



# MATHS

# **BOOKS - MBD MATHS (ODIA ENGLISH)**

# **APPLICATION OF DERIVATIVES**

**Question Bank** 

**1.** Find the velocity and acceleration at the end of 2 seconds of the particle moving according to the following rules.  $s=2t^2+3t+1$ 



to the following rules. $s=\sqrt{t}+1$ 



3. Find the velocity and acceleration at the end

of 2 seconds of the particle moving according

to the following rules.  $s=rac{3}{2}t+1$ 





5. The sides of an equilateral triangle are increasing at the rate of  $\sqrt{3}$  cm/sec. find the

rate at which the area of the triangle is increasing when the side is 4 cm long.



**6.** Find the rate at which the volume of a spherical balloon will increase when its radius is 2 meters if the rate of increase of its redius is 0.3 m/min.

**7.** The surface area of a cube is decreasing at the rate of 15 sq. cm/sec. Find the rate length of the edge is 5 cm.



8. Find the equations to the tangents and normals to the following curves at the indicated points.  $Y = 2x^2 + 3 atx = -1$ 

**9.** Find the equations to the tangents and normals to the following curves at the indicated points.  $y = x^3 - x \; atx = 2$ 

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10. Find the equations to the tangents and normals to the following curves at the indicated points.  $y = \sqrt{x} + 2x + 6 \ atx = 4$ 



12. Find the equations to the tangents and normals to the following curves at the indicated points.  $y = (\log x)^2$  at x = 1/e`

**13.** Find the equations to the tangents and normals to the following curves at the indicated points.  $y = \frac{1}{\log x}$  at x = 2`

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14. Find the equations to the tangents and normals to the following curves at the indicated points.  $y = xe^x atx = 0$ 

15. Find the equations to the tangents and normals to the following curves at the indicated points.

$$y=a(thita-\sin heta), y=a(1-\cos heta)$$

 $at heta=rac{\pi}{2}$ 

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**16.** Find the equations to the tangents and normals to the following curves at the indicated points.  $(x/a)^2/3 + (y/b) 2/3 = 1at(a cos^3 theta, b sin^3 theta)`$ 



# 17. Find the point on the curve $y^2-x^2+2x-1=0$

where the tangent is parallel to the x - axis.

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#### 18. Find the point (S) on the curve

$$x=rac{3at}{1+t^2}$$
,  $y=rac{3at^2}{1+t^2}$ 

where the tangent is perependicular to the

line 4x+3y+5=0.



19. Find the point on the curve

$$x^2 + y^2 - 4xy + 2 = 0$$

where the normal is paralell to the x-asis.

**20.** Show that the line y = mx + c touchees

$$y = 4ax$$
 if  $c = rac{a}{m}$ .

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**21.** Show that the line y = mx + c touches the

ellips

$$rac{x^2}{a^2}+rac{y^2}{b^2}=1 \ \ ext{if} \ \ c^2=a^2m^2+b^2.$$

22. Show that the sum of the intercepts on the

coordinate axes of any tangent to the curve

 $\sqrt{x} + \sqrt{y} = \sqrt{a}$  is constant.

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**23.** Show that the curves  $y = 2^x$  and  $y = 5^x$ 

intersect at an angle 
$$an^{-1} \left| rac{1n \left( rac{5}{2} 
ight)}{1 + 1n21n5} \right|$$

Note Angle between two curves is the angle between their tangents at the point of intersection.



**25.** Find the equasion of the tangents drawn from the point (1,2) to the curve.

$$y^2 - 2x^3 - 4y + 8 = 0$$

**26.** Show that the equation of the normal to  $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$  is y cos theta - x sin theta = a cos 2"theta"*where*theta` is the inclination of the normal to x-axis.



27. Show that the length of the portion of the

tangent to  $x^{(2/3)} + y^{(2/3)} = a^{(2/3)}$ 

intercepted between the axes is constant.



28. Find the tangent to the curve

$$y = \cos(x+y), 0 < x < 2\pi$$

which is parallel to the line x + 2y = 0.



29. If tangents are drawn from the origin to the curve y = sin x then show that the locus of the point of contact is  $x^2y^2 = x^2 - y^2$ .





**30.** Find the equation of the normal to the curve given by

 $x=3\cos heta-\cos^3 heta$ y = 3 sin theta - sin^3

theta`

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**31.** If  $x \cos lpha + y \sin lpha = p$  is a tangent to the

curve



$$x=a(t-\sin t), y=at(1+\cos t)$$
 at

$$t=rac{\pi}{2}$$
 has slope.(1- pi/2)`

**33.** Find the intervals where the following functions are (a) increasing and (b) decreasing.  $y = \sin x, \ \in [0, 2\pi]$ 



**34.** Find the intervals where the following functions are (a) increasing and (b)

decreasing.  $y = Inx, x \in R_+$ 

35. Find the intervals where the following functions are (a) increasing and (b) decreasing.  $y = a^x, a > 0, x \in R$ 

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**36.** Find the intervals where the following functions are (a) increasing and (b) decreasing.  $y = \sin x + \cos x, x \in [0, 2\pi]$ 

**37.** Find the intervals where the following functions are (a) increasing and (b) decreasing.  $y = 2x^3 + 3x^2 - 36x - 7$ 

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**38.** Find the intervals where the following functions are (a) increasing and (b) decreasing.  $y = \frac{1}{x-1}, x \neq 1$ 

**39.** Find the intervals where the following functions are (a) increasing and (b) decreasing.

$$y = egin{cases} x^2 + 1 & x \leq \ -3 \ x^3 - 8x + 13 & x > \ -3 \end{cases}$$

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40. Find the intervals where the following functions are (a) increasing and (b) decreasing.  $y = 4x^2 + \frac{1}{x}$ 

**41.** Find the intervals where the following functions are (a) increasing and (b) decreasing.  $y = (x - 1)^2 (x + 2)$ 

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**42.** Find the intervals where the following functions are (a) increasing and (b) decreasing.  $y = \frac{Inx}{x}, x > 0$ 



44. Find the intervals where the following functions are (a) increasing and (b) decreasing.  $y = \sin 2x - \cos 2x, x \in [0, 2\pi]$ 

**45.** Give a rough sketch of the functions given in question 1.



**47.** Show that  $2 \sin x +$ than x ge 3x all x in (0,

pi/20).

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48. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values. $y = x^2 + 2x + 3$ 

49. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values.  $y = 5x^2 - 2x^5$ 

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50. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values.  $y = \frac{3x}{x^2 + 1}$ .

**51.** Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values.  $y = x^2 \sqrt{1-x^2}$ 

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**52.** Find the extreme points of the following functions. Specify if the extremum is a

maximum or minimum. Find the extreme values.  $y=2x^3-15x^2-36x+18$ 

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**53.** Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values.

$$y=60\,/\left(x^4-x^2+25
ight)$$

54. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values.  $y = (x - 1)^3$ 

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55. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values. $y = (x - 2)^3 (x + 3)^4$ 

56. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values.  $y = x + \frac{1}{x}$ 

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**57.** Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme

values. y =

$$4\cos 2x-3\sin 2x, x\in \Big(-rac{\pi}{2},rac{\pi}{2}\Big)$$



**58.** Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values.  $y = \sin x \cos x, x \in \left[\frac{\pi}{8}, \frac{\pi}{2}\right]$ 

59. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values. $y=\cos x(1+\sin x),\,x\in[0,2\pi]$ 

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60. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values.  $y = xe^{-x}$ ,  $x \in (-2, 2)$ 





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62. Shows that the following functions do not

possess maximum or minimum.  $x^5$ 



65. Shows that the following functions do not

possess maximum or minimum. In|x|, x 
eq 0

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**66.** Use the function  $f(x) = x^{1/x}, x > 0$  to

show that  $e^{\pi} > \pi^{e}$ .
67. Prove the inequality

$$x^2e^{-x^2}\leq e^{-1}, x\in R.$$

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**68.** If  $f(x) = a \ln x + bx^2 + x$  has extreme values at x = -1 and x = 2 then find a and b.

**69.** Show that  $rac{x}{1+x an x,}x\in \left(0,rac{\pi}{2}
ight)$ is

maximum when  $x = \cos x$ .

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**70.** Determine the absolute maximum and absolute minimum of the following function on [-1, 1].

$$f(x) = egin{cases} {(x+1)}^2 & x \leq 0 \ {(x-1)}^2 & x > 0 \end{cases}$$



**72.** Find two numbers x and y whose sum is 15 such that  $xy^2$  is maximum.



**73.** If then sum of two positive numbers in constant then show that their product is maximum when they are equal.



### 74. Determine a rectangle of area 25sq. Units

which has minimum perimeter.



**75.** Find the altitude of a right circular cylinder of maximum volume inscribed in a sphere of radius r.



**76.** Show that the radius of the right cicular cylinder of greatest curved surface that can be inscribed in a given cone is half the redius of the base of the cone.



**77.** Show that the semivertical angle of a cone of given slant height is  $\tan^1 \sqrt{2}$  when its volume is maximum.



**78.** A cylindrical open water tank with a circular base is to be made out of 30 sq metres of metal sheet. Find the dimensions so that it can hold maximum water. (Neglect thickness of sheet).

**79.** A cylindrical vessel of capacity 500 cubic metres open at the top is to be constructed. Find the dimensions of the vessel if the material used is minimum given that the thickness of the material used is 2 cm.



80. Find the coordinates of the point on the

curve  $x^2y - x + y = 0$ 

where the slope of the tangent is maximum.



**81.** Find the points on the curve  $y = x^2 + 1$ 

which are nearest to the point (0,2).



82. Show that the minimum distance of a point on the curve  $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 1$  from the origin is a + b.



83. Show that the semi-vertical angle of a right

circular cone of minimum volume that

circumscribes a given sphere is  $\sin^{-1}\left(\frac{1}{3}\right)$ .

84. Show that the shrtest distance of the point

(0, 8a) from the curve  $ax^2=y^3$  is  $2a\sqrt{11}.$ 



85. Shows that the triangle of greatest area

that can be inscribed in a circle is equilateral.



following cases.  $y = x^3 - 1$ 

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87. Determine the differentials in each of the

following cases.  $y = \sin^2 x$ 

following cases. 
$$y=rac{1+\sqrt{x}}{1-\sqrt{x}}$$

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89. Determine the differentials in each of the

following cases.  $z = \cos 2t - 2 \cot t$ 

following cases.  $r=rac{4}{1+\sin heta}$ 

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91. Determine the differentials in each of the

following cases.  $x^2y=2$ 

following cases.  $xy^2 + yx^2 = 1$ 





**94.** Find  $\delta f$  and df when  $f(x)=\sqrt{x}, x=16, \delta x=0.3$ Watch Video Solution **95.** Find  $\delta f$  and df when  $f(x) = (x+1)^3, x = 8, \delta x = 0.04$ 

**96.** Find  $\delta f$  and df when f(x) = In (1+x), x = 1, deltax = 0.04 **Vatch Video Solution** 

### 97. Find approximate values of the following :





98. Find approximate values of the following :

 $\sqrt[6]{63}$ 



### 99. Find approximate values of the following :

 $\sqrt{48.96}$ 



100. Find approximate values of the following :

 $(1.99)^7$ 



### 101. Find approximate values of the following :

 $2^{3.02}$ 



102. Find approximate values of the following :

 $\sin 59^{\circ}$ 

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**103.** find the percentage of error in calculation of the surface area of a sherical ballon of diameter 14.02 m. if the true diameter is 14m.



**104.** Find approximately the difference between the volumes of two cubes of sides 3cm and 3.04 cm.



**105.** The height of a regular cone is 3 times the radius of its base. The radius of the base was wrongly measured to be 5 cm. where as its true radius is 4.88 cm. Find the relative error in

measuring the curved surface area of the

cone.











**115.** Find the following limits:  

$$\lim_{x \to \infty} \frac{x^3 - 3x + 1}{2x^3 - 7x^2 + 5}$$
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117. Find the following limits:  $\lim_{x o 0+} \frac{\ln \tan x}{\ln \sin 2x}$ 









128. Find the following limits:

 
$$\lim_{x \to 1} \left( \frac{1}{x-1} - \frac{1}{\ln x} \right)$$

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 129. Find the following limits:

 
$$\lim_{x \to \infty} \left( x - \sqrt{x^2 - 1} \right)$$

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**130.** Find the following limits:
$$\lim_{x \to \infty} \left( x - \sqrt{x^2 + 2x} \right)$$
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**134.** Find the following limits:  

$$\lim_{x \to \infty} \left(1 + \frac{3}{x}\right)^x$$
**Vatch Video Solution 135.** Find the following limits:  

$$\lim_{x \to \infty} \left(e^{\frac{1}{x}}\right)^{\ln(1+x)}$$
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**136.** Find the following limits:  $\lim_{x o 0} \left( \frac{\sin x}{x} \right)^{rac{1}{x}}$ 



**139.** Find the following limits:  
$$\lim_{x \to 1} (2 - x)^{\csc \pi x}$$
**Watch Video Solution 140.** Find the following limits:

$$\lim_{x \to 0} \left( \frac{x^2 + 2x - 1}{x^2 - 1} \right)^{\frac{1}{x}}$$
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