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## MATHS

## BOOKS - USHA MATHS (ODIA ENGLISH)

## RELATION AND FUNCTION INVERSE TRIGONOMETRIC FUNCTIONS LINEAR PROGRAMMING

Exercise

1. If a set has $n$ elements, how many relations are there from $A$ to $A$ ?

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2. Let $A=\{1,2,3\}, B=\{4,5\}$. Determine all relations from $A$ to $B$.
3. Let $A=\{1,2,3\}, B=\{4,5\}$. Determine all relations from $B$ to $A$.

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4. Let $A=\{1,2,3\}, B=\{4,5\}$. Is there any relation which both a relation from $A$ to $B$ and from $B$ to $A$ ?

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5. Let $A=\{1,2,3\}, B=\{4,5\}$. Determine all relations from $A$ to $A$.

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6. Let $A=\{1,2,3\}, B=\{4,5\}$. Determine all relations from $B$ to $B$.

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7. Mark the following as true (T) or false (F). $\phi$ is a relation from set A to set $B$.

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8. Mark as true (T) or false ( F ). $A \times B$ is a relation from A to B

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9. Mark as true (T) or false (F). $\phi \times \phi$ is a relation from $A$ to $B$

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10. Mark as true (T) or false (F). A relation cannot be both symmetric ad anti-systemetic.

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11. Mark as true ( $T$ ) or false ( $F$ ). For any equivalence relation $R$ on a set $A$, $\operatorname{dom} \cdot(R)=A$

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12. Mark as true ( T ) or false ( F ). The relation $R=\{(1,1),(2,2),(3,3)\}$ on the set $A=\{1,2,3\}$ is symmetric only.

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13. What is the total number of relations from a set $A$ to set $B$, where $|A|=3$ and $|B|=2$ ?

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14. What is the partition of the set $\{1,2\}$ induced by the equivalence relation $\{(1,1),(2,2)\}$ ?
15. Let $R=\{(x, y): x<y, x, y \in N\}$. Check if R is reflexive, symmetric or transitive.

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16. If a relation is symmetric and transitive on any non-empty set, then it is reflexive. Is it true or false? State with reason.

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17. Let $R$ be a relation in the set of natural numbers $N$ defined by xRy if and only if $x+y=18$. Is R an equivalence relation?

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18. Write true or false in the following with reason or by suitable example.

If $A=\{5,7,8\} \quad$ and $\quad B=\{10,11,12,13,15\} \quad$ then
$R=\{(5,10),(5,11),(7,12),(8,15)\}$, is a function from A to B .

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19. Write true or false in the following with reason or by suitable example.

Sum of two characteristic function is a characteristic function.

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20. Compute the following remainder function. $r_{3}$ (185)

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21. Compute the following remainder function. $r_{6}(-500)$
22. Mark as true (T) or false (F). The function $f: R \rightarrow[-1,1]$ defined by $f(x)=\sin x$ is onto.

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23. Mark as true (T) or false (F). The function $f: R \rightarrow R$ defined by $f(x)=\sin x$ is bijective.

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24. Mark as true (T) or false (F). The greatest integer function is not onto.

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25. Mark as true ( $T$ ) or false ( F ). The identity function is an odd function.
26. Mark as true (T) or false (F). Let $f: R \rightarrow R$ is defined by $f(x)=x^{2}+1$. Then $f^{-1}(17)=\{4,-4\}$.

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27. Mark as true (T) or false (F). Let $|A|=3,|B|=5$. Then the number of one function from A to B is 120 .

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28. Mark as true (T) or false (F). If $|A|=5,|B|=2$, then the number of one-one function from $A$ to $B$ is 20 .

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29. Let $f(x)=x^{2}$ and $g(x)=\sqrt{x}$. Then find (gof)(4).
30. Find a linear function $f$ which maps $[-1,1]$ onto $[0,2]$.

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31. Find the range of the function $f(x)=\frac{1}{2-\sin 3 x}$

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32. If $A=\{a, b, c\}$ and $B=\{1,2\}$, then find all functions from A to B . How many of these are surjective?

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33. If $A=\{a, b, c\}$ and $B=\{1,2\}$, then find all functions from A to B .

How many of these are injective?
34. If $A=\{a, b, c\}$ and $B=\{1,2\}$, then find all functions from A to B . How many of these are bijective?

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35. Give an example of a function which is onto but not one-one.

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36. Give an example of a function which is one-one but not onto.

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37. Give an example of a function which is neither one-one nor onto
38. Give an example of a function which is onto but not one-one.

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39. If $f$ and $g$ are even functions, what can you say about fog?

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40. Fill in the blanks choosing correct answer from the brackets. If $\sin \left(\cos ^{-1} x\right)=1$ then $\mathrm{x}=\ldots\left(0, \frac{1}{2}, 1\right)$

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41. Fill in the blanks choosing correct answer from the brackets. If $x^{-1} \tan \left(\cot ^{-1} \frac{1}{x}\right)=\ldots\left(x^{\wedge}(-1), 1, \mathrm{x}\right)$
42. Fill in the blanks choosing correct answer from the brackets. The principal value of $\sin ^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is $\ldots\left(-\frac{2 \pi}{3},-\frac{\pi}{3}, \frac{4 \pi}{3}\right)$

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43. $\tan ^{-1} 2+\tan ^{-1} 3=-\quad\left(-\frac{\pi}{4}, \frac{3 \pi}{4}, \frac{\pi}{4}\right)$

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44. Fill in the blank choosing correct answer from the brackets if the value of $\sin ^{-1} x=\frac{\pi}{5}$ for some $x \in(-1,1)$ then the value of $\cos ^{-1} x i s_{-} \quad$.
$\left(\frac{3 \pi}{10}, \frac{5 \pi}{10}, \frac{7 \pi}{10}\right)$

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45. Find the value of the following: $\sin \left(\sin ^{-1} \frac{1}{2}+\cos ^{-1} \frac{1}{2}\right)$

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46. If $\sin ^{-1} \frac{1}{3}=A$, find $\cos A$

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47. Express the value of the following in simplest form.
$\tan \left\{\frac{1}{2} \sin ^{-1} \frac{2 x}{1+x^{2}}+\frac{1}{2} \cos ^{-1} \frac{1-y^{2}}{1+y^{2}}\right\}$

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48. Fill up the blanks. $\sin ^{-}\left(\sin \frac{2 \pi}{3}\right)=$

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49. Fill up the blanks. $\tan ^{-1}\left(\tan \frac{3 \pi}{4}\right)=$
50. What is the value of $\cos ^{-1}\left(\cos \frac{7 \pi}{6}\right)$ ?

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51. Write the given LPP in matrix form: Maximize $Z=50 x+60 y$ subject to the constraints $5 x+8 y=200,10 x+8 y=240, x \geq 0, y \geq 0$.

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52. Let an LPP be as follows: Maximize $Z=3 x_{1}+5 x_{2}$ subject to $x_{1}+2 x_{2} \leq 12,2 x_{1}+5 x_{2} \leq 20$ and $x_{1}, x_{2} \geq 0$. Test whether the points
$(2,3)$ and $(-3,4)$ are feasible solutions or not.

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53. Write the solution of the following LPP

Maximise $Z=2 x+3 y$
Subject to $x, y \geq 0, x+y \leq 1$.

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54. Write the solution of the following LPP

Maximise $Z=x+y$
Subject to $3 x+4 y \leq 12, x \geq 0, y \geq 0$

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55. Write the solution of the following LPP: Maximize $Z=3 x+5 y$ subject to $2 x+3 y \leq 6, x, y \geq 0$.

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56. What is the domain of $f(x)=\sqrt{x^{2}-5 x+6}$.

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57. Find least non negative integer $r$ such that $7 \times 13 \times 23 \times 413 \equiv r(\bmod 11)$

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58. Find least non negative integer $r$ such that $6 \times 18 \times 27 \times(-225) \equiv r(\bmod 8)$

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59. Find least non negative integer $r$ such that $1237(\bmod 4)+985(\bmod 4) \equiv r(\bmod 4)$
60. Find least positive integer $x$, satisfying
$276 x+128=4(\bmod 7)$.

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61. State true or false with reason. "The operation $*$ defined by $a * b=\sqrt{a^{2}+b^{2}}$ on $Q^{+"}$ is a binary operation.

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62. What does the solution set of the inequation $2 x+y>5$ represents?

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63. If $\sin ^{-1} \frac{x}{5}+\operatorname{cosec}-\frac{5}{4}=\frac{\pi}{2}$, then what is the value of x ?
64. What is the value of $\sin \left(\tan ^{-1} x+\tan ^{-1} \frac{1}{x}\right), x>0$ ?

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65. Express each of $e^{x}$ function as the sum of an even function and an odd function.

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66. Express each of $x^{2}$ function as the sum of an even function and an odd function.

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67. Let $\mathrm{f}(\mathrm{x})=\left\{\begin{array}{ll}1 & x \in Q \\ 0 & x \in R-Q\end{array}\right.$ and $\mathrm{g}(\mathrm{x})=\left\{\begin{array}{ll}1 & x \in R-Q \\ 0 & x \in Q\end{array}\right.$, find $(\mathrm{f}+\mathrm{g})(\mathrm{x})$ and $(f g)(x)$.
68. Prove that the following sets are equivalent
$\{1,2,3,4,5,6 \ldots . .\},.(2,4,6,8,10, \ldots)$
$(1,3,5,7,9, . .),.\{1,4,9,16,25 \ldots)$

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69. Determine if the binaary operation o on the set $R$ of real numbers defined by $a o b=a+b-3 \forall a, b \in R$ is commutative/ associative.

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70. On the set $R$ of real numbers, a binary operation $o$ is defined by $a 0 b=a+b+a^{2} b \forall a, b \in R$. Prove that the operation is neither commutative nor associative.

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71. Find the idenity and inverse if any for the binary operation $*$ on $Q-\{0\}$ defined by $a * b=\frac{a b}{4}, a, b \in Q-\{0\}$

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72. Find the identity elements if any for the binary operations defined by $a * b=a+b+1, a, b \in Z$.

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73. Find the identity elements if any for the binary operations defined by $a * b=a+b+a b, a, b \in Q-\{-1\}$.

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74. Let $f: R \rightarrow(-1,1)$ be defined by $f(x)=\frac{x}{1+x^{2}}, x \in R$. Find $f^{-1}$ if exists.
75. Show that $A \times A$ is an equivalence relation on a non-empty set A . What is the corresponding equivalence class.

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76. Show that the relation S defined on set $N \times N$ by $(a, b) S(c, d) \Rightarrow a+d=b+c$ is an equivalence relation.

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77. Let $R$ be the relation on $Z$ defined by $a R b$ iff $a-b$ is an even integer. Show that $R$ is an equivalence relation.

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78. Prove that $\cos \tan ^{-1} \sin \cot ^{-1} x=\sqrt{\frac{1+x^{2}}{2+x^{2}}}$.

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79. Directions (Q. Nos. 16-25) Prove the following
$\cos \left(2 \tan ^{-1} \frac{1}{7}\right)=\sin \left(4 \tan ^{-1} \frac{1}{3}\right)$.

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80. Prove that $2 \tan ^{-1}\left(\operatorname{cosec} \tan -1 x-\tan \cot ^{-1} x=\tan ^{-1} x\right.$.

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81. Prove that $\frac{1}{2} \cos ^{-1} x=\sin ^{-1} \sqrt{\frac{1-x}{2}}=\cos ^{-1} \sqrt{\frac{1+x}{2}}$.
82. 

$x^{2}+y^{2}+z^{2}+2 x y z=1$.

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83. Slove: $\tan \left(\cos ^{-1} x\right)=\sin \left(\cot ^{-1} \frac{1}{2}\right)$

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84. Find the feasible region of the inequations $x+2 y<8,2 x+y<8$ and $x>0, y>0$.

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85. Find the feasible region of the inequations $2 x_{1}+4 x_{2}>40$, $3 x_{1}+2 x_{2}>60, x_{1}>0, x_{2}>0$
86. Shade the feasible region satisfying ineuations $x_{1}+2 x_{2} \geq 10$, $2 x_{1}+2 x_{2} \geq 12,3 x_{1}+x_{2} \geq 8$ and $x_{1}, x_{2} \geq 0$.

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87. Shade the feasible region satisfying ineuations $x+2 y \leq 10$, $x+y \geq 1, x-y<0$ and $x \geq 0, y \geq 0$.

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88. Shade the feasible region satisfying ineuations $4 x+3 y \leq 60$, $2 x-y \leq 0, x-3 \geq 0$ and $x \geq 0, y \geq 0$.

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89. Prove that the following sets $A, B, C D$ are equivalent. Where $A=\{1,2,3,4, \ldots\}, B=\{2,4,6,8, \ldots\}, C=\{1,3,5,7\} D=,\{1,8,27,64$,
90. Let n be a positive integer,and a function f be defined as follows:
$\left\{\begin{array}{ll}0 & n=1 \\ f\left(\left[\frac{n}{2}\right]\right)+1 & n>1\end{array}\right.$ Then find $f(35)$.

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91. Let f be an invertible function. Then prove that $\left(f^{-1}\right)^{-1}=\mathrm{f}$.

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92. If $P\left(\frac{A}{B}\right)>P(A)$ then prove that $P\left(\frac{B}{A}\right)>P(B)$

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93. If $: f(x)=\log \left(\frac{1+x}{1-x}\right)$, then show that $\mathrm{f}(\mathrm{a})+\mathrm{f}(\mathrm{b})=\mathrm{f}\left(\frac{a+b}{1+a b}\right)$ and $2 \mathrm{f}(\mathrm{x})=\mathrm{f}\left(\frac{2 x}{1+x^{2}}\right)$.

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## 94.

$\sin ^{-1}\left(\frac{x}{a}\right)+\sin ^{-1}\left(\frac{y}{b}\right)=\alpha$ prove that $\frac{x^{2}}{a^{2}}+\frac{2 x y}{a b} \cos \alpha+\frac{y^{2}}{b^{2}}=\sin ^{2} \alpha$

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95. 

$\sin ^{-1}\left(\frac{x}{a}\right)+\sin ^{-1}\left(\frac{y}{b}\right)=\alpha$ prove that $\frac{x^{2}}{a^{2}}+\frac{2 x y}{a b} \cos \alpha+\frac{y^{2}}{b^{2}}=\sin ^{2} \alpha$

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96. Solve: $\sin ^{-1} x+\sin ^{-1}(1-x)=\cos ^{1} x$

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97. Solve $\sin ^{-1} x+\sin ^{-1}(1-x)=\frac{\pi}{2}$.
98. If: $\tan \left(\pi \cos \frac{\theta}{2}\right)=\cot \left(\pi \sin \frac{\theta}{2}\right)$, then show that $\theta=\sin ^{1}\left(\frac{3}{4}\right)$.

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99. Maximize $\quad \mathrm{Z}=6 x_{1}+10 x_{2}$ subject to
$x_{1}+2 x_{2}>10,2 x_{1}+2 x_{2}>12,3 x_{1}+x_{2}>8, x_{1}, x_{2}>0$

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100. Solve graphically $4 x+7 y \leq 28$

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101. Let an LPP be as follows: Maximize $Z=3 x_{1}+5 x_{2}$ subject to $x_{1}+2 x_{2} \leq 12,2 x_{1}+5 x_{2} \leq 20$ and $x_{1}, x_{2} \geq 0$. Test whether the points
$(2,3)$ and $(-3,4)$ are feasible solutions or not.

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102. 11.Minimize $\mathrm{Z}=20 x_{1}+40 x_{2}$ subject to the constraints $36 x_{1}+6 x_{2} \geq 108,3 x_{1}+12 x_{2} \geq 36,20 x_{1}+10 x_{2} \geq 100, x_{1}, x_{2} \geq 0$

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103. Solve the following LPP graphically Maximize $z=20 x+10 y$

Subject to $x+2 y \leq 40$
$3 x+y \geq 30$
$4 x+3 y \geq 60$
$x, y \geq 0$

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