MATHS

BOOKS - USHA MATHS (ODIA ENGLISH)

CONTINUITY AND DIFFERENTIABILITY APPLICATION OF DERIVATIVES

Exercise

1. If a function is continuous at x=a, then find

$$\lim_{h \to 0} + \frac{1}{2}\{f(a+h) - f(a-h)\}$$

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2. If a function is continuous at x=a, then find

$$\lim_{h \to 0} + \frac{1}{2}\{f(a+h) - f(a-h)\}$$

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3. Find the value of k so that the function f defined below is continuous at $x = 0$

$$f(x) = \begin{cases} \frac{\sin 2x}{5x}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

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4. If $f(x+y)=f(x)f(y)$ for all $x, y \in R$ and $f(5)=4$ and $f'(0)=2$, then find $f'(5)$

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5. If $f(x)=|\cos 2x|$, then find $f'\left(\frac{\pi}{4}+0\right)$

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6. If $f(4)=4, \quad f'(4)=1,$ then find $\lim_{x \to 4} \frac{2-\sqrt{f(x)}}{2-\sqrt{x}}$

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7. Find the values of x, for which $\frac{d}{dx}\cot\left(\cot^{-1}x\right) = 1.$

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8. Find x, for which $\frac{d}{dx}\cos\left(\cos^{-1}x\right) = 1.$

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9. Write the value x for which $\frac{d}{dx}\sin^{-1}(\sin x) = 1$

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10. Find x, for which $\frac{d}{dx}\cos\left(\cos^{-1} x\right) = 1.$

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11. Find x, for which $\frac{d}{dx}\sec\left(\sec^{-1} x\right) = 1.$

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12. If $y = \sin^{-1} x$ and $z = 2\cos^{-1} x$, find $\frac{dy}{dz}$.

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13. If $y = t^2$ and $x = t^3$ find, $\frac{dy}{dx}$ at $t = 1$.

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14. If $x = \log t$, $y = \sin t$, find $\frac{dy}{dx}$ at $t = \frac{\pi}{2}$.

State true or false in exercises $14 - 21$.

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15. Differential coefficient of $\sin^{-1}\left(\frac{2x^3}{1+x^6}\right)$ with respect to x is $\frac{6x^2}{1+x^6}$

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16. There is no function which is its own derivative

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17. The derivative of a non constant even function is always an odd function.

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18. If $x = \cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$, $y = \sin^{-1}\left(\frac{1}{\sqrt{t^2+1}}\right)$ then $\frac{dy}{dx}$ is independent of t.

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19. The derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right) w.r.t. \sqrt{1-x^2}$ is 3

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20. Derivative of $\log_{10}\sin x$ w.r.to x is $\cot x . \log_{10} e$

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21. If

$$y = \frac{\sec^{-1}\left(\sqrt{x}+1\right)}{\sqrt{x}} + \sin^{-1}\frac{\sqrt{x}}{\sqrt{x}+1} then \frac{dy}{dx} =$$

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22. $f(x) = x|x|$ is derivable at $x = 0$.

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23. Fill in the blanks : The derivative of $\sec^{-1} x$ w.r.to x if $x < -1$ is ____

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24. Fill in the blanks : The derivative of $\sec^{-1} x$ w.r.to x if $x > 1$ is ____

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25. Fill in the blanks : The derivative of $\cos ec^{-1} x$ w.r.to x if $x > 1$ is ____

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26. Fill in the blanks : The derivative of $\cos ec^{-1}x$ w.r.to x if $x < -1$ is ____

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27. Fill in the blanks : The derivative of $\sin^{-1}x$ w.r.to x if $-1 < x < 1$ is ____

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28. Fill in the blanks : The derivative of $\cos^{-1} x$ w.r. to x if $-1 < x < 1$ is ____

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29. Fill in the blanks : The derivative of $|2x - 5|$ when $x < \frac{5}{2}$ is ____

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30. Give an example of a function which is neither one-one nor onto

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31. Check the injectivity of the following function

f: $R \to R$ is given by f(x) = x^3

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32. What is the maximum value of $\sin x \cos x$ for $x \in \left(\frac{\pi}{6}, \frac{\pi}{2}\right)$?

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33. Find the extreme points of the function $y = x + \frac{1}{x}$.

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34. Does the. normal at any point on a circle passes through the centre of the circle ?

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35. Examine the continuity of the following functions at the indicated points:

$$f(x) = \begin{cases} x + \frac{1}{2} & x \leq \frac{1}{2} \\ \frac{3}{2} - xa & x > \frac{1}{2} \end{cases} atx = \frac{1}{2}$$

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36. examine the contiunity of the following functions at indicated points . $f(x)$=

$$\begin{cases} (1+2x)^{\frac{1}{x}} & \text{if } x \neq 0 \\ (e^2) & \text{if } x = 0 \end{cases} \text{ at } x = 0.$$

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37. Examine the contiunity of the following functions at the indicated points .$f(x)$=

$$\begin{cases} 2x+1 & \text{if } x \leq 0 \\ x & \text{if } 0 < x < 1 \\ 2x-1 & \text{if } x \geq 1 \end{cases} \text{ at } x = 0, 1.$$

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38. Examine the continuity of the following functions at indicated points.

$$f(x) = \begin{cases} \dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1} & \text{if } x \neq 0 at x = 0 \\ 0 \text{ if } x = 0 \end{cases}$$

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39. Examine the contiunity of the the following functions at indicated points. $f(x)=$

$$\begin{cases} \frac{1}{e^{\frac{1}{x}}-1} & \text{if } x>0 \\ 0 & \text{if } x \leq 0 \end{cases} \quad \text{at } x=0$$

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40. Find the derivative of the following functions from definition $\sqrt{x}\cos x$.

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41. Find the derivative of the following functions from definition $\sin^2 x$.

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42. Find the derivative of the following functions from definition $\sqrt{\tan x}$.

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43. $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases} at x = 0$

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44. Test the differentiability of the following functions $f(x) = \frac{x}{1+|x|} atx = 0.$

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45. If $y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$, then find $\frac{dy}{dx}$

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46. If $y = \tan^{-1}(\sec x + \tan x)$, then find $\frac{dy}{dx}$.

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47. Find $\frac{dy}{dx}$, if $y = \sin^{-1}\left(\frac{\sqrt{1+x} + \sqrt{1-x}}{2}\right), -1 < x < 1$

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48. Find $\frac{dy}{dx}$, if $x = \sqrt{\sin 2u}$ and $y = \sqrt{\cos 2u}$

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49. Find $\frac{dy}{dx}$, if $x = a(t - \sin t), y = a(1 - \cos t) att = \frac{\pi}{2}$

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50. If $y\sqrt{x^2+1} = \log\left\{\sqrt{x^2+1} - x\right\}$ then prove that $\left(x^2+1\right)\frac{dy}{dx} + xy + 1 = 0$

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51. If cos y = x cos(a+y) then prove that

$$\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$$

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52. Write the subinterval of $(0, \pi)$ in which sin $\left(x + \frac{\pi}{4}\right)$ is increasing.

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53. Find the intervals where the following functions are (a) increasing and (b) decreasing.

$y = \tan x - 4(x - 2), x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

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54. Show that the line y = mx + c touches the parabola y^2 = 4ax if c $= \frac{a}{m}$.

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55. Find the points on the curve $x^2 + y^2 - 2x - 4y + 1 = 0$, where the tangent is parallel to y-axis

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56. Find the equations of the tangent and the normal to the curve $y = \frac{x - 7}{(x - 2)(x - 3)}$ at the point, where it cuts the X-axis.

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57. Find the angle of intersection of two curves $y = 2^x$ and $y = 5^x$

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58. Show that $\frac{x}{1 + x \tan x,} x \in \left(0, \frac{\pi}{2}\right)$ is maximum when x = cos x.

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59. Show that $f(x) = \frac{\log x}{x}$ has minimum value at x=e

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60. Find $\frac{dy}{dx}$ if $x = \frac{3at}{1+t^3}, y = \frac{3at^2}{1+t^3}$ at $t = \frac{1}{2}$

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61. Find the extreme values of the function $f(x) = x^2 e^{-x^2}$ and show that $f(x) = \leq e^{-1} \forall x \in R.$

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62. A curve passes through the point (2, 0) and slope of tthe tangent at any point (x, y) is $x^2 - 2x$ for all $x \in R$. Show that the point where ordinate is maximum is $\left(0, \frac{4}{3}\right)$.

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63. A figure consists of a semi-circle with a rectangle on its diameter. Given that the perimeter of the figure is 20 cm. Find the dimensions in order that its area may be maximuum.

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64. what is the direction of cosine of the line passing through (4,2,3) and (6,8,4).

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65. Show that the line y=mx+c touches the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if $c^2 = a^2m^2 - b^2$

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66. Find the equation of the tangent and normal to the curve y(x-2)(x-3)-x+7=0 at the point where it cuts the x-axis.

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67. Find $\frac{dy}{dx}$ if $y = \frac{1}{\sqrt{a^2 - x^2}}$

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68. Find $\frac{dy}{dx}$ if $y = \frac{(5x-1)^{\frac{2}{3}}(2x-5)^{\frac{3}{5}}}{(x-1)^{\frac{1}{3}}(3x-2)^3}$

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69. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, then prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

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