MATHS

BOOKS - USHA MATHS (ODIA ENGLISH)

MODEL QUESTION SET

MODEL QUESTION SET

1. What is $\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx$?

Watch Video Solution

2. Write the minimum value of n such that $\frac{d^n}{dx^n}\left(3x^4 + 5\right)^{20} = 0$

Watch Video Solution

3. Write the direction cosines of the normal to the plane $x - 2y + 2z - 10 = 0$

Watch Video Solution

4. What is the principal value of $\sin^{-1}\left(\sin\frac{5\pi}{6}\right)$

Watch Video Solution

5. For what value of λ, the vectors $\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = 8\hat{i} + \lambda\hat{j}$ are perpendicular to each other.

Watch Video Solution

6. If $\underset{n}{I}$ is an identity matrix of order n, then k being a natural number, write the matrix $\underset{n}{I}^k$.

Watch Video Solution

7. What is the value of $\int_0^{\pi} \cos^{101} x dx$?

Watch Video Solution

8. If A is a singular matrix, then what is $A(adjA)$?

Watch Video Solution

9. Minimize: $Z = 6x_1 + 7x_2$

Subject to: $x_1 + 2x_2 \geq 4$

$x_1, x_2 \geq 0$

Watch Video Solution

10. Show that if R is an equivalence relation on X then dom R=rngR =X.

Watch Video Solution

11. Prove that: $\cos^{-1}\left(\frac{12}{13}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{56}{65}\right)$

Watch Video Solution

12. Solve $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$

Watch Video Solution

13. Let f be a real function. Prove that $f(x)-f(-x)$ is an odd function and $f(x)+f(-x)$ is an even function.

Watch Video Solution

14. If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$ then prove that $A^2-5A+7I=O$

Watch Video Solution

15. Express as a sum of a symmetric and a skew symmetric matrix:

$$\begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix}$$

Watch Video Solution

16. If ABC is a triangle, then prove that $\begin{vmatrix} \tan A & 1 & 1 \\ 1 & \tan B & 1 \\ 1 & 1 & \tan C \end{vmatrix} = 2$

Watch Video Solution

17. Evaluate $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} - \begin{vmatrix} 1 & a & b^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

Watch Video Solution

18. Find the inverse of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Watch Video Solution

19. Find the number of critical points of $f(x) = \frac{|x-1|}{x^2}$

Watch Video Solution

20. If $f(x) = \begin{cases} ax^2 + b & \text{if } x < 1 \\ 1 & \text{if } x = 1 \\ 2ax - b & \text{if } x > 1 \end{cases}$

is continuous at x=1, then find a and b.

Watch Video Solution

21. Find the equation of the tangent to the curve $x = y^2 - 1$ at the point where the slope of the normal to the curve is 2.

Watch Video Solution

22. If $y = \sin^{-1}(x^3)$, then find that dy/dx

Watch Video Solution

23. If $y = e^{x^{e^{x^{e^x}}}}$, then find $\frac{dy}{dx}$.

Watch Video Solution

24. Evaluate $\int_0^2 |x^2 + 2x - 3| dx$

Watch Video Solution

25. Find the area of the region bounded by the curve $y = \sin^3 x$ and the straight lines

$x = -\frac{\pi}{4}, x = \frac{\pi}{4}$ and $y = 0$.

Watch Video Solution

26. $\int \frac{dx}{1 + \tan x}$

Watch Video Solution

27. Evaluate $\int e^{\tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right)dx.$

Watch Video Solution

28. Find the solution of the following differential equations:

(4x+6y+5)dx-(2x+3y+4)dy=0

Watch Video Solution

29. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $\left|\vec{a}\right|=3, \left|\vec{b}\right|=5, \left|\vec{c}\right|=7$. Then prove that the angle between $\vec{a}$ and $\vec{b}$ is 60°.

Watch Video Solution

30. Find the image of the point $(2, -1, 3)$ in the plane $3x-2y+z-9=0$

Watch Video Solution

31. Find the co-ordinates of the foot of the perpendicular from the point $(1, 1, 1)$ on the line joining $(1, 4, 6)$ and $(54, 4)$.

Watch Video Solution

32. Resolve the vector $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ into vectors parallel and perpendicular to the vector $\vec{a} = \hat{i} + \hat{j}$.

Watch Video Solution

33. Find the foot of the perpendicular drawn from the point (5,7,3) to the line $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$. Find the length of the perpendicular.

Watch Video Solution

34. If $x = \frac{1-\cos^2\theta}{\cos\theta}, y = \frac{1-\cos^{2n}\theta}{\cos^n\theta}$ then show that $\left(\frac{dy}{dx}\right)^2 = n^2\left(\frac{y^2+4}{x^2+4}\right)$

Watch Video Solution

35. Show that the sum of the intercepts on the coordinate axes of any tangent to the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ is constant.

Watch Video Solution

36. Find $\frac{dy}{dx}$ if y=([sinx+x^2]/[cot2x])`.

Watch Video Solution

37. Integrate $\int\left[\frac{(\sin^2 x+\cos 2x)}{\cos^2 x}\right]dx$

Watch Video Solution

38. Solve the following differential equations

$(x + \tan y)dy = \sin 2ydx$

Watch Video Solution

39. Evaluate $\int_0^1 \frac{\ln(1+x)}{1+x^2}dx$

Watch Video Solution

40. Let $f: X \to Y$ and $g: Y \to Z$. Prove that gof is bijective if both f and g are bijective. Also prove that $(gof)^{-1} = f^{-1}og^{-1}$.

Watch Video Solution

41. If $\sin^{-1}\left(\frac{x}{a}\right) + \sin^{-1}\left(\frac{y}{b}\right) = \sin^{-1}\left(\frac{c^2}{ab}\right)$,

then prove that $b^2x^2 + 2xy\sqrt{a^2b^2 - c^4} + a^2y^2 = c^4$

Watch Video Solution

42. Solve the following LPP graphically Maximize$z = 20x + 10y$

Subject to $x + 2y \leq 40$

$3x + y \geq 30$

$4x + 3y \geq 60$

$x, y \geq 0$

Watch Video Solution

43. A variable plane meets the coordinate axes at A, B, C and is at a constant distance d from origin. Prove that the locus of the centroid of the triangle ABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{9}{d^2}$

Watch Video Solution

44. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitude, show that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}. \vec{b}. \vec{c}.$

Watch Video Solution

Watch Video Solution

45. If 2s=a+b+c show that

$$\begin{bmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{bmatrix} = 2s^3(s-a)(s-b)(s-c)$$

Watch Video Solution

46. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then show that $A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$, $k \varepsilon N$

Watch Video Solution

47. Solve the following system of equations by matrix method

$x + 2y + 3z = 8$, $2x + y + z = 8$ and $x + y + 2z = 6$

Watch Video Solution

48. What is the number of solution of the following system: $2x + 3y = 6, x + y = 3$

Watch Video Solution

49. What is $\int \frac{2x^2 + 3}{(x^2 - 1)(x^2 + 4)} dx$?

Watch Video Solution

50. Write the equation of the plane passing through the point(1,0,0), (0,2,0)and (0,0,3).

Watch Video Solution

51. Given an example of a function which is both odd and even function.

Watch Video Solution

52. If $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}-\vec{b}\right|$ then what is the angle between $\vec{a}$ and $\vec{b}$?

Watch Video Solution

53. Find the principal value of $\sin^1\left(\sin\frac{2\pi}{3}\right)$

Watch Video Solution

54. Find `d/dx[e^(x)+sinx/1+logx]

Watch Video Solution

55. What is the interval in which $y=\sin x, x\in[0,2\pi]$ is increasing?

Watch Video Solution

56. What is the value of $\int_{-1}^{1}x|x|dx$

Watch Video Solution

57. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, prove that $x^2 + y^2 + z^2 + 2xyz = 1$.

Watch Video Solution

58. Find the domain of the function $f(x) = \sqrt{\log\left(\frac{12}{x^2 - x}\right)}$.

Watch Video Solution

59. Solve the pair of these equations`x+y=10,3x+y=12,

Watch Video Solution

60. Show that $\sin^{-1}\left(\frac{1}{\sqrt{10}}\right) + \cos^{-1}\left(\frac{2}{\sqrt{5}}\right) = \frac{\pi}{4}$.

Watch Video Solution

61. If a relation R:$A \to A$is an equivalence relation then prove that R^-1:A toA is also an equivalence relation.

Watch Video Solution

62. Verify that $[AB]^T = B^T A^T$ where $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$

Watch Video Solution

63. Verify that $[AB]^T = B^T A^T$ where $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$

Watch Video Solution

64. α = 0.9. The value of β is

Watch Video Solution

65. If Aand B are both symmetric matrices of same oder,then prove that AB is symmetric if and only if AB=BA.

Watch Video Solution

66. If ABC is a triangle, then prove that $\begin{vmatrix} \tan A & 1 & 1 \\ 1 & \tan B & 1 \\ 1 & 1 & \tan C \end{vmatrix} = 2$

Watch Video Solution

67. Solve by matrix method$2x + y = 5, x - y = 1$

Watch Video Solution

68. Find the point on the curve $y^2 - x^2 + 2x - 1 = 0$

where the tangent is parallel to the x - axis.

Watch Video Solution

69. If $2y = x\left(1 + \frac{dy}{dx}\right)$, then show that y_2 is a constant.

Watch Video Solution

70. If $y = \tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right)$, then prove that $\frac{dy}{dx} = -\frac{1}{2}$

Watch Video Solution

71. Examine the continuity of the following functions at indicated points.

$f(x) = [3x + 11] at x = -\frac{11}{3}$

Watch Video Solution

72. If f$(x+y)=f(x)f(y)$for all real $x, y, f(2)=3, f'(0)=1$then what is the value of $f'(2)$?

Watch Video Solution

73. Find the area of the circle

$x^2+y^2=2ax.$

Watch Video Solution

74. $\int \frac{dx}{\sqrt{x}-\sqrt[3]{x}}(x=t^6)$

Watch Video Solution

75. Solve $\frac{dy}{dx}+\frac{y}{x}=xy^2.$

Watch Video Solution

76. Evaluate $\int_0^2 |x^2 + 2x - 3| dx$

Watch Video Solution

77. Evaluate $\int_{-2}^{2} [x^2 + 3x + 2] dx.$

Watch Video Solution

78. Find the point where the line $\frac{x-2}{1} = \frac{y}{-1} = \frac{z-1}{2}$ meets the plane $2x + y + z = 2.$

Watch Video Solution

79. Find the vector projection of $\vec{a} = \hat{i} - \hat{j} + \hat{k} on \vec{b} = 3\hat{i} + \hat{j} + 3\hat{k}.$

Watch Video Solution

80. Show that the line passing through the points (a_1, b_1, c_1) and (a_2, b_2, c_2) passes through the origin , if $a_1a_2 + b_1b_2 + c_1c_2 = p_1p_2$. where p_1 and p_2 are distances of the points from origin.

Watch Video Solution

81. What is the number of line which are equally inclined to the axes?

Watch Video Solution

82. Prove that `(d)/(dx)[(sin2x+1/(cos^2x)]

Watch Video Solution

83. If $y = e^x + a^x + x^x$, then find dy/dx at x=a

Watch Video Solution

84. Show that the semivertical angle of a cone of given slant height is $\tan^1 \sqrt{2}$ when its volume is maximum.

Watch Video Solution

85. If $a = 2\hat{i} + \hat{k}, b = \hat{i} + \hat{j} + \hat{k}$ and $c = 4\hat{i} - 3\hat{j} + 7\hat{k}$, then find the vector $\overrightarrow{r}$ which satisfies $r \times b = c \times b$ and $r.\, a = 0$.

Watch Video Solution

86. If a line in the space makes angles α, β and γ with the coordinate axes, then find the value of $\cos 2\alpha + \cos 2\beta + \cos 2\gamma + \sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$.

Watch Video Solution

87. If x,y,z are positive and are the pth, qth and rth terms of a G.P. then prove that

$$\begin{vmatrix} \log x & p & 1 \\ \log y & q & 1 \\ \log z & r & 1 \end{vmatrix} = 0$$

Watch Video Solution

88. If $A = \begin{bmatrix} 0 & -\tan\left(\frac{\alpha}{2}\right) \\ \tan\left(\frac{\alpha}{2}\right) & 0 \end{bmatrix}$ show that

$(I + A) = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Watch Video Solution

89. Prove that $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

Watch Video Solution

90. Prove that

$$\tan\left\{\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right\}=\frac{2b}{a}.$$

Watch Video Solution

91. $\int\frac{x^2+1}{x^4+x^2+1}dx$

Watch Video Solution

92. Show that

$$\int_0^1\frac{Inx}{\sqrt{1-x^2}}dx=\frac{\pi}{2}In\frac{1}{2}$$

Watch Video Solution

93. Solve the following differential equations

$$\left(1-x^2\right)\frac{dy}{dx}+2xy=x\sqrt{1-x^2}$$

Watch Video Solution

94. If A is a non-singular square matrix of order n, then what is $|adjA|$?

Watch Video Solution

95. Write the subinterval of $(0, \pi)$ in which $\sin\left(x + \frac{\pi}{4}\right)$ is increasing.

Watch Video Solution

96. Find least non negative integer r such that $1936 \times 8789 \equiv r(\mathrm{mod}4)$

Watch Video Solution

97. What is the number of independent constants in the equations of a plane ?

Watch Video Solution

98. What is the antiderivative of 5^{5+x}?

Watch Video Solution

99. What is the order of the matrix B if [2 3 5 -1] B=[1 3 2 5 -4 1] .

Watch Video Solution

100. What is $\left|\vec{a}-\vec{b}\right| =$ if $\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 4$ and $\left|\vec{a}+\vec{b}\right| = 5$?

Watch Video Solution

101. Find the principal value of $\sin^1\left(\sin\frac{2\pi}{3}\right)$

Watch Video Solution

102. What is the value of $\frac{d}{dx}\int_{150}^{300}\left(x^3+3x^2\right)^3 dx$?

Watch Video Solution

103. What is the value of $\frac{d}{dx}\left[\ln \sin^{-1}\left\{\cos\left(\frac{\pi - 2e^x}{2}\right)\right\}\right]$?

Watch Video Solution

104. Let f= {(1,3),(2,4),(3,7)} and g ={(3,2),(4,3),(7,1)}

Determine gof and fog if possible . Test whether fog =gof.

Watch Video Solution

105. If y=sinx+cosx then what is [dy/dx]^2.

Watch Video Solution

106. Prove that $f: X \to$ Y is surjective iff for all $B \subseteq Y, f\left(f^{-1}(B)\right) = B.$

Watch Video Solution

107. Find the maximum value of Z = 2x + 3y subject to conditions $x + y \leq 1$ and $x \geq 0, y \geq 0$.

Watch Video Solution

108. Prove that $\sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right) = \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x, 0 < x < 1$

Watch Video Solution

109. Show that the relation ~ on $\mathbb{Z} - \{0\} \times \mathbb{Z} - \{0\}$ defined by $(m, n)\sim(p, q) \Leftrightarrow mq = np$ is an equivalence relation.

Watch Video Solution

110. Solve by matrix method. 2x + y = 5 , x-y =1

Watch Video Solution

111. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $n \in N$, then $A^n = 2^{n-1}A$.

Watch Video Solution

112. If $\begin{bmatrix} x & y \\ x & \frac{x}{2} + x \end{bmatrix} + \begin{bmatrix} y & x + t \\ \frac{x}{2} & \frac{x}{2} \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 2 & 2 \end{bmatrix}$ then find the values of x,y,z and t.

Watch Video Solution

113. Evaluate the determinant $|(4, 6, -2), (1, -4, 3), (7, -2, 1)|$

Watch Video Solution

114. If $\begin{vmatrix} a & b & a-b \\ b & c & b-c \\ 2 & 1 & 0 \end{vmatrix} = 0$, then prove that a,b,c are in GP.

Watch Video Solution

115. evaluate $\lim_{x \to 1} \frac{x^2 - 3x + 2}{x - 1}$.

Watch Video Solution

116. If $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$ then prove that $x^3 y \frac{dy}{dx} = 1$.

Watch Video Solution

117. Find the maximum and minimum of f(x)=|x+3|,for all x belongs to R.

Watch Video Solution

118. Find the interval on which the function f(x)=x^x decreases.

Watch Video Solution

119. Find the solution of the following differential equations:

(4x+6y+5)dx-(2x+3y+4)dy=0

Watch Video Solution

120. Integrate $\int\left(\frac{1-\cos x}{1+\cos x}\right)dx$.

Watch Video Solution

121. Solve $(x+y)^2\frac{dy}{dx}=a^2$.

Watch Video Solution

122. If f is periodic with period 3 and f(1) = 1 then what is the value of $\int_1^3 f(x)e^x dx + \int_1^3 f'(x)e^x dx$?

Watch Video Solution

123. Evaluate $\int_{-2}^{2} |[x]| dx$.

Watch Video Solution

124. Prove that $\left(\vec{a} \times \vec{b}\right)^2 = a^2b^2 - \left(\vec{a}.\ \vec{b}\right)^2$.

Watch Video Solution

125. Find the perpendicular distance of the point $(-1, 3, 9)$ from the line $\frac{x-13}{5} = \frac{y+8}{-8} = \frac{z-31}{1}$

Watch Video Solution

126. Prove that $\left(\vec{a} \times \hat{i}\right)^2 + \left(\vec{a} \times \hat{j}\right)^2 + \left(\vec{a} \times \hat{k}\right)^2 = 2\vec{a}^2$.

Watch Video Solution

127. Find the equation of planes passing throught the points $(6, -1, 1)$, (5,1,2) and (1,-5,-4)`

Watch Video Solution

128. Find the co-ordinates of the foot of the perpendicular drawn from the point (1,3,4) to the line joining the points $(3, 0, -1)$ and $(0, 1, -2)$.

Watch Video Solution

129. If $e^{y/x} = \frac{x}{a + bx}$ then show that $x^3 \frac{d}{dx}\left(\frac{dy}{dx}\right) = \left(x\frac{dy}{dx} - y\right)^2$

Watch Video Solution

130. If $2x = y^{\frac{1}{m}} + y^{-\frac{1}{m}}$, then prove that $(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$.

Watch Video Solution

131. Show that the rectangle of maximum area that can be inscribed in a given circle is a square.

Watch Video Solution

132. Prove the following by vector method. Altitudes of a triangle are concurrent.

Watch Video Solution

133. Find the magnitude and equation of the line of shotest distance between the lines $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$ and

$$\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$$

Watch Video Solution

134. Solve the following by matrix method x-y+z=4, 2x+y-3z=0, x+y+z=2

Watch Video Solution

135. Prove that $\begin{vmatrix} -2a & a+b & c+a \\ a+b & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix} = 4(b+c)(c+a)(a+b)$

Watch Video Solution

136. Show that the function $f: R^+ \to R^+$ defined by $f(x) = x + \frac{1}{x}$ is injective, but not surjective .

Watch Video Solution

137. Solve

$$\sin^{-1}x - \cos^{-1}x = \cos^{-1}\frac{\sqrt{3}}{2}$$

Watch Video Solution

138. Solve: $2\sin^{-1}x + \sin^{-1}(1-x) = \frac{\pi}{2}$

Watch Video Solution

139. Evaluate $\int \ln(x^2 + x + 2)\,dx$

Watch Video Solution

140. $\int_0^{\pi} \left(\frac{x\tan x}{\sec x + \tan x}\right) \text{dx}$

Watch Video Solution

141. Solve the differential equation $\frac{dy}{dx} = \frac{y - x + 1}{y + x + 5}$.

Watch Video Solution

142. Differentiate $\cos^{-1}(\sin x)$ w.r.t.x.

Watch Video Solution

143. $\int \frac{\cot x dx}{\ln \sin x} = ?$

Watch Video Solution

144. How many independent constants are there in the general equation of a plane $ax + by + cz + d = 0$?

Watch Video Solution

145. What is the range of the signum function?

Watch Video Solution

146. If the vectors $\hat{i}+2\hat{j}+\hat{k}$ & $2\hat{i}+3\hat{j}+\alpha\hat{k}$ are perpendicular, find α

Watch Video Solution

147. Between tan 1 and $\tan^{-1}1$, which is greater?

Watch Video Solution

148. Answer the following: In which inverval does the determinant

$$A=\begin{bmatrix}1 & \sin\theta & 1\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{bmatrix}$$ lie?

Watch Video Solution

149. What is AB – BA if A, B and AB are all symmetric matrices .

Watch Video Solution

150. If y =(cosx+sinx)/cosx, then what is dy/dx ?

Watch Video Solution

151. Write the order of the differential equation whose general solution is

$y = ax^2 + b$

Watch Video Solution

152. Consider the binary operation * on the set

{1,2,3,4,5} defined by $a * b$= min{a,b}. Write operation table of operation *.

Watch Video Solution

153. Let ~ be defined by (m,n)~(p,q) if mq=np where m, n, p,$q \in Z$-{0}. Show that it is an equivalence relation.

Watch Video Solution

154. Find the feasible solution region of the following system of inequations: $x + 2y \leq 8, x - 2y \geq 4, x \geq 0, y \geq 0$

Watch Video Solution

155. Show that $\sin^{-1}\frac{4}{5} + 2\tan^{-1}\frac{1}{3} = \frac{\pi}{2}$.

Watch Video Solution

156. Construct an example to show that $f(A \cap B) \neq f(A) \cap f(B)$ where $A \cap B \neq \theta$.

Watch Video Solution

157. Express $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \\ -1 & 5 & -2 \end{bmatrix}$ as a sum of a symmetric and a skew – symmetric matrix.

Watch Video Solution

158. Without expanding prove $\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$

Watch Video Solution

159. Find the inverse of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Watch Video Solution

160. Find A and B where

2A+B= $\begin{bmatrix} 2 & 2 & 5 \\ 5 & 4 & 3 \\ 1 & 1 & 4 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} 1 & 6 & 5 \\ 5 & 2 & -1 \\ -2 & -2 & 2 \end{bmatrix}$

Watch Video Solution

161. Solve for x,

$$\begin{vmatrix} 15-2x & 11 & 10 \\ 11-3x & 17 & 16 \\ 7-x & 14 & 13 \end{vmatrix} = 0$$

Watch Video Solution

162. Find absolute maximum and absolute minimum of $f(x) = \begin{cases} (x+1)^2 & x \leq 0 \\ (x-1)^2 & x > 0 \end{cases}$ in [-1,1]

Watch Video Solution

163. If $x^7 y^3 = (x+y)^{10}$, then find $\frac{d^2y}{dx^2}$

Watch Video Solution

164. If $y = e^{ax} \sin bx$ show that $y_2 - 2ay_1 + \left(a^2 + b^2\right)y = 0.$

Watch Video Solution

165. Find the equation of tangent to the curve $x = y^2 - 2$ at the points where slope of the normal equal to (-2).

Watch Video Solution

166. Find the intervals in which the function $y = \frac{\ln x}{x}$ is increasing and decreasing.

Watch Video Solution

167. Solve $\frac{dy}{dx} = x + y$

Watch Video Solution

168. $\int \frac{2\sin x + 3\cos x}{3\sin x + 4\cos x} dx$=?

Watch Video Solution

169. Solve $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$.

Watch Video Solution

170. Integrate $\int_{-3/5}^{3/5} [2x + 1] dx$

Watch Video Solution

171. Evaluate $\int_0^1 \frac{x^5}{\sqrt{1 - x^2}} dx$

Watch Video Solution

172. Find the co-ordinates of the point where the perpendicular from the origin meets the line joining the points $(-9, 4, 5)$ and $(11, 0, -1)$.

Watch Video Solution

173. Resolve the vector $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ into vectors parallel and perpendicular to the vector $\vec{a} = \hat{i} + \hat{j}$.

Watch Video Solution

174. Find the equation of the plane passing through the line x = y = z and the point (3,2,1).

Watch Video Solution

175. Find the equation of the line through (-1,0,1) and perpendicular to the plane x + 2y + 1=0.

Watch Video Solution

176. Prove by vector method that in a ΔABC, $c^2 = a^2 + b^2 - 2ab\cos C$.

Watch Video Solution

177. Show that

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

Watch Video Solution

178. Prove the following by vector method. Altitudes of a triangle are concurrent.

Watch Video Solution

179. Prove that the measure of the angle between two main diagonals of a cube is $\cos^{-1}\frac{1}{3}$.

Watch Video Solution

180. Let f : R $\rightarrow$ R be defined by f(x)=3x+5. Show that f is bijective. Find $f^{-1}(1)$ and $f^{-1}(0)$.

Watch Video Solution

181. Prove that $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$

Watch Video Solution

182. Find the extreme values of the function $f(x)=x^2e^{-x^2}$ and show that $f(x)\leq e^{-1}\forall x\in R$.

Watch Video Solution

183. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values.

$y = (x-2)^3(x+3)^4$

Watch Video Solution

184. Show that $f(x) = [3x + 11]$ is discontinuous at $x = -\frac{11}{3}$

Watch Video Solution

185. Find the solution of the following differential equations:

(4x+6y+5)dx-(2x+3y+4)dy=0

Watch Video Solution

186. $\int_0^{\pi} \frac{x dx}{1+\sin x}$

Watch Video Solution

187. Show that

$$\int_0^{\pi} x In \sin x = \frac{\pi^2}{2} In \frac{1}{2}$$

Watch Video Solution

188. If A is a 4×5 matrix and B is a matrix such that $A^T B$ and BA^T both are defined, then write the order of B.

Watch Video Solution

189. What is $F'(x)$ if $F(x) = \int_0^x e^{3t} \cos 4t dt$?

Watch Video Solution

190. Find r if $-501 \in [r]_5$.

Watch Video Solution

191. To which coordinate axis is the plane 2y – 5z=0 parallel ?

Watch Video Solution

192. What does the solution set of the inequation $2x + y > 5$ represents?

Watch Video Solution

193. What is the value of $\int \frac{d}{dx} f(x)dx - \frac{d}{dx}\int f(x)dx$?

Watch Video Solution

194. IF $\begin{bmatrix} 3 & 5 & 3 \\ 2 & 4 & 2 \\ \lambda & 7 & 6 \end{bmatrix}$ is a singular matrix, then write the value of λ

Watch Video Solution

195. What is the angle between $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$?

Watch Video Solution

196. Write a logarithmic functions which is differentiable atevery point in R.

Watch Video Solution

197. Write the equation of the tangent to the curve $y=\sqrt{x}$ at the point (-4, 4).

Watch Video Solution

198. Solve:$\sin^{-1}x+\sin^{-1}(1-x)=\cos^{1}x$

Watch Video Solution

199. Find the following congruence classes as a set $[1]_3$,

Watch Video Solution

200. Find the following congruence classes as a set $[3]_7$

Watch Video Solution

201. Find the least positive integer r such that $114 \in [r]_7$

Watch Video Solution

202. Find the least positive integer r such that $506 \in [r]_5$

Watch Video Solution

203. Solve graphically: Maximize Z=3x+ 5y Subject to $x + y \leq 1, x \geq 0, y \geq 0$

Watch Video Solution

204. Express the value of $\sin^{-1}\frac{1}{\sqrt{5}} + \cos(-1)\frac{3}{\sqrt{10}}$ in simplest form.

Watch Video Solution

205. If f:X $\rightarrow$ Y and g:Y $\rightarrow$ Z be two bijective functions, then prove that $(gof)^{-1}$ =f^(-1)og(-1)`.

Watch Video Solution

206. Show that x=2 is a root of

$$\begin{bmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{bmatrix} = 0$$

Solve this completely,

Watch Video Solution

207. Find A if $A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

Watch Video Solution

208. If $A = \begin{bmatrix} 1 & -2 & 2 \\ 3 & 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 \\ 1 & 2 \\ 3 & -1 \end{bmatrix}$, then verify that $(AB)^T = B^T A^T$.

Watch Video Solution

209. Find B if $B^2 = \begin{bmatrix} 17 & 8 \\ 8 & 17 \end{bmatrix}$

Watch Video Solution

210. Prove that: $\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix}$ is a perfect square.

Watch Video Solution

211. If $y\sqrt{x^2+1} = \log\left\{\sqrt{x^2+1} - x\right\}$ then prove that $\left(x^2+1\right)\frac{dy}{dx} + xy + 1 = 0$

Watch Video Solution

212. If $y = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$, then prove that dy/dx=-1

Watch Video Solution

213. Let $f(x) = \begin{cases} \frac{1}{x+[x]} & \text{if } x < 0 \\ -1 & \text{if } x \geq 0 \end{cases}$ Examine the continuity of f(x) at x=0.

Watch Video Solution

214. Show that the function $\frac{e^x}{x^p}$ is strictly increasing for $x > p > 0$.

Watch Video Solution

215. Find the equation of tangents and normals to the parabola $y^2 = 4ax$ at the ends of its latus rectum.

Watch Video Solution

216. Find the solution of the following differential equations:

(4x+6y+5)dx-(2x+3y+4)dy=0

Watch Video Solution

217. Evaluate $\int e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) dx.$

Watch Video Solution

218. Evaluate the integral: $\int \frac{x^2 - 4}{x - 2} dx$

Watch Video Solution

219. Solve the following differential equations

$(x + \tan y)dy = \sin 2y dx$

Watch Video Solution

220. Evaluate $\int_0^7 \left[\frac{x}{3}\right] dx$

Watch Video Solution

221. Prove that the point (1,2,3),(-1,1,0),(2, 1, 3) and (1, 1, 2) are coplanar.

Watch Video Solution

222. If $\vec{a}$ and $\vec{b}$ are unit vectors represented by the adjacent sides of a regular hexagon, taken in order, what are the vectors represented by the other sides taken in order?

Watch Video Solution

223. Find the distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$, measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$

Watch Video Solution

224. Find the angle which a diagonal of a cube makes with one of its edges.

Watch Video Solution

225. Find the value of k for which A (1,0,3), B`(-1,2,4), C(1,2,1) and D(k,2,5) are coplaner.

Watch Video Solution

226. Show that $\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)=2\left(\vec{a}\times\vec{b}\right)$. Interpret this result geometrically.

Watch Video Solution

227. Find the area of the parallelogram whose diagonals are vectors $3\hat{i}+\hat{j}-2\hat{k}$ and $\hat{i}-3\hat{j}+4\hat{k}$.

Watch Video Solution

228. Prove that the lines $\frac{x+4}{3}=\frac{y+6}{5}=\frac{z-1}{-2}$ and $3x-2y+z+5=0=2x+3y+4z-4$ are co-planar.

Watch Video Solution

229. Find the equation of the tangent and normal to the curve y(x-2)(x-3)-x+7=0 at the point where it cuts the x-axis.

Watch Video Solution

230. Show that the semivertical angle of a cone of given slant height is $\tan^1 \sqrt{2}$ when its volume is maximum.

Watch Video Solution

231. Let f be defined by $f(x) = 2x + 1$ for all ξnR. Show that f is bijective and determine the inverse of f. Find $f^{-1}(0)$, $f^{-1}(2)$ and $f^{-1}(-1)$.

Watch Video Solution

232. Show that the relation ~ on $\mathbb{Z} - \{0\} \times \mathbb{Z} - \{0\}$ defined by $(m, n) \sim (p, q) \Leftrightarrow mq = np$ is an equivalence relation.

Watch Video Solution

233. If A+B+C = π, prove that

$$\begin{bmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{bmatrix} = 0$$

Watch Video Solution

234. Find the inverse of the following matrix using elementary transformation : $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 1 & 0 & 2 \end{bmatrix}$

Watch Video Solution

235. Express $A = ((1, 2), (2, 3)$ as the sum of a symmetric and a skew symmetric matrix.

Watch Video Solution

236. Prove that $\int \frac{dx}{1 + \cos x + \sin x} = \ln\left|1 + \tan\frac{x}{2}\right| + c$.

Watch Video Solution

237. Solve (x-y+1)dx-(x+y+5)dy=0

Watch Video Solution

238. Give example of a function whose is continuous but not differentiable at x=2.

Watch Video Solution

239. If $\left|\vec{a}\right| = 10, \left|\vec{b}\right| = 1$ and $\vec{a}.\vec{b} = 0$ then what is the value of $\left|\vec{a} \times \vec{b}\right|$?

Watch Video Solution

240. Express the value of $\sin\cos^{-1}\tan\sec^{-1}\sqrt{2}$in simplest form.

Watch Video Solution

241. Give example of a function which is increasing in $(-\infty, 2)$ and $(3, \infty)$ and decreases in (2,3).

Watch Video Solution

242. For a 2×2 matrix A, if A.adjA=$\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then what is the value of |A| ?

Watch Video Solution

243. What is the distance of point (3,-4, 5) from z-axis ?

Watch Video Solution

244. What is the value of $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$?

Watch Video Solution

245. What is the value of $\int_0^{100} \left[\tan^{-1} x\right] dx$?

Watch Video Solution

246. Write the value of $\int e^x e^{e^x} e^{e^{e^x}} dx.$

Watch Video Solution

247. Prove the $\sin^{-1}\sqrt{\frac{x-q}{p-q}} = \cos^{-1}$sqrt((p-x)/(p-q))="cot"^(-1) sqrt((p-x)/(x-q))`

Watch Video Solution

248. Let X and Y be sets containing m and n elements respectively.What is the total number of functions from X to Y.

Watch Video Solution

249. Solve the following LPP graphically: maximize Z=5x+7y subject to $3x + y \leq 3, x \geq 0, y \geq 0$

Watch Video Solution

250. Solve:$\sin^{-1}2x + \sin^{-1}x = \frac{\pi}{3}$.

Watch Video Solution

251. Show that the operation * given by x*y=x+y+ -xy is a binary oeration on Z,Q and R but not on N.

Watch Video Solution

252. If ax+hy+g=0, hx+by+f=0 and gx+fy+c=λ, find the value of λ in the form of a determinant.

Watch Video Solution

253. If A+B+C = π, prove that

$$\begin{bmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{bmatrix} = 0$$

Watch Video Solution

254. Find the adjoint of the following matrice.

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & -1 & 2 \\ 1 & 3 & -2 \end{bmatrix}$$

Watch Video Solution

255. If A and B are matrices of the same order and AB=BA, Then prove that

$A^2 - B^2 = (A - B)(A + B)$

Watch Video Solution

256. Solve by matrix method: 2x + 3y=1,4x+5y=3

Watch Video Solution

257. If $e^x + e^y = e^{x+y}$, find $\frac{dy}{dx}$.

Watch Video Solution

258. If $x^7y^3 = (x+y)^{10}$, then find $\frac{d^2y}{dx^2}$

Watch Video Solution

259. Determine the values of x for which the function $f(x) = x^x, x > 0$ is increasing or decreasing.

Watch Video Solution

260. If $x = \cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right), y = \sin^{-1}\left(\frac{1}{\sqrt{t^2+1}}\right)$ then $\frac{dy}{dx}$ is independent of t.

Watch Video Solution

261. Find the angle between the tangents to the curve $y = x^2 - 5x + 6$ at the points (2,0) and (3,0).

Watch Video Solution

262. Find the area of the region bounded by $y = 6x - x^2$x-axis and between ordinates x = 0 and x = 6.

Watch Video Solution

263. $\int \sin^{-1} \sqrt{\frac{x}{a+x}} dx =$ _____.

Watch Video Solution

264. Solve : $x \frac{dy}{dx} + y = xy^2$

Watch Video Solution

265. Evaluate $\int_3^6 \frac{\sqrt{x}}{\sqrt{(9-x)} + \sqrt{x}} dx$

Watch Video Solution

266. Evaluate $\int_{-1}^{2} \{|x| + [x]\} dx$

Watch Video Solution

267. The plane $4x + 7y + 4z + 81 = 0$ is rotated through a right angle about its line of intersection with the plane $5x + 3y + 10z - 25 = 0$. Find the equation of the plane in new position.

Watch Video Solution

268. If G is the centroid of a triangle ABC, prove that $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = 0$

Watch Video Solution

269. Find the image of the point $(2, -1, 3)$ in the plane $3x - 2y + z - 9 = 0$

Watch Video Solution

270. Prove that the measure of the angle between two main diagonals of a cube is $\cos^{-1}\frac{1}{3}$.

Watch Video Solution

271. Show that the vectors $4\hat{i}+4\hat{j}+4\hat{k}$, $7\hat{i}+6\hat{j}-\hat{k}$ and $3\hat{i}+2\hat{j}-5\hat{k}$ form a right angled triangle and find its area.

Watch Video Solution

272. Integrate $\int\frac{dx}{x^4+1}$

Watch Video Solution

273. Solve $x^2ydx-\left(x^3+y^3\right)dy=0$.

Watch Video Solution

274. Prove by vector method that in any triangle ABC, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

Watch Video Solution

275. If a line in the space makes angles α, β and γ with the coordinate axes, then find the value of $\cos 2\alpha + \cos 2\beta + \cos 2\gamma + \sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$.

Watch Video Solution

276. If the mapping f and g are given by f= {(1,2), (3, 4), (5, 6), (7, 8)}, g={(2, 5),(4, 7),(6, 3),(8, 1)} then find (i) gof (ii) fog. Hence show that composition of functions is not commutative.

Watch Video Solution

277. Prove that $\cos^{-1}\left(\frac{b+a\cos x}{a+b\cos x}\right)=2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)$

Watch Video Solution

278. Show that the line y=mx+c touches the curve $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ if $c^2=a^2m^2-b^2$

Watch Video Solution

279. Show that the shrtest distance of the point (0, 8a) from the curve $ax^2=y^3$ is $2a\sqrt{11}$.

Watch Video Solution

280. Show that the curves $y=2^x$ and $y=5^x$ intersect at an angle $\tan^{-1}\left|\frac{1n\left(\frac{5}{2}\right)}{1+1n21n5}\right|$.

Note Angle between two curves is the angle between their tangents at the point of intersection.

Watch Video Solution

281. Prove the following:

$$\begin{bmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{bmatrix}$$

$$=\left(a^2+b^2+c^2\right)(a+b+c)(b-c)(c-a)(a-b)$$

Watch Video Solution

282. Find the inverse of the following matrix using elementary transformation : $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 1 & 0 & 2 \end{bmatrix}$

Watch Video Solution

283. If A is a singular matrix, then what is $A(adjA)$?

Watch Video Solution

284. Give example of a function which is increasing in $(-\infty, 2)$ and $(3, \infty)$ and decreases in (2,3).

Watch Video Solution

285. If the direction angles of a line are $\alpha = 30^\circ$ and $\beta = 60^\circ$, find the other direction angle γ

Watch Video Solution

286. Give an example of a relation which is not a function.

Watch Video Solution

287. Write the integrating factor of the differential equation $\frac{dy}{dx}(x+y+1)=1$.

Watch Video Solution

288. $\tan\left\{\left(\frac{1}{2}\right)\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\frac{1}{2}\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right\}$.

Watch Video Solution

289. What is the angle between $\vec{a}$ and $\vec{b}$ if $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}-\vec{b}\right|$?

Watch Video Solution

290. If $M=\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$,then what is M^{50} ?

Watch Video Solution

291. Write the value x for which $\frac{d}{dx}\sin\left(\sin^{-1}x\right) = 1$

Watch Video Solution

292. What is the value of $\int_{-1}^{1} \ln\left(\frac{4-x}{4+x}\right)dx$?

Watch Video Solution

293. Prove the following statements

$\cot^{-1}9 + \operatorname{cosec}^{-1}\frac{\sqrt{41}}{4} = \frac{\pi}{4}$

Watch Video Solution

294. Show that if R is an equivalence relation on X then dom R=rngR =X.

Watch Video Solution

295. Solve graphically: Maximize Z=5x+6y subject to $2x + 3y \leq 6, x, y \geq 0$

Watch Video Solution

296. Solve $\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$

Watch Video Solution

297. Let f = {(1,3), (2,4), (3,7)} and g = {(3,2), (4,3), (7,1)} determine gof?

Watch Video Solution

298. Prove that $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

Watch Video Solution

299. If $\begin{vmatrix} a & b & a-b \\ b & c & b-c \\ 2 & 1 & 0 \end{vmatrix} = 0$, then prove that a,b,c are in GP.

Watch Video Solution

300. Verify that $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ satisfies the equation $A^2 - (a+d)A + (ad-bc)I = 0$ where I is the 2 x2 unit matrix.

Watch Video Solution

301. Solve : $\begin{bmatrix} 7 & 6 & x \\ 2 & x & 2 \\ x & 3 & 7 \end{bmatrix} = 0$

Watch Video Solution

302. Verify that $[AB]^T = B^T A^T$ where $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$

Watch Video Solution

303. Prove the inequality

$x^2 e^{-x^2} \leq e^{-1}, x \in R.$

Watch Video Solution

304. Prove that $\frac{d}{dx} \ln \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) = \sec x.$

Watch Video Solution

305. Determine the point on the curve y = ln x, at which the tangent will be parallel to the chord joining the points P(1, 0) and Q(e, 1).

Watch Video Solution

306. Find the angle of intersection of two curves $y = 2^x$ and $y = 5^x$

Watch Video Solution

307. If $2y = x\left(1 + \frac{dy}{dx}\right)$, then show that y_2 is a constant.

Watch Video Solution

308. Find the solution of the following differential equations:

(4x+6y+5)dx-(2x+3y+4)dy=0

Watch Video Solution

309. $\int \frac{xe^x}{1+x^2} dx$

Watch Video Solution

310. Solve $\frac{dy}{dx} - y = e^x$.

Watch Video Solution

311. Evaluate $\int_{-2}^{2} |[x]| dx$.

Watch Video Solution

312. $\int \frac{dx}{\sqrt{x} - \sqrt[3]{x}} (x = t^6)$

Watch Video Solution

313. Find the co-ordinates of the foot of the perpendicular from the point $(1, 1, 1)$ on the line joining $(1, 4, 6)$ and $(54, 4)$.

Watch Video Solution

314. Show that $\left[\vec{a} + \vec{b}\,\vec{b} + \vec{c}\,\vec{c} + \vec{a}\right] = 2\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

Watch Video Solution

315. Find the points of intersection of the line $\frac{x-1}{1}=\frac{y+2}{3}=\frac{z-1}{-1}$ and the plane $2x+y+z=9$.

Watch Video Solution

316. If the magnitude of the difference of two unit vectors is $\sqrt{3}$ then find the magnitude of their sum.

Watch Video Solution

317. Using vector method prove that $\cos A=\frac{b^2+c^2-a^2}{2bc}$

Watch Video Solution

318. If 2s=a+b+c show that

$$\begin{bmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{bmatrix} = 2s^3(s-a)(s-b)(s-c)$$

Watch Video Solution

319. Solve the following by matrix method x-y+z=4, 2x+y-3z=0, x+y+z=2

Watch Video Solution

320. In any triangle ABC, prove by vector method that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Watch Video Solution

321. Find the distance of the point (3,-4,5)from the plane 2x+5y-6z-19=0 measured parallel to the line $\frac{x-1}{2} = \frac{y}{1} = \frac{z+3}{-2}$.

Watch Video Solution

322. Prove that the two lines whose direction cosines are connected by the equations $l + 2m + 3n = 0, 3lm - 4\ln + mn = 0$ are perpendicular to each other.

Watch Video Solution

323. Integrate the following $\int e^{\cos^2 x} \sin 2x dx$

Watch Video Solution

324. Solve the differential equation $\frac{dy}{dx} = \frac{y - x + 1}{y + x + 5}$.

Watch Video Solution

325. Prove that $\int_0^{\frac{\pi}{2}} \ln \sin x dx = \frac{\pi}{2} \ln\left(\frac{1}{2}\right)$

Watch Video Solution

326. If $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$ and $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$, then find $\frac{dy}{dx}$.

Watch Video Solution

327. Let $f: X \to Y$ and $g: Y \to Z$. Prove that gof is bijective if both f and g are bijective. Also prove that $(gof)^{-1} = f^{-1}og^{-1}$.

Watch Video Solution

328. If $\cos^{-1}\left(\frac{x}{a}\right) = \cos^{-1}\left(\frac{y}{b}\right) = \theta$, prove that $\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\theta + \frac{y^2}{b^2} = \sin^2\theta$.

Watch Video Solution

329. Minimize: $Z = 20x_1 + 10x_2$

Subject to: $x_1 + 2x_2 \leq 40$

$3x_1 - x_2 \geq 30$

$4x_1 + 3x_2 \geq \ = 60$

$x_1, x_2 \geq 0$

Watch Video Solution

330. If $y = x \sin^{-1} x + x \cos^{-1} x$, then what is $\frac{dy}{dx}$?

Watch Video Solution

331. Give an example of a relation which is symmetric and anti-symmetric.

Watch Video Solution

332. What is the value of $\int_0^1 \sin^2 t dt + \int_0^1 \cos^2 t dt - \int_0^1 dr$?

Watch Video Solution

333. What is the principal value of $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$?

Watch Video Solution

334. What is the value of $\begin{vmatrix} i^{103} & 3 & i^{101} \\ i^{56} & 5 & i^{54} \\ i^{23} & 7 & i^{21} \end{vmatrix}$

Watch Video Solution

335. What is the value of $\hat{a}\cdot\hat{b}+\hat{b}\cdot\hat{c}+\hat{c}\cdot\hat{a}$ if $\hat{a}+\hat{b}+\hat{c}=\vec{0}$?

Watch Video Solution

336. The distance between the parallel planes $2x-3y+6z+1=0$ and $4x-6y+12z-5=0$ is____

Watch Video Solution

337. If is a 3×3 matrix and $|A| = 7$, then which matrix is represented by $A \times adjA$?

Watch Video Solution

338. What is the value of $\int \sin^2 x d(\sin x)$?

Watch Video Solution

339. Write the subinterval of $(0, \pi)$ in which $\sin\left(x + \frac{\pi}{4}\right)$ is increasing.

Watch Video Solution

340. Write whether the equation $\tan^{-1}(\cot x) = 2x$ has exactly two real solutions statements are true or false.

Watch Video Solution

341. Find the feasible solution region of the following:

$3x + y \geq 3, x + y \geq 1, x \geq 0, y \geq 0$

Watch Video Solution

342. Prove statement $\tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right) + \frac{7}{17} = 0$

Watch Video Solution

343. Prove that for any $f: X \to Y$, $foid_x = f = id_Y$ of.

Watch Video Solution

344. If $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, then find A^{-1}

Watch Video Solution

345. Evaluate $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} - \begin{vmatrix} 1 & a & b^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

Watch Video Solution

346. If $\begin{bmatrix} x+y & y-z \\ 5-t & 7+x \end{bmatrix} = \begin{bmatrix} t-x & z-t \\ z-y & x+z+t \end{bmatrix}$, then find the values of x, y, z and t.

Watch Video Solution

347. Express as a sum of a symmetric and a skew symmetric matrix:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \\ -1 & 5 & -2 \end{bmatrix}$$

Watch Video Solution

348. Prove the following : $\begin{bmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{bmatrix} = \left(1-x^3\right)^2$

Watch Video Solution

349. Find the intervals where the following functions are (a) increasing and (b) decreasing. $y = \tan x - 4(x - 2), x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Watch Video Solution

350. Find $\frac{dy}{dx}$, if $y = \tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right)$.

Watch Video Solution

351. Show that the line y = mx + c touches the parabola y^2 = 4ax if c $= \frac{a}{m}$.

Watch Video Solution

352. Use the function $f(x) = x^{1/x}, x > 0$ to show that $e^{\pi} > \pi^{e}$.

Watch Video Solution

353. Let $f(x) = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ Examine the continuity of f(x) at x=0.

Watch Video Solution

354. $\int \frac{1 + x^2}{x\sqrt{x^4 + 1}} dx$

Watch Video Solution

355. Determine the area within the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Watch Video Solution

356. $\int_0^{\pi} \frac{x \sin x dx}{1+\cos^2 x}$

Watch Video Solution

357. Evaluate the following integrals $\int \frac{3\sin x + 28\cos x}{5\sin x + 6\cos x} dx$

Watch Video Solution

358. Find the equation of the plane passing through the line $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and the point $(1, 2, -4)$.

Watch Video Solution

359. Prove that the sum of the vectors directed from the vertices to the mid points of opposite sides of a triangle is zero

Watch Video Solution

360. If A (1,0,-1), B (-2,4,-2) and C(1,5,10) be the vertices of a triangle and the bisector of the angle BAC, meets BC at D, then find the coordinates of the point D.

Watch Video Solution

361. Find the image of the point $(3, -2, 1)$ in the plane $x - y + 3z = 2$.

Watch Video Solution

362. Prove the following by vector method. The diagonals of a rhombus are at right angles.

Watch Video Solution

363. Prove that $\int \frac{dx}{(x-1)^2(x+2)} = \frac{1}{9}\ln\left|\frac{x+2}{x-1}\right| - \frac{1}{3(x-1)} + c$

Watch Video Solution

364. $\int_0^{\pi} \left(\frac{x \tan x}{\sec x + \tan x} \right) dx$

Watch Video Solution

365. Solve: $\left(1 + x^2\right) \frac{dy}{dx} + 2xy - x^3 = 0$

Watch Video Solution

366. Find $\frac{dy}{dx}$ if $x = \frac{3at}{1 + t^3}, y = \frac{3at^2}{1 + t^3}$ at $t = \frac{1}{2}$

Watch Video Solution

367. If $x \cos \alpha + y \sin \alpha = p$ is a tangent to the curve

$\left(\frac{x}{a}\right)^{\frac{n}{n}-1} + \left(\frac{y}{b}\right)^{\frac{n}{n}-1} = 1$then so that

$(a \cos \alpha)^n + (b \sin \alpha)^n = p^n.$

Watch Video Solution

368. Prove the following:

$$\begin{bmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{bmatrix}$$

$$=(a^2+b^2+c^2)(a+b+c)(b-c)(c-a)(a-b)$$

Watch Video Solution

369. If $A=\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$, then show that $A^3-A^2+I_3=\begin{bmatrix} 2 & -1 & 0 \\ 1 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

Watch Video Solution

370. If $\cos^{-1}\left(\frac{x}{a}\right)=\cos^{-1}\left(\frac{y}{b}\right)=\theta$, prove that $\frac{x^2}{a^2}-\frac{2xy}{ab}\cos\theta+\frac{y^2}{b^2}=\sin^2\theta$.

Watch Video Solution

371. Solve the following LPP graphically: Maximize: $Z = 4x_1 + 3x_2$ subject to $x_1 + x_2 \leq 50, x_1 + 2x_2 \leq 80, 2x_1 + x_2 \geq 20, x_1, x_2 \geq 0$

Watch Video Solution

372. Prove by vector method that in any triangle ABC, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

Watch Video Solution

373. A line makes angles $\alpha, \beta, \gamma, \delta$ with the four main diagonals of a cube. Prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$

Watch Video Solution

374. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitude, show that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}, \vec{b}, \vec{c}$.

Watch Video Solution

375. The equation of a plane passing through $(1, 1, 2)$ and parallel to $x + y + z - 1 = 0$ is____

Watch Video Solution

376. What is the value of $\int_0^{100} [\tan^{-1} x] dx$?

Watch Video Solution

377. What is the trigonometric function f(x) such that $f''(x) + f(x) = 0$?

Watch Video Solution

378. Write whether the following statements are true or false.

$\sec^{-1}\frac{1}{2} + \operatorname{cosec}^{-1}\frac{1}{2} = \frac{\pi}{2}$

Watch Video Solution

379. What is the minimum value of $\begin{vmatrix} \sin x & \cos x \\ -\cos x & 1+\sin x \end{vmatrix}$?

Watch Video Solution

380. What is the value of $\hat{a}\cdot\hat{b} + \hat{b}\cdot\hat{c} + \hat{c}\cdot\hat{a}$ if $\hat{a} + \hat{b} + \hat{c} = \vec{0}$?

Watch Video Solution

381. if A is a 3×3 matrix and |A| = 3, then write the matric represented by $A \times adjA$.

Watch Video Solution

382. What is the slope of the normal to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 20$ at the point (8, 64)?

Watch Video Solution

383. Find the degree and the order of the differential equation $\left(\frac{dy}{dx}\right) + 1 + \frac{d^2y}{dx^2} = 0.$

Watch Video Solution

384. Prove statement$\tan^2\cos^{-1}\frac{1}{\sqrt{3}} + \cot^2\sin^{-1}\frac{1}{\sqrt{5}} = 6$

Watch Video Solution

385. Let $R = \{(m, n) : 2$ divides $m + n\}$ on Z. Show that R is an equivalence relation on Z.

Watch Video Solution

386. Find the feasible solution region of $x + y \leq 1, 2x + 3y \leq 6, x \geq 0, y \geq 0.$

Watch Video Solution

387. Prove statement $\tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right) + \frac{7}{17} = 0$

Watch Video Solution

388. Prove that for any $f: X \to Y$, $foid_x = f = id_Y$ of.

Watch Video Solution

389. If ABC is a triangle, then prove that $\begin{vmatrix} \tan A & 1 & 1 \\ 1 & \tan B & 1 \\ 1 & 1 & \tan C \end{vmatrix} = 2$

Watch Video Solution

390. Find B if $B^2 = \begin{bmatrix} 17 & 8 \\ 8 & 17 \end{bmatrix}$

Watch Video Solution

391. If $f(x) = \begin{bmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{bmatrix}$

what is the maximum value of f(x).

Watch Video Solution

392. Solve the following : $\begin{bmatrix} x & 1 & 3 \\ 1 & x & 1 \\ 3 & 6 & 3 \end{bmatrix}$ =0

Watch Video Solution

393. If f(x) = a In $x + bx^2 + x$ has extreme values at $x = -1$ and $x = 2$

then find a and b.

Watch Video Solution

394. If cos y = x cos(a+y) then prove that

$$\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$$

Watch Video Solution

395. If $y\sqrt{x^2+1} = \log\left\{\sqrt{x^2+1} - x\right\}$ then prove that $(x^2+1)\frac{dy}{dx} + xy + 1 = 0$

Watch Video Solution

396. Find the equation of normal to the curve $x^3 = 4y$ which passes through (1,2).

Watch Video Solution

397. If $y = x^2 \cos^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) + x^2 \cos ec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)$ then prove that $\frac{d^2y}{dx^2} = \pi$

Watch Video Solution

398. Integrate $\int\left(\frac{x-1}{2x+1}\right)dx$

Watch Video Solution

399. $\int_0^2 \left[x^2\right]dx$

Watch Video Solution

400. $\int_0^{\frac{\pi}{2}} e^x \cos x dx$

Watch Video Solution

401. Solve$(x \log x) \frac{dy}{dx}$+y=$2 \log x$.

Watch Video Solution

402. Find the area of the portion of the parabola $y^2 = 4x$ bounded by the double ordinate through(3,0).

Watch Video Solution

403. If the sum of two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

Watch Video Solution

404. Prove that : $\left|\vec{a} + \vec{b}\right| \leq \left|\vec{a}\right| + \left|\vec{b}\right|$.

Watch Video Solution

405. If A (1,0,-1), B (-2,4,-2) and C(1,5,10) be the vertices of a triangle and the bisector of the angle BAC, meets BC at D, then find the coordinates of the point D.

Watch Video Solution

406. Find a symmetric form of the equation to the lines $x + 2y - z - 2 = 0$ and $2x - y + 3z - 4 = 0$.

Watch Video Solution

407. Find the points of intersection of the line $\frac{x-1}{1} = \frac{y+2}{3} = \frac{z-1}{-1}$ and the plane $2x + y + z = 9$.

Watch Video Solution

408. Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left(\frac{\sqrt{a^2+x^2}+\sqrt{a^2-x^2}}{\sqrt{a^2+x^2}-\sqrt{a^2-x^2}}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$, by substitution.

Watch Video Solution

409. A curve passes through the point (2, 0) and slope of tthe tangent at any point (x, y) is $x^2 - 2x$ for all $x \in R$. Show that the point where ordinate is maximum is $\left(0, \frac{4}{3}\right)$.

Watch Video Solution

410. Prove that $\int_0^{\pi} \frac{x dx}{1+\sin x} = \pi$

Watch Video Solution

411. Find the solution of the following differential equations:

(2x+3y-5)dy/dx+3x+2y-5-0

Watch Video Solution

412. Prove that $\cos^{-1}\left(\frac{b+a\cos x}{a+b\cos x}\right)=2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)$

Watch Video Solution

413. Maximize: $Z=10x_1+12x_2+8x_3$

Subject to: $x_1+2x_2\leq 30$

$5x_1-7x_3\geq 12$

$x_1+x_2+x_3=20$

$x_1,x_2\geq 0$

Watch Video Solution

414. Let f: $X \to Y$

If there exists a map g:Y $\to$ X such that gof = id_X and fog = id_y, then show that

f is bijective and (ii) $g = f^{-1}$

Watch Video Solution

415. Using vector method prove that $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Watch Video Solution

416. A variable plane meets the coordinate axes at A, B, C and is at a constant distance d from origin. Prove that the locus of the centroid of the triangle ABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{9}{d^2}$

Watch Video Solution

417. Show that the shortest distance between the lines $x + a = 2y = -12z$ and $x = y + 2a = 6z - 6a$ is 2a .

Watch Video Solution

418. Prove that the following. $\begin{bmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{bmatrix}$ =(b-c)(c-a)(a-b)(bc+ca+ab)

Watch Video Solution

419. Solve the following system of equations by matrix method $x + 2y + 3z = 8, 2x + y + z = 8$ and $x + y + 2z = 6$

Watch Video Solution

420. Find the distance between the parallel planes $2x - 2y + z + 1 = 0$ and $4x - 4y + 2z + 3 = 0$.

Watch Video Solution

Watch Video Solution

421. If $y = x^{-1} \sin \cos ec^{-1} \frac{1}{x}$, then what is $\frac{dy}{dx}$?

Watch Video Solution

422. Write down the smallest and the largest equivalence reaction on a set A ={1,2,3}..

Watch Video Solution

423. Differentiate twice $\int \sin^{-1} x dx + \int \cos^{-1} x dx$

Watch Video Solution

424. If x+y=4, xy=1 then what is the value of $\tan^{-1} x + \tan^{-1} y$?

Watch Video Solution

425. What is the value of $\hat{a} \cdot \hat{b} + \hat{b} \cdot \hat{c} + \hat{c} \cdot \hat{a}$ if $\hat{a} + \hat{b} + \hat{c} = \overrightarrow{0}$?

Watch Video Solution

426. What is the value of $\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$?

Watch Video Solution

427. If A is a 2×2 non-singular matrix and $|A| = \frac{1}{2}$ then which matrix is represented by $A \times adjA$?

Watch Video Solution

428. Write the subinterval of $(0, \pi)$ in which $\sin\left(x + \frac{\pi}{4}\right)$ is increasing.

Watch Video Solution

429. What is the value of $\int_0^2 |x-1|dx$?

Watch Video Solution

430. Solve for x, $\cos^{-1} x + \sin^{-1}\left(\frac{x}{2}\right) = \frac{\pi}{6}$.

Watch Video Solution

431. If R and S are two equivalence relation on the set then prove that $R \cap S$ is also an equivlaence relation on the set.

Watch Video Solution

432. Show that:$4\left(\cot^{-1}\left(\frac{3}{2}\right) + \operatorname{cosec}^{-1}\sqrt{26}\right) = \pi$.

Watch Video Solution

433. Prove that $f: R \to R$ such that $f(x) = \dfrac{2x^2}{x^2+1}$ is neither one-one nor onto function.

Watch Video Solution

434. Factorize the following. $\begin{bmatrix} a & b & c \\ b+c & c+a & a+b \\ a^2 & b^2 & c^2 \end{bmatrix}$

Watch Video Solution

435. Find B if $B^2 = \begin{bmatrix} 17 & 8 \\ 8 & 17 \end{bmatrix}$

Watch Video Solution

436. If x,y,z are positive and are the pth, qth and rth terms of a G.P. then prove that

$$\begin{vmatrix} \log x & p & 1 \\ \log y & q & 1 \\ \log z & r & 1 \end{vmatrix} = 0$$

Watch Video Solution

437. Find the adjoint matrix of the matrix $\begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix}$

Watch Video Solution

438. Solve the following : $\begin{bmatrix} x & 1 & 3 \\ 1 & x & 1 \\ 3 & 6 & 3 \end{bmatrix}$ =0

Watch Video Solution

439. Find the point on the curve $y^2 - x^2 + 2x - 1 = 0$

where the tangent is parallel to the x - axis.

Watch Video Solution

440. Find $\frac{dy}{dx}$, if $y = \tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right)$.

Watch Video Solution

441. Find the equation of the tangent and normal to the curve y(x-2)(x-3)-x+7=0 at the point where it cuts the x-axis.

Watch Video Solution

442. If f(x) = a In $x + bx^2 + x$ has extreme values at $x = -1$ and $x = 2$ then find a and b.

Watch Video Solution

443. If $y = x + \frac{1}{x + \frac{1}{x + \ldots \infty}}$ find $\frac{dy}{dx}$, the rhs being a valid expression.

Watch Video Solution

444. Solve $(x + y + 1)\frac{dy}{dx} = 1$

Watch Video Solution

445. Integrate $\int \frac{\tan x + \tan \alpha}{\tan x - \tan \alpha} dx$

Watch Video Solution

446. Integrate the following $\int_0^1 \frac{x^7}{\sqrt{1-x^2}} dx$

Watch Video Solution

447. Find the area of the circle

$x^2 + y^2 = 2ax.$

Watch Video Solution

448. $\int_0^2 [x^2] dx$

Watch Video Solution

449. Show that the line passing through the points (a_1, b_1, c_1) and (a_2, b_2, c_2) passes through the origin , if $a_1a_2 + b_1b_2 + c_1c_2 = p_1p_2$. where p_1 and p_2 are distances of the points from origin.

Watch Video Solution

450. If the sum of two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

Watch Video Solution

451. Calculate the area of the triangle ABC (by vector method) where A(1,2,4), B(3,1,-2), C(4,3,1)

Watch Video Solution

452. The plane $4x + 7y + 4z + 81 = 0$ is rotated through a right angle about its line of intersection with the plane $5x + 3y + 10z - 25 = 0$. Find the equation of the plane in new position.

Watch Video Solution

453. Find the equation of lines joining the points. (4,-6,1) and (0,3,-1)

Watch Video Solution

454. Evaluate $\int \ln\left(x^2 + x + 2\right) dx$

Watch Video Solution

455. Evaluate $\int_0^1 \frac{\ln(1+x)}{1+x^2} dx$

Watch Video Solution

456. Solve $x\frac{dy}{dx} + y = x^4$

Watch Video Solution

457. If $f: X \to Y$ is onto, then prove that $f\left(f^{-1}(B)\right) = B$ for all $B \subseteq Y$

.

Watch Video Solution

458. Prove that $f: X \to Y$ is injective iff $f^{-1}(f(A)) = \text{A for all} A \subseteq X.$

Watch Video Solution

459. Prove that $\cos^{-1}\left(\frac{b + a\cos x}{a + b\cos x}\right) = 2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)$

Watch Video Solution

460. Prove that the measure of the angle between two main diagonals of a cube is $\cos^{-1}\frac{1}{3}$.

Watch Video Solution

461. Show that the shortest distance between the lines $x+a=2y=-12z$ and $x=y+2a=6z-6a$ is 2a .

Watch Video Solution

462. If $e^{y/x}=\frac{x}{a+bx}$ then show that $x^3\frac{d}{dx}\left(\frac{dy}{dx}\right)=\left(x\frac{dy}{dx}-y\right)^2$

Watch Video Solution

463. Find the extreme values of the function $f(x)=x^2e^{-x^2}$ and show that $f(x)\leq e^{-1}\forall x\in R$.

Watch Video Solution

464. Shows that the triangle of greatest area that can be inscribed in a circle is equilateral.

Watch Video Solution

465. Find the inverse of $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ using elementary operations.

Watch Video Solution

466. If $\begin{bmatrix} x & x^2 & x^3-1 \\ y & y^2 & y^3-1 \\ z & z^2 & z^3-1 \end{bmatrix} = 0$

then prove that xyz=1 when x,y,z are non zero and unequal.

Watch Video Solution

467. If A $\begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 0 \\ -1 & 4 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$ and$C = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & -1 \\ 2 & 2 & 2 \end{bmatrix}$

Show that AB=AC

though B '!=C`.Verify that: A(B+C)=AB+AC

Watch Video Solution

468. Let $f: R \to R$ be defined as $f(x) = 3x$. Then f is one-one and onto

Watch Video Solution

469. Let $f: R \to R$ be defined as $f(x) = 3x$. Then f is one-one and onto

Watch Video Solution

470. Let $f: R \to R$ be defined as $f(x) = 3x$. Then f is one-one and onto

Watch Video Solution

471. Let $f: R \to R$ be defined as $f(x) = 3x$. Then f is one-one and onto

Watch Video Solution

472. Find the number of binary operations on the set {a,b}.

Watch Video Solution

473. If A, B are symmetric matrices of same order, then find $AB - BA$.

Watch Video Solution

474. Let A be square matrix of order 3×3, then find $|KA|$ where, k is a scalar.

Watch Video Solution

475. If A is an invertible matrix of order 2, then find $\det\left(A^{-1}\right)$.

Watch Video Solution

476. Find $\int x^2 e^{x^3} dx$

Watch Video Solution

477. Give an example of a homogeneous differential equation.

Watch Video Solution

478. In triangle ABC, find $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$

Watch Video Solution

479. Let the vectors $\vec{a}$ and $\vec{b}$ such that $\left|\vec{a}\right| = 3$ and $\left|\vec{b}\right| = \frac{\sqrt{2}}{3}$. If $\vec{a} \times \vec{b}$ is a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.

Watch Video Solution

480. Are the planes $2x - y + 4z = 5$ and $5x - 25y + 10z = 6$ are perpendicular?

Watch Video Solution

481. Find $fog(2)$ and $gof(1)$, when $f: R \to R$ defined by $f(x) = x^2 + 8$ and $g: R \to R$, $g(x) = 3x^3 + 1$.

Watch Video Solution

482. Solve $\cos^{-1} x + \sin^{-1} \frac{x}{2} = \frac{\pi}{6}$.

Watch Video Solution

483. Solve LPP: Maximize $Z = 5x_1 + 7x_2$ subject to the constrains $3x_1 + 4x_2 \leq 12, x_1, x_2 \geq 0$

Watch Video Solution

484. Show the $f: R - \{-1) \to R - \{1\}$ given by $f(x) = \frac{x}{x-1}$ is invertible. Also find f^{-1}.

Watch Video Solution

485. Prove the following

$$\tan^{-1}\frac{2a-b}{b\sqrt{3}} + \tan^{-1}\frac{2b-a}{a\sqrt{3}} = \frac{\pi}{3}$$

Watch Video Solution

486. Express $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

Watch Video Solution

487. Using the properties of determinants, show that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$

Watch Video Solution

488. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}, n \in N.$

Watch Video Solution

489. Find the minimum value of $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$

Watch Video Solution

490. Find the values of x and y such that $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 9 \end{bmatrix}$

Watch Video Solution

491. Discuss the continuity of the function defined by $f(x) = \begin{cases} x+1 & \text{if } x \leq 1 \\ x-2 & \text{if } x > 1 \end{cases}$

Watch Video Solution

492. Differentiate the functions w.r.t x $x^{x\cos x} + \dfrac{x^2+1}{x^2-1}$

Watch Video Solution

493. Find the tangent to the curve $y = \cos(x + y), 0 \leq x \leq 2\pi$ which is parallel to the line x + 2y = 0

Watch Video Solution

494. Find the intervals in which the function f given by $f(x) = \sin x + \cos x, 0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

Watch Video Solution

495. If $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$ and $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$, then find $\frac{dy}{dx}$.

Watch Video Solution

496. Find the integral $\int \frac{x^3 \sin\left(\tan^{-1} x^4\right)}{1 + x^8} dx$

Watch Video Solution

497. Find the integral $\int \frac{x+3}{(5-4x+x^2)} dx$

Watch Video Solution

498. Evaluate $\int_0^{\pi/2} \log \sin x dx.$

Watch Video Solution

499. Form the differential equation of the family of all circles of radius r.

Watch Video Solution

500. Solve the differential equation $\frac{dy}{dx} = \log(x+1)$

Watch Video Solution

501. The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. Find the unit vector parallel to its diagonal. Also find its area.

Watch Video Solution

502. Prove the following by vector method. In a triangle AOB, $m\angle AOB$ = 90°. If P and Q are the points of trisection of AB, prove that $OP^2 + OQ^2 = \frac{5}{9}AB^2$

Watch Video Solution

503. Find the distance of the point (2,3,4) from the plane $3x + 2y + 2z + 5 = 0$ measured parallel to the line $\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$.

Watch Video Solution

504. Prove by vector method that in a parallelogram, the line joining a vertex to the midpoint of an oppositeside trisects the other diagonal.

Watch Video Solution

505. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$prove that$x^4 + y^4 + z^4 + 4x^2y^2z^2 = 2\left(x^2y$

Watch Video Solution

506. Maximize Z=$6x_1 + 10x_2$subject to $x_1 + 2x_2 > 10, 2x_1 + 2x_2 > 12, 3x_1 + x_2 > 8, x_1, x_2 > 0$

Watch Video Solution

507. Show the $f: R - \{-1) \to R - \{1\}$ given by $f(x) = \frac{x}{x-1}$ is invertible. Also find f^{-1}.

Watch Video Solution

508. Prove that the following.

$$\begin{vmatrix} (v+w)^2 & u^2 & u^2 \\ v^2 & (w+u)^2 & v^2 \\ w^2 & w^2 & (u+v)^2 \end{vmatrix} = 2uvw(u+v+w)^3$$

Watch Video Solution

509. Show that $P(m,1)+P(n,1)=P(M+n,1)$ for all positive integers m, n.

Watch Video Solution

510. Prove that the curves $x=y^2$ and $xy=k$ cut at right angles, if $8k^2=1$.

Watch Video Solution

511. Show that the semivertical angle of a cone of given slant height is $\tan^1 \sqrt{2}$ when its volume is maximum.

Watch Video Solution

512. A ladder 5m long is leaning against a wall. The bottom of ladder is pulled along the ground, away from the wall, at the rate of $2cm/\sec$. How fast is its height on the wall is decreasing when the foot of ladder is 4m away from the wall.

Watch Video Solution

513. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight line $\frac{x}{a} + \frac{y}{b} = 1$

Watch Video Solution

514. Find the co-ordinates of the foot of the perpendicular from the point (1,1,2) to the plane $2x - 2y + 4z + 5 = 0$. Also,find the length of the perpendicular.

Watch Video Solution

515. If with reference to the eight handed system of mutually perpendicular unit vectors $\hat{i}, \hat{j}, \hat{k}, \vec{\alpha} = 3\hat{i} - \hat{j}, \beta = 2\hat{i} + \hat{j} - 3\hat{k}$, then express $\vec{\beta}$ in the form $\vec{\beta} = \vec{\beta_1} + \vec{\beta_2}$, where $\vec{\beta_1}$ is parallel to $\vec{\alpha}$ and $\vec{\beta_2}$ is perpendicular to $\vec{\alpha}$.

Watch Video Solution

516. Prove that the lines $\frac{x+4}{3} = \frac{y+6}{5} = \frac{z-1}{-2}$ and $3x - 2y + z + 5 = 0 = 2x + 3y + 4z - 4$ are co-planar.

Watch Video Solution

517. What is $\int 5^x . e^{3x}$?

Watch Video Solution

518. If $|X| = m$ and $|Y|$= n, then what is the total number of functions from X to Y.

Watch Video Solution

519. If the tangent to the curve $y = x^2 + 3x$ at P has slope 5, then what are the coordinates of P?

Watch Video Solution

520. Fill in the blank choosing correct answer from the brackets

$$\cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\left(\sqrt{1}+\sin x\right)}\right] =_{\_} \ _{\_}{}_{\_}$$

$$\left(2\pi - \frac{x}{2}, \frac{x}{2}, \pi - \frac{x}{2}\right)$$

Watch Video Solution

521. What is the symmetric form of the equation of z-axis ?

Watch Video Solution

522. If A is a non singular square matrix of order n, then what is $|adjA|$?

Watch Video Solution

523. Answer any three questions

if $\hat{a}, \hat{b}$ and $\hat{c}$ are unit vectors such that $\hat{a} + \hat{b} + \hat{c} = 0$, then find the value of $\hat{a}.\ \hat{b} + \hat{b}.\ \hat{c} + \hat{c}.\ \hat{a}$.

Watch Video Solution

524. If A is a symmetric matrix, then A- A is both symmetric and skew symmetrix. State true or false.

Watch Video Solution

525. If $f(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 2 - x & 1 \leq x \leq 2 \end{cases}$, then what is the value of $\int_0^2 f(x)dx$?

Watch Video Solution

526. Solve $\sin^{-1} x + \sin^{-1}(1 - x) = \frac{\pi}{2}$.

Watch Video Solution

527. Let f be a real function. Show that h(x) =f(x)=f(-x) is always an even function and g(x) = f(x) -f(-x) is always an odd fuction.

Watch Video Solution

528. Minimize: $Z = 6x_1 + 7x_2$

Subject to: $x_1 + 2x_2 \geq 4$

$x_1, x_2 \geq 0$

Watch Video Solution

529. If $r^2 = x^2 + y^2 + z^2$, Prove that

$$\tan^{-1}\frac{yz}{xr} = \tan^{-1}\frac{zx}{yr} + \tan^{-1}\frac{xy}{zr} = \frac{\pi}{2}$$

Watch Video Solution

530. Find A^{-1} if $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ by using elementary row operations.

Watch Video Solution

531. Evaluate $\begin{bmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{bmatrix} - \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$

Watch Video Solution

532. Define involuntary matrix and show that $\begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ is an involuntary matrix.

Watch Video Solution

533. If $\begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 7 \end{bmatrix}$, then find xand y.

Watch Video Solution

534. If ABC is a triangle, then prove that $\begin{vmatrix} \tan A & 1 & 1 \\ 1 & \tan B & 1 \\ 1 & 1 & \tan C \end{vmatrix} = 2$

Watch Video Solution

535. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, then prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

Watch Video Solution

536. If $y=e^{m\cos^{-1}x}\left(1-x^2\right)y_2-xy_1=m^2y$

Watch Video Solution

537. Differentiate with respect to x: $y=3^{x^2}+\tan^{-1}\left(\frac{\cos x+\sin x}{\cos x-\sin x}\right)$

Watch Video Solution

538. Show that $f(x)=x^3-6x^2+24x+4$ has neither a maximum nor a minimum value.

Watch Video Solution

Watch Video Solution

539. Find Integrating factor of: $(x \log x)\frac{dy}{dx} + y = 2\log x$

Watch Video Solution

540. Integrate $\int \frac{x+7}{x^2+8x} dx$

Watch Video Solution

541. Solve $\frac{dy}{dx} + \frac{y}{x} = xy^2$.

Watch Video Solution

542. Prove that $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan X}} dx = \frac{\pi}{12}$

Watch Video Solution

543. Prove that the measure of the angle between two main diagonals of a cube is $\cos^{-1}\frac{1}{3}$.

Watch Video Solution

544. If the sum of two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

Watch Video Solution

545. Find the foot of the perpendicular drawn from the point (5,7,3) to the line $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$. Find the length of the perpendicular.

Watch Video Solution

546. Show that the vector area of the triangle whosse vertices have position vector $\vec{a}, \vec{b}, \vec{c}$ is $\frac{1}{2}\left(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right)$.

Watch Video Solution

Watch Video Solution

547. passing through the point $(-1, 3, 2)$ perpendicular to the planes $x + 2y + 2z = 5$ and $3x + 3y + 2z = 8$.

Watch Video Solution

548. If $y = x^{\sin x} + (\sin x)^{\cos x}$, then find $\frac{dy}{dx}$.

Watch Video Solution

549. Show that $\sin^P \theta \cos^q \theta$ attains a maximum value, when $\theta = \tan^{-1} \sqrt{\frac{p}{q}}$.

Watch Video Solution

550. Show that the sum of the intercepts on the coordinate axes of any tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is constant.

Watch Video Solution

551. Prove by vector method that the mid-point of the hypotentise of a right angles triangle is equidistance from the vertices.

Watch Video Solution

552. Find the shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y-7}{2}=\frac{z-6}{4}$ Find also the equation of the line of shortest distance.

Watch Video Solution

553. Let f be defined by f(x)=4x + 3 for all `x""inR. Show that it is bijective and determine the inverse of f.

Watch Video Solution

554. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$, show that $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$

Watch Video Solution

555. Prove that $\int_0^{\frac{\pi}{2}} \ln \sin x dx = -\frac{\pi}{2}\ln 2$

Watch Video Solution

556. Evaluate $\int \frac{dx}{\cos x(1+2\sin x)}$

Watch Video Solution

557. Solve the following differential equations

$(x + \tan y)dy = \sin 2y dx$

Watch Video Solution

558. Express $\begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix}$ in the form of a perfect square.

Watch Video Solution

559. If $A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 & 2 \\ -2 & 3 & 0 \end{bmatrix}$ verify

(i) $(A+B)C = AC + BC$

(ii) $(AB)C = A(BC)$

Watch Video Solution

560. If $f(x) = \int_0^x e^{2t}.\sin 3t dt$then what is $f^1(x)$?

Watch Video Solution

561. What does the solution set of the inequation $2x + y > 5$ represents?

Watch Video Solution

Watch Video Solution

562. What is the value of $\tan\left(\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}\right)$.

Watch Video Solution

563. What is the general solution of $\tan^{-1}\sqrt{\frac{dy}{dx}}=x$?

Watch Video Solution

564. Give an example of symmetric matrix.

Watch Video Solution

565. If $\vec{a}$ and $\vec{b}$ are non-collinear and $\vec{p}=x\vec{a}+(y+1)\vec{b}$, $\vec{q}=y\vec{a}+x\vec{b}$ then what is the value of x and y if $\vec{p}=2\vec{q}$?

Watch Video Solution

Watch Video Solution

566. If $y = \cos^{-1}\left(\frac{x - \frac{1}{x}}{x + \frac{1}{x}}\right)$, then what is $\frac{dy}{dx}$?

Watch Video Solution

567. What is the restriction on the formula $\int a^x dx = \frac{a^x}{Ina}$?

Watch Video Solution

568. Write down the symmetric from of the equation of y-axis.

Watch Video Solution

569. Solve:$\sin^{-1}2x + \sin^{-1}x = \frac{\pi}{3}$.

Watch Video Solution

570. Find all possible equivalence relations on $X = \{1, 2, 3\}$

Watch Video Solution

571. Find the fesible region of the following system: $3x - y \geq 0, x \geq 0, y \geq 0$

Watch Video Solution

572. In a ΔABC, if $m\angle A = 90^\circ$, prove that $\tan^{-1}\frac{b}{a+c} + \tan^{-1}\frac{c}{a+b} = \frac{\pi}{4}$, where a,b, c are sides of the triangle.

Watch Video Solution

573. If $f: X \to Y$ is onto, then prove that $f\left(f^{-1}(B)\right) = B$ for all $B \subseteq Y$

.

Watch Video Solution

574. If A is a 3×3 matrix and $|A| = 2$, then which matrix is represented by $A \times adjA$?

Watch Video Solution

575. $\begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x & 2 \\ 1 & y \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ -1 & 4 \end{bmatrix}$, find x and y .

Watch Video Solution

576. Solve : $\begin{bmatrix} 7 & 6 & x \\ 2 & x & 2 \\ x & 3 & 7 \end{bmatrix} = 0$

Watch Video Solution

577. Prove that: $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

Watch Video Solution

578. If A is square matrix,then prove that $A.\ (adj \cdot A) = |A|I.$

Watch Video Solution

579. If $y = \sqrt{x+1}$Find δy and dy when $X = 8$ and $dx = 0.02.$

Watch Video Solution

580. Shows that the following functions do not possess maximum or minimum. $3x^3 - 12x^2 + 16x - 15$

Watch Video Solution

581. Show that $\frac{x}{1 + x\tan x,} x \in \left(0, \frac{\pi}{2}\right)$ is maximum when x = cos x.

Watch Video Solution

582. If x =sin t,y = sin 2t then prove that $\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+4y=0$

Watch Video Solution

583. Show that the line y = mx + c touches the parabola y^2 = 4ax if c $=\frac{a}{m}$.

Watch Video Solution

584. Solve $x\sin\frac{y}{x}dy=\left(y\sin\frac{y}{x}-x\right)dx$

Watch Video Solution

585. Evaluate $\int_0^{\frac{\pi}{2}}\frac{\sin x}{\sin x+\cos x}dx.$

Watch Video Solution

586. Determine the area within the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$$

Watch Video Solution

587. Integrate the following $\int\left[\frac{2x+1}{\sqrt{x^2+10x+29}}\right]dx$

Watch Video Solution

588. If $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$,then find the angle between $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$.

Watch Video Solution

589. Find the equation of plane passing through the intersection of the planes $\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=1$ and $\vec{r}\cdot\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0$and parallel x-axis.

Watch Video Solution

590. Find the co-ordinates of the foot of the perpendicular from the point $(1, 1, 1)$ on the line joining $(1, 4, 6)$ and $(54, 4)$.

Watch Video Solution

591. Prove that $\left(\vec{a} \times \hat{i}\right)^2 + \left(\vec{a} \times \hat{j}\right)^2 + \left(\vec{a} \times \hat{k}\right)^2 = 2\vec{a}^2$

Watch Video Solution

592. Prove that the line $\frac{x+5}{3} = \frac{y+4}{1} = \frac{z-7}{-2}$ and $\frac{x}{1} = \frac{y+7}{5} = \frac{z-16}{-13}$ are coplanar.

Watch Video Solution

593. Evaluate the following integrals $\int \frac{3}{x^3 - 1} dx$

Watch Video Solution

594. Show that

$$\int_0^{\pi} x In \sin x = \frac{\pi^2}{2} In \frac{1}{2}$$

Watch Video Solution

595. Solve $\frac{dy}{dx} + \frac{y}{x} = \frac{\sin x}{x}, y(0) = 0$

Watch Video Solution

596. If $x = \frac{1 - \cos^2 \theta}{\cos \theta}, y = \frac{1 - \cos^{2n} \theta}{\cos^n \theta}$ then show that $\left(\frac{dy}{dx}\right)^2 = n^2 \left(\frac{y^2 + 4}{x^2 + 4}\right)$

Watch Video Solution

597. A figure consists of a semi-circle with a rectangle on its diameter. Given that the perimeter of the figure is 20 cm. Find the dimensions in order that its area may be maximuum.

Watch Video Solution

598. Find the points on the curve $y = x^2 + 1$ which are nearest to the point (0,2).

Watch Video Solution

599. Prove that $a \equiv b \mod 3$ is an equivalence relation on the set of integers Z. Find its congruence classes.

Watch Video Solution

600. If $\cos^{-1}\left(\frac{x}{a}\right) = \cos^{-1}\left(\frac{y}{b}\right) = \theta$, prove that $\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\theta + \frac{y^2}{b^2} = \sin^2\theta$.

Watch Video Solution

601. A factory uses three different respurce for the manufacture of two different products, 20 units of the resource A, 12 units of B and 16 unit of C being available. One unit of the first product requires 2,2 and 4 units of the resources and one unit of the second product requires 4,2 and 0 units of the resources taken in order. It is known that the first product gives a profit of ₹20 per unit and the second ₹ 30 prt uniy. Formulate the LPP so as to earn maximum profit.

Watch Video Solution

602. If the edges of a rectangular parallelopiped are of lengths a, b, c, then the angle between four diagonals are $\cos^{-1}\left(\frac{\pm a^2 \pm b^2 \pm c^2}{a^2 + b^2 + c^2}\right)$.

Watch Video Solution

603. Find the distance of the point (3,-4,5)from the plane 2x+5y-6z-19=0 measured parallel to the line $\frac{x-1}{2}=\frac{y}{1}=\frac{z+3}{-2}$.

Watch Video Solution

604. A variable plane is at a constant distance p from the origin and meets the axes at A,B,C. Through A,B,C plane are drawn parallel to the co-ordinate planes. Show that the locus of their points of intersection is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$.

Watch Video Solution

605. Without expanding show that the following determinant is equal to Ax+B where A and B are determinants of order 3 not involning x.

$$\begin{bmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+3x+3 & 2x-1 & 2x-1 \end{bmatrix}$$

Watch Video Solution

606. If $A = \begin{bmatrix} 0 & -\tan\left(\frac{\alpha}{2}\right) \\ \tan\left(\frac{\alpha}{2}\right) & 0 \end{bmatrix}$ show that $(I + A) = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Watch Video Solution

607. Find the probability distribution of number of doublets in four throws of a pair of dice. Find also the mean and the variance of the number of doublets.

Watch Video Solution

608. What is the value of $\int_0^{\pi} \cos^{101} x dx$?

Watch Video Solution

609. If $f(x) = \begin{cases} 3x+2 & x \leq 0 \\ 2-3x & x > 0 \end{cases}$

Watch Video Solution

610. What is the equation of the plane $\perp r$ to y-axis and passing through(2,-3,5)?

Watch Video Solution

611. Find least non negative integer r such that $7 \times 13 \times 23 \times 413 \equiv r(\mathrm{mod}\ 11)$

Watch Video Solution

612. For a 2xx2 matrix A,if Axxadj$A = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ then what is the value of $|A|$?

Watch Video Solution

613. Write the Order and degree of the differential equation whose general solution is y=A sinx +Bcosx.

Watch Video Solution

614. With 4 different elements how many different determinant of order 2 can be constructed.

Watch Video Solution

615. What is the value of a so that $6\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-4\hat{j}+a\hat{k}$ are orthogonal to each other ?

Watch Video Solution

616. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$, show that $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$

Watch Video Solution

617. Find the feasible solution region of the following system: $-x + y \geq -1, x + y \leq 6, y \leq 5, x \geq 0, y \geq 0.$

Watch Video Solution

618. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z - \pi$

show that x+y+z=xyz.

Watch Video Solution

619. If p is a prime and $ab \equiv 0$ (mod p) then show that either a=0 (mod p) or $b \equiv 0$ (mod p).

Watch Video Solution

620. If A+B+C=pi,then show that $\begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix} = 0$

Watch Video Solution

621. Prove that the following. $\begin{bmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{bmatrix}$

= abc(1+1/a+/b+1/c)

Watch Video Solution

622. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \end{bmatrix}$, then verify that $(AB)^T = B^T A^T$.

Watch Video Solution

623. Find A^-1 if A= $\begin{bmatrix} 4 & -1 \\ 2 & 3 \end{bmatrix}$

Watch Video Solution

624. Express $\begin{bmatrix} 3 & -1 & 0 \\ 2 & 2 & 5 \\ 1 & 3 & 2 \end{bmatrix}$ as sum of a symmetric and a skew symmetric matrix.

Watch Video Solution

625. Find the point on the curve $x^2 + y^2 - 4xy + 2 = 0$

where the normal is paralell to the x-asis.

Watch Video Solution

626. If sin (x + y) = y cos (x + y) then prove that

$$\frac{dy}{dx} = -\frac{1 + y^2}{y^2}$$

Watch Video Solution

627. Find the interval where $f(x) = \sin x, x \in [0, 2\pi]$ increasing.

Watch Video Solution

628. If $\sin y = x \cos(a + y)$, show that $\frac{dy}{dx} = \frac{\cos^2(a + y)}{\cos a}$ and find the value of $\frac{dy}{dx} at x = 0$

Watch Video Solution

629. Determine the area of the region bounded by $y^2 = x^3$ and the double ordinate through (2,0).

Watch Video Solution

630. $\int \frac{xe^x}{1 + x^2} dx$

Watch Video Solution

631. Evaluate $\int_0^1 \frac{\ln(1+x)}{1+x^2}dx$

Watch Video Solution

632. Find the solution of the following differential equations:

$$x\frac{dy}{dx} + \sqrt{x^2+y^2} = y$$

Watch Video Solution

633. $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\cot x}}$

Watch Video Solution

634. Prove the following by vector method. The diagonals of a rhombus are at right angles.

Watch Video Solution

635. Show that the vector area of the triangle whosse vertices have position vector $\vec{a}, \vec{b}, \vec{c}$ is $\frac{1}{2}\left(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right)$.

Watch Video Solution

636. Find the perpendicular distance of the point $(-1, 3, 9)$ from the line $\frac{x-13}{5} = \frac{y+8}{-8} = \frac{z-31}{1}$

Watch Video Solution

637. If the sum of two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

Watch Video Solution

638. Find the image of the point (3,5,7) with respect to the plane $2x + y + z = 6$.

Watch Video Solution

639. Prove that $\int x \log\left(1 + \frac{1}{x}\right) dx = \frac{x^2}{2}\log\left(\frac{x+1}{x}\right) - \frac{x^2}{2}\log x - \frac{1}{2}\log(x+1) + \frac{1}{2}x +$

Watch Video Solution

640. Prove that $\int \frac{dx}{2 + \sin x} = \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x/2 + 1}{\sqrt{3}}\right) + c.$

Watch Video Solution

641. Solve $\frac{dy}{dx} - \frac{y}{x} = xy^2.$

Watch Video Solution

642. Let f: $X \to Y$

If there exists a map g:Y $\to$ X such that gof = id_X and fog = id_y, then

show that

f is bijective and (ii) $g = f^{-1}$

Watch Video Solution

643. If

$\cos^{-1}\left(\frac{x}{y}\right) + \cos^{-1}\left(\frac{y}{3}\right) = \theta$, prove that $9x^2 - 12xy\cos\theta + 4y^2 = 36\sin^2\theta$

.

Watch Video Solution

644. Maximize: $Z = 10x_1 + 12x_2 + 8x_3$

Subject to: $x_1 + 2x_2 \leq 30$

$5x_1 - 7x_3 \geq 12$

$x_1 + x_2 + x_3 = 20$

$x_1, x_2 \geq 0$

Watch Video Solution

645. Prove by vector method that the medians of a triangle are concurrent.

Watch Video Solution

646. If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines show that the d.cs. Of the line perpendicular to both of them are $m_1n_2 - n_1m_2, n_1l_2 - l_1n_2, l_1m_2 - m_1l_2$

Watch Video Solution

647. If 2s=a+b+c show that

$$\begin{bmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{bmatrix} = 2s^3(s-a)(s-b)(s-c)$$

Watch Video Solution

648. If $A = \begin{bmatrix} 1 & -2 & 2 \\ 3 & 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 \\ 1 & 2 \\ 3 & -1 \end{bmatrix}$, then verify that $(AB)^T = B^T A^T$.

Watch Video Solution

649. Let A={1,2,3}, which of the following functions on A is invertible?

Watch Video Solution

650. Let A={1,2,3}, which of the following functions on A is invertible?

Watch Video Solution

651. Let A={1,2,3}, which of the following functions on A is invertible?

Watch Video Solution

652. Let A={1,2,3}, which of the following functions on A is invertible?

Watch Video Solution

653. Find $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

Watch Video Solution

654. Find the number of all possible matrices of order 3×3 with each of entry 0 or 1.

Watch Video Solution

655. Find the value of x if $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

Watch Video Solution

656. If A is a square matrix of order 2, then find $\det(adjA)$.

Watch Video Solution

657. Evaluate $\int e^x(\sin x + \cos x)dx$.

Watch Video Solution

658. Write the differential equation having $y = C_1e^x + C_2e^x$ as its general solution.

Watch Video Solution

659. If $\vec{a}$ is a non zero vector of magnitude $\left|\vec{a}\right|$ and λ is a non zero scalar, then find the value of λ for which $\lambda\vec{a}$ is a unit vector.

Watch Video Solution

660. Find the value of $\hat{i} \cdot \left(\hat{j} \times \hat{k}\right) + \hat{j} \cdot \left(\hat{i} \times \hat{k}\right) + \hat{k} \cdot \left(\hat{i} \times \hat{j}\right)$.

Watch Video Solution

661. Are the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x-1}{-2} = \frac{y-2}{-4} = \frac{z-3}{-6}$ parallel?

Watch Video Solution

662. Solve LPP : Maximize $Z = 20x_1 + 30x_2$ subject to constraints $3x_1 + 5x_2 \leq 15, x_1, x_2 > 0$

Watch Video Solution

663. Solve $\sin^{-1} x + \sin^{-1}(1-x) = \frac{\pi}{2}$.

Watch Video Solution

664. Let $*$ be a binary operation on the set of real numbers, defined by $a * b = \frac{ab}{5}$, for all $a, b \in R$. Show that $*$ is both commutative and associative.

Watch Video Solution

665. Let $*$ be a binary operation on the set of real numbers, defined by $a * b = \frac{ab}{5}$, for all $a, b \in R$. Find the identity element.

Watch Video Solution

666. Let $*$ be a binary operation on the set of real numbers, defined by $a * b = \frac{ab}{5}$, for all $a, b \in R$. Find the inverse elements.

Watch Video Solution

667. If a relation R is defined on the set Z of integers as following $(a, b) \in R \Leftrightarrow a^2 + b^2 = 25$, then find domain R.

Watch Video Solution

668. If $A = \begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix}$ is expressed as sum of a symmetric and skew symmetric matrix , then find the symmetric matrix.

Watch Video Solution

669. Given the equations

x=cy+bz, y=az+cx and z=bx+ay

where x,y and z are not all zero, prove that $a^2 + b^2 + c^2 + 2abc = 1$ by determinant method.

Watch Video Solution

670. Find $A^2 - 5A + 6$ if $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

Watch Video Solution

671. Let a,b and c are real number and

$$\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$$

Watch Video Solution

672. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ and $A + A' = I$, then find the value of α

.

Watch Video Solution

673. Find the value of a and b if the function given below is continuous.

$$f(x) = \begin{cases} 5 & \text{if } x \leq 2 \\ ax + b & \text{if } 2 < x < 10 \\ 21 & \text{if } x \geq 10 \end{cases}$$

Watch Video Solution

674. If $y = (\sin x)^{\tan x}$, find $\frac{dy}{dx}$

Watch Video Solution

675. If $y = \log\left[x + \sqrt{x^2 + 1}\right]$, then prove that $\left(x^2 + 1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0.$

Watch Video Solution

676. Prove that all normals to the curve $x = a\cos t + at\sin t, y = a\sin t - at\cos t$ are at a distance a from the

origin.

Watch Video Solution

677. If $y = \left(\tan^{-1} x\right)^2$ then show that $\left(x^2+1\right)^2 \frac{d^2y}{dx^2} + 2x\left(x^2+1\right)\left(\frac{dy}{dx}\right) = 2.$

Watch Video Solution

678. Find $\int \sec^3 x dx$

Watch Video Solution

679. $\int_0^{\frac{\pi}{2}} \sin 2x \log(\tan x) dx$

Watch Video Solution

680. Solve $\left(1+x^2\right)\frac{dy}{dx}+2xy-4x^2=0$ subject to the initial condition $y(0)=0$

Watch Video Solution

681. Evaluate: $\int\frac{x+3}{\sqrt{5-4x+x^2}}dx$

Watch Video Solution

682. If with reference to the eight handed system of mutually perpendicular unit vectors $\hat{i},\hat{j},\hat{k},\vec{\alpha}=3\hat{i}-\hat{j},\beta=2\hat{i}+\hat{j}-3\hat{k}$, then express $\vec{\beta}$ in the form $\vec{\beta}=\vec{\beta_1}+\vec{\beta_2}$, where $\vec{\beta_1}$ is parallel to $\vec{\alpha}$ and $\vec{\beta_2}$ is perpendicular to $\vec{\alpha}$.

Watch Video Solution

683. Let $\vec{a}$ and $\vec{b}$ be two unit vector and α be the angle between them and $\vec{a} + \vec{b}$ is unit vector. Find α.

Watch Video Solution

684. Find the distance between the two planes $2x + 3y + 4z = 4$ and $4x + 6y + 8z = 12$.

Watch Video Solution

685. Solve for x: $\cot^{-1}(x-1) + \cot^{-1}(x-2) + \cot^{-1}(x-3) = 0$

Watch Video Solution

686. Show that $\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{16} = \pi$

Watch Video Solution

687. Prove that the following. $\begin{bmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{bmatrix}$ =(b-c)(c-a)(a-b)(bc+ca+ab)

Watch Video Solution

688. It is given that for the function f given by $f(x) = x^3 + bx^2 + ax, x \in [1, 3]$. Rolle's theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$. Find the values of a and b.

Watch Video Solution

689. If $x = \frac{1 - \cos^2 \theta}{\cos \theta}, y = \frac{1 - \cos^{2n} \theta}{\cos^n \theta}$ then show that $\left(\frac{dy}{dx}\right)^2 = n^2 \left(\frac{y^2 + 4}{x^2 + 4}\right)$

Watch Video Solution

690. Evaluate $\int_0^{\frac{\pi}{2}} \sin 2x \cdot \log(\tan x) dx$

Watch Video Solution

691. Evaluate $\int(\sqrt{\tan x}+\sqrt{\cot x})\,dx$

Watch Video Solution

692. Prove by vector method that the medians of a triangle are concurrent.

Watch Video Solution