MATHS

BOOKS - MBD MATHS (ODIA ENGLISH)

COMPLEX NUMBERS AND QUADRATIC EQUATIONS

Question Bank

1. Multiply $(2\sqrt{-3}+3\sqrt{-2})$ by $(4\sqrt{-3}-5\sqrt{-2})$

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2. Multiply $(3\sqrt{-7}-5\sqrt{-2})(3\sqrt{-2}+5\sqrt{-2}$

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3. Multiply $\left(\sqrt{-1}+\sqrt{-1}\right)\left(a-b\sqrt{-1}\right)$

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4. Multiply $\left(x-\frac{1+\sqrt{-3}}{2}\right)\left(x-\frac{1-\sqrt{-3}}{2}\right)$

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5. Express with rational denominator $\frac{1}{3-\sqrt{-2}}$

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6. Express with rational denominator $\frac{3\sqrt{-2}+2(-5)}{3\sqrt{-2}-2\sqrt{-2}}$

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7. Express with rational denominator $\frac{3+2\sqrt{-1}}{2-5\sqrt{-1}}+\frac{3-2\sqrt{-1}}{2+5\sqrt{-1}}$

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8. Express with rational denominator $\frac{a+x\sqrt{-1}}{a-x\sqrt{-1}}-\frac{a-x\sqrt{-1}}{a+x\sqrt{-1}}$

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9. Express with rational denominator $\frac{\left(x+\sqrt{-1}\right)^2}{x-\sqrt{-1}}-\frac{\left(x-\sqrt{-1^2}\right)}{x+\sqrt{-1}}$

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10. Express with rational denominator $\frac{\left(a+\sqrt{-1}\right)^3-\left(a-\sqrt{-1}\right)^3}{\left(a+\sqrt{-1}\right)^2-\left(a-\sqrt{-1}\right)^2}$

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11. Find the value of $(-i)^{4n+3}$, when n is positive.

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12. Find the square of $(a+40i)+\sqrt{9-40\sqrt{-i}}$

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13. Express in the form $a+ib$: $\frac{3+5i}{2-3i}$

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14. Express in the form a+bi $\frac{\sqrt{3}-i\sqrt{2}}{2\sqrt{3}-i\sqrt{3}}$

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15. Express in the form $\frac{(I+i)^2}{3-i}$

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16. Express in the form $\frac{(a+ib)^3}{a-ib} - \frac{(a-ib)^3}{a+ib}$

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17. Express in the form a+bi $\frac{1+i}{1-i}$

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18. Express the 1 points geometrically in the Argrand plane.

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19. Express the $3i$ points geometrically in the Argrand plane.

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20. Express the -2 points geometrically in the Argrand plane.

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21. Express the $3+2i$ points geometrically in the Argand plane.

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22. Express the $-3+i$points geometrically in the Argand plane.

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23. Express the $1 - i$ points geometrically in the Argand plane.

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24. Show that the following number are equidistant from the origin :

$\sqrt{2} + i, 1 + i\sqrt{2}, i\sqrt{3}$

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25. Express $\sqrt{2} + i = r\cos\theta + ir\sin\theta$complex numbers in the polar form.

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26. Express $1 + i\sqrt{2} = r\cos\theta + \sin\theta$ complex numbers in the polar form.

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27. Express $i\sqrt{3} = r\cos\theta + ir\sin\theta$ complex numbers in the polar form.

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28. If $1, \omega, \omega^2$ are the three cube roots of unity, prove that $\left(1+\omega^2\right)^4 = \omega.$

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29. $\left(1-\omega+\omega^2\right)\left(1+\omega-\omega^2\right) = 4$

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30. $(1-\omega)\left(1-\omega^2\right)\left(1-\omega^4\right)\left(1-\omega^5\right) = 9$

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31. $\left(2+5\omega+2\omega^2\right)^6=\left(2+2\omega+5\omega^2\right)^6=729$

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32. $\left(1-\omega+\omega^2\right)\left(1-\omega^2+\omega^4\right)\left(1-\omega^4+\omega^2\right)\ldots.$ to $2nFac\rightarrow rs=2^{2n}$

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33. Prove that $x^3+y^3+z^3-3xyz$

$=(x+y+z)\left(x+\omega y+\omega^2 z\right)\left(x+y\omega^2+z\omega\right)$

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34. If $x = a + b, y = a\omega + b\omega^2, z = a\omega^2 + b\omega$ show that $xyz = a^3 + b^3$

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35. If $x = a + b, y = a\omega + b\omega^2, z = a\omega^2 + b\omega$ show that $x^2 + y^2 + z^2 = 6ab$

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36. If $x = a + b, y = a\omega + b\omega^2, z = a\omega^2 + b\omega$ show that $x^3 + y^3 + z^3 = 3(a^3 + b^3)$

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37. If $ax + by + cz = X, cx + by + az = Y, bx + ay + cz = Z$ show that

$$\left(a^2+b^2+c^2-ab-bc-ca\right)\left(x^2+y^2+z^2-xy-yz-zx\right)$$

$$=X^2+Y^2+Z^2-YZ-ZX-XY$$

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38. If $|z_1| \leq 1, |z_2| \leq 1$show that

$$|1-z_1z_2|^2-|z_1-z_2|^2=\left(1-|z_1|^2\right)\left(1-|z_2|^2\right.$$

"Hence or otherwise show that."

$$\left|\frac{z_1-z_2}{1-z_1z_2}\right|<1\text{if}|z_1|<1, |z_2|<1$$

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39. If a, b, c are complex numbers satisfying a+b+c=0 and $a^2+b^2+c^2=0$ then show that |a|=|b|=|c|.

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40. What do the $\{z: |z-a|+|z+a|=2c\}$where$|a|<c$ represent?

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41. What do the`{z:|z-a|-|z+a|=c} represent?

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42. What happen in (i)$|a| \geq c$?

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43. Given $\cos\alpha+\cos\beta+\cos\gamma=\sin\alpha+\sin\beta+\sin\gamma=0$

Show that $\cos 3\alpha+\cos 3\beta+\cos 3\gamma$=$3\cos(\alpha+\beta+\gamma)$

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44. Use Binomial theorem and De Moivre's theorem to show

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$,

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$Express cos ntheta as sum of product of power of $\sin\theta$and $\cos\theta$. Do the same thing for$\sin n\theta$.

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45. Find square root of $-5 + 12\sqrt{-1}$

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46. Find square root of $-11 - 60\sqrt{-1}$

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47. Find square root of $-47 + 8\sqrt{-1}$

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48. Find square root of $-8+\sqrt{-1}$

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49. Find square root of $a^2-1+2a\sqrt{-1}$

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50. Find square root of $4ab-2\left(a^2-b^2\right)\sqrt{-1}$

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51. Find the values of $\cos 72^\circ$ .....

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52. Find the value of $\cos 36^\circ$.

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53. Evaluate $\cos \frac{2\pi}{17}$ using the equation $x^{17} - 1 = 0$

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54. Solve the equation $z^7 = 1$

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55. Solve the equation $z^3 = i$

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56. Solve the equation $z^6 = -i$

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57. Solve the equation $z^3 = 1 + i$

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58. Given $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$

Show that $3\alpha + 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$

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59. If $\sin\alpha + \sin\beta + \sin\gamma = 0$

$= \cos\alpha + \cos\beta + \cos\gamma = 0$

Show that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = \cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{3}{2}$

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60. If $x + \frac{1}{x} = 2\cos\theta$, Show that $x^n + \frac{1}{x^n} = 2\cos n\theta$

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61. $x_r = \cos\alpha_r + i\sin\alpha_r$.

$r = 1, 2, 3 \text{and} x_1 + x_2 + x_3 = 0$

Show that $\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} = 0$

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62. Show that $\left(\frac{1 + \sin\theta + i\cos\theta}{1 + \sin\theta - i\cos\theta}\right)^n$
$= \cos\left(\frac{n\pi}{2} - n\theta\right) + i\sin\left(\frac{n\pi}{2} - n\theta\right)$

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63. If

$\alpha \text{and} \beta \text{are roots} \quad x^2 - 2x + 4 = 0 \text{then show that} \alpha^n + \beta^n = 2^{n+1}\cos\frac{n\pi}{3}$

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64. For a positive integer n show that $(1+i)^n + (1-i)^n = 2^{\frac{n+2}{2}} \cos\left(\frac{n\pi}{4}\right)$

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65. For a positive integer n show that $\left(1+i\sqrt{3}\right)^n + \left(1-i\sqrt{3}\right)^n = 2^{n+1}\cos\frac{n\pi}{3}$

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66. Let$x + \frac{1}{x} = 2\cos\alpha, y + \frac{1}{y} = 2\cos\beta$ $z + \frac{1}{z} = 2\cos\gamma$. Show that$2\cos(\alpha+\beta+\gamma) = xyz + \frac{1}{xyz}$

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67. Let $x + \frac{1}{x} = 2\cos\alpha, y + \frac{1}{y} = 2\cos\beta$

$z + \frac{1}{z} = 2\cos\gamma$. Show that $2\cos(p\alpha + q\beta + r\gamma) = x^p y^q z^r + \frac{1}{x^p y^q z^r}$

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68. Solve $x^9 + x^5 - x^4 = 1$

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69. Find the general value of

θ if $(\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta)\ldots\ldots\ldots$

$(\cos n\theta + i\sin n\theta) = 1$

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70. If $z = x + iy$ show that $|x| + |y| \leq \sqrt{2}|z|$

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71. Show that $Re(z_1 z_2) = Rez_1 Rez_2 - Imz_1 Imz_2$

$Im(z_1 z_2) = Rez_1 Imz_2 + Rez_2 Imz_1$

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72. What is the value of $arg\omega + arg\omega^2$?

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73. If $|z_1| \leq 1, |z_2| \leq 1$show that

$|1 - z_1 z_2|^2 - |z_1 - z_2|^2 = \left(1 - |z_1|^2\right)\left(1 - |z_2|^2\right.$

"Hence or otherwise show that."

$\left|\frac{z_1 - z_2}{1 - z_1 z_2}\right| < 1\text{if}|z_1| < 1, |z_2| < 1$

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74. If $z_1^2 + z_2^2 + z_3^2 - z_2 z_3 - z_3 z_4 = 0$

Show that

$|z_1 - z_2| = |z_2 - z_3| = |z_3 - z_1|$

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75. Solve $\frac{(1-i)x + 3i}{2+i} + \frac{(3+2i)y + i}{2-i} = -i \text{where} x, y, \in R.$

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76. Find the region on the Argand plane on which z satisfies

$1 < |z - 2i| < 3$

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