,\vec{b} + \vec{c}\,\vec{c} + \vec{a}\right] = 2\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

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83. If the vertices A,B,C of a triangle ABC are at (1,1,2),(2,2,3), (3,-1,-1) respectively, then using vector method find the area of the triangle.

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84. If $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} \neq \vec{0}$, prove that $\vec{a} + \vec{c} = m\vec{b}$, where m is a scalar.

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85. Prove analytically : The perpendicular bisector of the sides of a triangle are concurrent.

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86. Show that the following vector are co-planar. $\hat{i} + 2\hat{j} + 3\hat{k}, -2\hat{i} - 4\hat{j} + 5\hat{k}, 3\hat{i} + 6\hat{j} + \hat{k}$

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87. Prove by vector method that the medians of a triangle are concurrent.

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88. Show that $\left(\vec{a} \times \vec{b}\right)^2 = a^2b^2 - \left(\vec{a}.\ \vec{b}\right)^2$.

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89. Prove by vector method that the medians of a triangle are concurrent.

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90. Show that the following vector are co-planar.

$\hat{i} - 2\hat{j} + 2\hat{k}, 3\hat{i} + 4\hat{j} + 5\hat{k}, \ -2\hat{i} + 4\hat{j} - 4\hat{k}.$

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