MATHS

BOOKS - USHA MATHS (ODIA ENGLISH)

VECTORS THREE DIMENSIONAL GEOMETRY

Exercise

1. Write the values of a and b, for which the vectors $(a-1)\hat{i}+(b+2)\hat{j}+4\hat{k}$ and

$(a+1)\hat{i}+(b-2)\hat{j}+8\hat{k}$ will be parallel.

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2. Find the value of 'a' do the which the pointA,B,C with position vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $a\hat{i}-3\hat{j}+\hat{k}$ respectively are the vertices of a right angled triangle with $\angle C=\frac{\pi}{2}$.

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3. find the unit vector in the direction of 'PQ' where P and Q are the points (1,2,3) and (4,5,6).

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4. If A, B, C and D are the vertices of a square, find $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA}$.

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5. If $\overrightarrow{OP_1} = 4\hat{i} + 3\hat{j}$ and $\overrightarrow{OP_2} = 8\hat{i} - 5\hat{j}$ find $\overrightarrow{p_1p_2}$

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6. If A, B, C and D are the vertices of a square, find $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA}$.

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7. Find the magnitude: $\frac{5}{2}$ and is parallel to the vector$3\hat{i} + 4\hat{j}$.

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8. Find the vector from the origin O to the centroid of the triangle whose vertices are (1,-1,2),(2,1,3),and (-1,2,-1).

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9. If G is centroid of the $\triangle ABC$, then find $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC}$.

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10. Find $\vec{a} \cdot \left(\vec{a} \times \vec{b}\right)$.

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11. Find $\vec{i} \cdot \left(\vec{k} \times \vec{j}\right)$

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12. Find $\vec{i} \cdot \left(\vec{j} \times \vec{k} \right)$

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13. find the position vector of the mid point of the vector joining the points P(2,3,4) and Q(4,1,-2).

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14. Evaluate:

$$\hat{i} \times \left(\hat{j} \times \hat{k}\right) + \hat{j} \times \left(\hat{k} \times \hat{i}\right) + \hat{k} \times \left(\hat{i} \times \hat{j}\right)$$

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15. Using vector method find the area of the triangle with vertices (1, 0, 0) (0, 1, 0) and (0, 0, 1)

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16. What is the distance of the point (4, 5, -3) from y-axis ?

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17. What is the distance of point (1,2,3,)from yz-plane?

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18. What is the distance of the point (x, y, z) from x-axis?

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19. Find the image of the point (5,3,-2)w.r.t yz-plane.

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20. Find the number of points (x, y, z) in space other than the point (1,-2, 3), such that $|x| = 1$, $|y| = 2$ and $|z| = 3$.

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21. Find the ratio in which the line segment through (1,3,-1) and (2,6,-2) is divided by zx-plane.

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22. Write the value of y so that the points (1,y,2),(3,2,-1) and $(-4, 6, 3)$ are collinear.

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23. If O be the origin and P is the point (3,4,5),what are the direction cosine of OP?

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24. If a line makes angles α, β and γ with the positive direction of coordinate axes, then write the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.

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25. If the d.cs of a straight line be $\left\langle \frac{2}{7}, \frac{3}{7}, \frac{k}{7} \right\rangle$, then what is the value of k ?

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26. Fill in the blanks in the length of the projection of the line segment joining (1,3,-1) and (3,2,4)on z-axis is _____.

$[1, 3, 4, 5]$

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27. The projection of a line segment axes are 3,4,12.Find the length and direction cosines of the line.

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28. Find the equation of the plane passing through the line x = y = z and the point (3,2,1).

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29. State true or False .The planes $2x + 4y - z + 1 = 0$ and $x - 2y - 6z + 3 = 0$ are perpendicular to each other.

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30. Write the equation of the plane $3x - 4y + z + 5 = 0$ in normal form.

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31. Write the equation of the plane $x + 3y - 7z + 2 = 0$ in the intercept from.

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32. Find the equation of the plane passing through the point (2,3,1) and direction ratios of the normal to the plane being $< 3, 5, 7 >$.

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33. Show that the ax+by+d =0 is perpendicular to xy-plane.

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34. What are the direction cosines of the normal to the plane X+y+1=0?

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35. Write the equation of the plane passing through (3,-6,-9) and parallel to xy-plane.

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36. Write the equation of the plane passes through y-axis and z-axis.

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37. What is the distance of the point (1,1,1)from the plane y=x?

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38. Find the direction cosines of the line segment joining (3,6,1) and (4,-1,5).

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39. What is the number of line which are equally inclined to the axes?

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40. If the equation of x-axis is $\frac{x}{a} = \frac{y-d}{b} = \frac{z}{c}$, what is the value of a,b,c,d?

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41. What are the direction cosines of the line $\frac{x-2}{2} = \frac{y+4}{3} = \frac{z-1}{6}$

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42. Write the symmetrical from of the line X=5, y=4.

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43. If the plane ax+by+cz=1 meets the co-ordinate axes at A,B,C, what is the centroid of the triangle ABC?

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44. If the points P(2,-1,1),Q(1,-3 c)and R(3,-4,-4) are the vertices of a right triangle PQR, then find c if any.

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45. Show by vector method that the point P(3,-2, 4),Q(1,1 1)and R(-1,4,-2)are collinear.

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46. If the vertices A,B,C of a triangle ABC are A(1,1,8),B(4,-3,-4)and C(-3,1,5) respectively then find $\angle BAC$.

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47. If $\vec{a}$ makes equal angles with $\hat{i}, \hat{j}$ and $\hat{k}$ has magnitude 3, prove that the angle between $\vec{a}$ and each of $\hat{i}, \hat{j}$ and $\hat{k}$ is $\cos^{-1}\frac{1}{\left(\sqrt{3}\right)}$.

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48. If $\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 1, \left|\vec{c}\right| = 4$ and $\vec{a} + \vec{b} + \vec{c} = 0$, find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

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49. Let $\hat{a}, \hat{b}, \hat{c}$ be unit vectors. Suppose that and the angle between $\hat{a} \cdot \hat{b} = \hat{a} \cdot \hat{c} = 0$ and the angle between $\hat{b}$ and $\hat{c}$ is $\frac{\pi}{6}$.

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50. Calculate the area of the triangle ABC (by vector method) where A(1,1,2), B(2,2,3), C(3,-1,-1)

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51. If the co-ordinates of the two given point s A and B are (3,-1,7)and (4,-3,-1) respectively,find the magnitude and direction cosines of $\overline{AB}$.

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52. Find the co-ordinates of the foot of the perpendicular from the point $(1, 1, 1)$ on the line joining $(1, 4, 6)$ and $(54, 4)$.

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53. Write the vector equation of a line through the point (1,2,3) and parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$

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54. Show that the line through the point(4,7,8), (2,3,4)is parallel to the line through the points(-1,-2,1)and (1,2,5)

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55. If P $(1, y, z)$ lies on the line through $(3, 2, -1)$ and $(-4, 6, 3)$ find y & z.

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56. Find the perpendicular distance of the point $(-1, 3, 9)$ from the line $\frac{x-13}{5} = \frac{y+8}{-8} = \frac{z-31}{1}$

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57. Find the direction cosines of the unit vector perpendicular to the plane $\vec{r}.\left(2\hat{i} + 3\hat{j} - 6\hat{k}\right) - 21 = 0,$ through the origin.

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58. Find the equation of the plane through the points (1,0,-1),(3,2,-2)and parallel to the line $x-1=\frac{y-1}{-2}=\frac{z-2}{3}$.

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59. A variable plane passes through a fixed point (a,b,c) and meets the co-ordinate axes at A,B,C. Show that the locus of the point common to the planes drawn through A,B and

C parallel to the co-ordinate planes is

$$\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=1$$

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60. Find the equation plane passing through the point (2,3,1)and perpendicular to the line

$$\frac{x-1}{1}=\frac{y-2}{-2}=\frac{z+1}{3}$$

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61. Find the equation of the plane passing through the line $x = y = z$ and the point $(3,2,1)$.

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62. Prove the following by vector method. Median to the base of an isosceles triangle is perpendicular to the base.

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63. Prove the following by vector method. In a triangle AOB, $m\angle AOB = 90^\circ$. If P and Q are the points of trisection of AB, prove that $OP^2 + OQ^2 = \frac{5}{9}AB^2$

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64. show that the direction of cosines of a vector equally inclined to the axes OX,OY and OZ are '1/3^1/2,1/3^1/2,1/3^1/2.

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65. If veca=vecb+vecc, then write the value of $\vec{a} \cdot \left(\vec{b} \times \vec{c} \right)$.

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66. If D is the mid point of side BC of a $\triangle ABC$, show by vector method that $AB^2 + AC^2 = 2\left(AD^2 + BD^2\right)$

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67. Find the scalar components of a unit vector which is perpendicular to the vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $3\hat{i} - \hat{j} + 2\hat{k}$.

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68. Decompose the vector $6\hat{i} - 3\hat{j} - 6\hat{k}$ into vectors which are parallel and perpendicular to the vector $\hat{i} + \hat{j} + \hat{k}$.

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69. Express $2\hat{i} - \hat{j} + 3\hat{k}$ as the sum of a vector parallel, and a vector perpendicular to $2\hat{i} + 4\hat{j} - 2\hat{k}$.

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70. Resolve the vector $\overrightarrow{b} = \hat{i} + \hat{j} + \hat{k}$ into vectors parallel and perpendicular to the vector $\overrightarrow{a} = \hat{i} + \hat{j}$.

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71. Find the angle between the following pair of the planes.$2x+y+2z-4=0$and $3x+5y+z-8=0$

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72. Show that the join of the points (6,-4,4) and (0,0,-4) intersects the join of (-1,-2,-3) and (1,2,-5).

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73. Show that the points (2,3,-5) and (3,4,7) lie on the opposite side of the plane $x + 2y - 2z = 9$

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74. Find dy/dx if y=log(sec x+tan x).

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75. Obtain the equation of the line through the point $(1, 2, -3)$ and perpendicular to each of the lines $x + 4y - 3z = 0 = 2x - 5y + 7$ and $y + 3z - 2 = 0 = x + 2z + 5$

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76. Find the shortest distance between the lines $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$.

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77. Find the equation of the plane passing through the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and the point (0,7,-7). Also show that the line $x = \frac{7-y}{3} = \frac{z+7}{2}$ lies in this plane.

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78. Find the equation of the plane containing the line $\frac{x+3}{3} = \frac{y-1}{4} = \frac{z-2}{-2}$ and the point (0,2,4).

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79. Show that lines $\vec{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(3\hat{i} - \hat{j}\right)$ and $\vec{r} = \left(4\hat{i} - \hat{k}\right) + \mu\left(2\hat{i} + 3\hat{k}\right)$ intersect each other. Find their point of intersection.

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80. If P is the point (2,1,6), then find the point Q such that PQ is perpendicular to the plane

passing through the points (2,1,0), (5,0,1), (4,1,1) and the mid point of PQ lies on it.

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81. The plane $lx + my = 0$ is rotated about its line of intersection with the plane z=0 through angle measure alpha. Prove that the equation of the plane in new position is $lx + my \pm z\sqrt{l^2 + m^2}\tan\alpha = 0$

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