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## MATHS

# BOOKS - MBD MATHS (ODIA ENGLISH) 

## RELATION AND FUNCTION

Question Bank

1. If $A-\{a, b, c, d)$ mention the type of relations on $A$ given below, which of them are equivalence relations?
$\{(a, a),(b, b)\}$

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2. If $A-\{a, b, c, d)$ mention the type of relations on $A$ given below, which of them are equivalence relations?
$\{(a, a),(b, b),(c, c),(d, d)\}$

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3. If $A=\{a, b, c, d\}$ mention the type of relations on $A$ given below, which of them are equivalence relations? $\{(a, b),(b, a),(b, d),(d . b)\}$

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4. If $A=\{a, b, c, d\}$ mention the type of relations on $A$ given below, which of them are equivalence relations? $\{(b, c),(b, d),(c, d)\}$
5. If $A=\{a, b, c, d\}$ mention the type of relations on $A$ given below, which of them are equivalence relations? $\{(a, a), b, b),(c, c),(d, d),(a, d),(a, c),(d, a),(c, a),(c, d),(d, c)\}$

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6. Write relations in tabular form and determine their type for
$R=\{(x, y): 2 x-y=0\}$ on $A=\{1,2,3, \ldots, 13\}$

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7. Write relations in tabular form and determine their type for
$R=\{(x, y): x$ divides $y\}$ on $A=\{1,2,3,4,5,6\}$

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8. Write relations in tabular form and determine their type for
$R=\{(x, y): x$ divides $2-y\}$ on $A=\{1,2,3,4,5\}$

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9. Write the relations in tabular form and determine their type.

$$
\mathrm{R}=\{(\mathrm{x}, \mathrm{y}): \mathrm{y} \leq \mathrm{x} \leq 4\} \text { on } \mathrm{A}=\{1,2,3,4,5)
$$

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10. Test whether the relations are reflexive, symmetric or transitive on the sets specified.
$R=\{(m, n): m-n \geq 7)$ on $Z$.

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11. Test wheter relations are reflexive, symmetric or transitive on the sets specified for $R=\{(m, n\}: 2 \mid(m+n)\}$ on $Z$.

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12. Test whether the relations are reflexive, symmetric or transitive on the sets specified.
$R=\{(m, n): m+n$ is not divisible by 3$)$ on $Z$.

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13. Test whether the relations are reflexive, symmetric or transitive on the sets specified.
$\mathrm{R}=\left\{(\mathrm{m}, \mathrm{n}): \frac{m}{n}\right.$ is a power of 5$\}$ on $\mathrm{Z}-\{0\}$.
14. Test whether the relations are reflexive, symmetric or transitive on the sets specified.
$R=(m, n\}$ : $m n$ is divisible by 2 ) on $Z$.

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15. Test wheter relations are reflexive, symmetric or transitive on the sets specified for
$R=\{(m, n): 3$ divides $m-n\} o n\{1,2,3 \ldots, 10\}$.

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16. List the members of the equivalence relation defined by $\{\{1\},\{2\},\{3,4\}\}$ partitins on $X=\{1,2,3,4\}$.Also find the equivalence classes of 1,2,3 and 4.
17. List the members of the equivalence relation defined by $\{\{1,2,3\},\{4\}\}$ partitins on $X=\{1,2,3,4\}$.Also find the equivalence classes of 1,2,3 and 4.

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18. List the members of the equivalence relation defined by $\{\{1,2,3,4\}\}$ partitins on $\mathrm{X}=\{1,2,3,4\}$. Also find the equivalence classes of 1,2,3 and 4.

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19. Show that if $R$ is an equivalence relation on $X$ then dom
$R=r n g R=X$.
20. Given an example of a relation which is
reflexive, symmetric but not transitive.

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21. Given an example of a relation which is
reflexive, transitive but not symmetric.

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22. Given an example of a relation which is
symmetric, transitive but not reflexive.
23. Given an example of a relation which is reflexive but neither symmetric nor transitive.

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24. Given an example of a relation which is
transitive but neither reflexive nor symmetric.

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25. Given an example of a relation which is
an empty relation.

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26. Given an example of a relation which is
a universal relation.

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27. Let R be a relation on X , If R is symmetric then $x R y \Rightarrow y R x$. If it is also transitive then xRy and $y R x \Rightarrow x R x$.So whenever a relation is symmetric and transitive then it is also reflexive. What is wrong in this argument ?

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28. Suppose a box contains a set of $n$ balls $(n>4)$ (denoted by $B$ )of four different colours (many have different sizes), viz,red, blue, green and yellow. Show that a relation $R$ defined on $B$ as $R=\left\{\left(b_{1}, b_{2}\right):\right.$ balls $b_{1} \operatorname{and} b_{2}$ have the same colour $\}$ is an
equivalence relation on B. How many equivalence classes can you find with respect ot R ?

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29. Find the number of equivalence, relations on $X=\{1,2,3)$,

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30. Let $R$ be the relation on the set $R$ of real numbers such that $a R b$ iff $a-b$ is and integer. Test whether $R$ is an equivalence relation. If so find the equivalence class of $\operatorname{land} \frac{1}{2}$ wrt. This equivalence relation.

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31. Find the least positive integer $r$ such that $185 \in[r]_{7}$

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32. Find the least positive integer $r$ such that $-375 \in[r]_{11}$

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33. Find the least positive integer $r$ such that $-12 \in[r]_{13}$

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34. Find least non negative integer $r$ such that
$7 \times 13 \times 23 \times 413 \equiv r(\bmod 11)$

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35. Find least non negative integer $r$ such that $6 \times 18 \times 27 \times(-225) \equiv r(\bmod 8)$

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36. Find least non negative integer $r$ such that $1237(\bmod 4)+985(\bmod 4) \equiv r(\bmod 4)$

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37. Find least non negative integer $r$ such that $1936 \times 8789 \equiv r(\bmod 4)$

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38. Find least positive integer $x$, satisfying
$276 x+128=4(\bmod 7)$.

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39. Find three positive integers
$x_{i}, i=1,1,3$ satisfying $3 x \equiv 2(\bmod 7)$

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40. Let $X=\{x, y\}$ and $Y=\{u, v\}$. Write down all the functions that can be defined from X To Y. How many of these are (i) one-one (ii) onto and (iii)one-one and onto ?

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41. Let X and Y be sets containing m and n elements respectively.What is the total number of functions from X to Y .

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42. Let $X$ and $Y$ be sets containing $m$ and $n$ elements respectively.How many functions from X to Y are one-one according as $m<n, m>n$ and $m=n ?$

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43. Examine $f: R \rightarrow R, f(x)=x^{2}$ functions if it is (i) injective (ii) surjective, (iii) bijective and (iv) none of the three.

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44. Examine $f: R \rightarrow[-1,1], f(x)=\sin x$ functions if it is (i) injective (ii) surjective, (iii) bijective and (iv) none of the three.

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45. Examine $f: R_{+} \rightarrow R+, f(x)=x+\frac{1}{x}$
where $R_{+}=\{x \in R: x>0\}$ functions if it is (i) injective
surjective, (iii) bijective and (iv) none of the three.

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46. Examine $f: R \rightarrow R, f(x)=x^{3}+1$ functions if it is injective (ii) surjective, (iii) bijective and (iv) none of the three.

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47. Examine $f:(-1,1) \rightarrow R, f(x)=\frac{x}{1-x^{2}}$ functions if it is (i) injective (ii) surjective, (iii) bijective and (iv) none of the three.

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48. Examine $f: R \rightarrow R, f(x)=[x]=$ the greatest integer $\leq x$.
functions if it is (i) injective (ii) surjective, (iii) bijective and (iv) none of the three.

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49. Examine $f: R \rightarrow R, f(x)=|x|$ functions if it is (i) injective
(ii) surjective, (iii) bijective and (iv) none of the three.

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50. Examine $f: R \rightarrow R, f(x)=\operatorname{sgn} x$ functions if it is (i) injective
(ii) surjective, (iii) bijective and (iv) none of the three.

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51. Examine $f: R \rightarrow R, f=i d_{R}=$ the identity function or R . functions if it is (i) injective (ii) surjective, (iii) bijective and (iv) none of the three.

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52. Show that $f(x)=\sin x$ on $[0, p i / 2]$ functions are injective.
53. Show that $f(x)=\cos x[0, \pi]$ functions are injective.

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54. Show that $f(x)=\log _{a} x$ on $(0, \infty),(a>0$ and $a \neq 1)$ functions are injective.

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55. Show that $f(x)=a^{x}$ on $R .(a>0$ and $a \neq 1)$ functions are injective.

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56. Show that functions $f$ and $g$ defined by $f(x)=2 \log x$ and $g(x)$ $=\log x^{2}$ are not equal even though $\log { }^{`} x^{\wedge} 2=2 \log \mathrm{x}$.

## D Watch Video Solution

57. Give an example of a function which is Surjective but not injective.

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58. Give an example of a function which is injective but not surjective.

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59. Give an example of a function which is neither injective nor surjective.

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60. Give an example of a function which is bijective

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61. Prove that $\{1,2,3,4,5,6, \ldots \ldots\}$ set are equivalent.

## D View Text Solution

62. Prove that the following sets are equivalent
$\{1,2,3,4,5,6 \ldots . .\},.(2,4,6,8,10, . .$.

## D Watch Video Solution

63. Prove that the following sets are equivalent
$\{1,2,3,4,5,6 \ldots . .\},.(2,4,6,8,10, . .$.
(1,3,5, 7, 9,...), \{1, 4, 9, 16, 25...)

## - Watch Video Solution

64. Prove that the following sets are equivalent
$\{1,2,3,4,5,6 \ldots . .\},.(2,4,6,8,10, . .$.
$(1,3,5,7,9, . .),.\{1,4,9,16,25 \ldots)$

## - Watch Video Solution

65. Let $\mathrm{f}=\{(1, \mathrm{a}),(2, \mathrm{~b}),(3, \mathrm{c}),(4, \mathrm{~d})\}$ and $\mathrm{g}=\{(\mathrm{a}, \mathrm{x}),(\mathrm{b}, \mathrm{x}),(\mathrm{c}, \mathrm{y}),(\mathrm{d}, \mathrm{x})\}$ Determine gof and fog if possible. Test whether fog=gof.

## D Watch Video Solution

66. Let $\mathrm{f}=\{(1,3),(2,4),(3,7)\}$ and $\mathrm{g}=\{(3,2),(4,3),(7,1)\}$

Determine gof and fog if possible. Test whether fog =gof.

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67. Let $f(x)=\sqrt{x} \operatorname{and} g(x)=1-x^{2}$. Find natural domains of f and $g$.
68. Let $f(x)=\sqrt{x} \operatorname{and} g(x)=1-x^{2}$. Compute fog and gof and find their natural domains.

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69. Let $f(x)=\sqrt{x} \operatorname{and} g(x)=1-x^{2}$. Find natural domain of $\mathrm{h}(\mathrm{x})$
$=1-x$.

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70. Let $\mathrm{f}(\mathrm{x})=\sqrt{x}$ and $\mathrm{g}(\mathrm{x})=1-x^{2}$.

Show that hgof only on $R_{0}=\{x \in R: x \geq 0\}$ and not on R..

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71. Find the composition fog and gof and test whether fog = gof when $f$ and $g$ are functions or $R$ given by $f(x)=x^{3}+1, g(x)=x^{2}-2$

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72. Find the composition fog and gof and test whether fog = gof when $f$ and $g$ are functions or $R$ given by $f(x)=\sin x, g(x), g(x)=x^{5}$

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73. Find the composition fog and gof and test whether fog = gof when $f$ and $g$ are functions or $R$ given by

$$
f(x)=\cos x, g(x)=\sin x^{2}
$$

74. Find the composition fog and gof and test whether fog = gof when $f$ and $g$ are functions or $R$ given by $f(x)=g(x)=\left(1-x^{3}\right)^{\frac{1}{3}}$

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75. Let $f$ be a real function. Show that $h(x)=f(x)=f(-x)$ is always an even function and $g(x)=f(x)-f(-x)$ is always an odd fuction.

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76. Express each of $1+x+x^{2}$ function as the sum of an even function and an odd function.
77. Express each of $x^{2}$ function as the sum of an even function and an odd function.

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78. Express each of $e^{x}$ function as the sum of an even function and an odd function.

## - Watch Video Solution

79. Express each of $e^{x}+\sin x$ function as the sum of an even function and an odd function.

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80. Let $X=\{1,2,3,4\}$ Determine whether $\mathrm{f}: X \rightarrow X$ defined as given below have inverses. Find $f^{-1}$ if it exist $f=\{(1,4),,(2,3),(3,2),(4,1)\}$

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81. Let $X=\{1,2,3,4\}$ Determine whether $\mathrm{f}: X \rightarrow X$ defined as given below have inverses. Find $f^{-1}$ if it exist
$f=\{(1,3)(2,1),(3,1),(4,2)\}$

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82. Let $X=\{1,2,3,4\}$ Determine whether $\mathrm{f}: X \rightarrow X$ defined as given below have inverses. Find $f^{-1}$ if it exist $f=\{(1,2),(2,3),(3,4),(4,1)\}$
83. Let $\mathrm{X}=\{1,2,3,4\}$ Determine whether $\mathrm{f}: X \rightarrow X$ defined as given below have inverses. Find $f^{-1}$ if it exist $f=\{(1,1),(2,2),(2,3),(4,4)\}$

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84. Let $\mathrm{X}=\{1,2,3,4\}$ Determine whether $\mathrm{f}: X \rightarrow X$ defined as given below have inverses. Find $f^{-1}$ if it exist $f=\{(1,2),(2,2),(3,2),(4,2)\}$

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85. Construct an example to show that
$f(A \cap B) \neq f(A) \cap f(B)$ where $A \cap B \neq \theta$.
86. Prove that for any $f: X \rightarrow Y$, foid $_{x}=f=i d_{Y}$ of.

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87. Prove that $f: X \rightarrow Y$ is surjective iff for all $B \subseteq Y, f\left(f^{-1}(B)\right)=B$.

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88. Prove that $f: X \rightarrow Y$ is injective iff
$f^{-1}(f(A))=\mathrm{A}$ for all $A \subseteq X$.

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89. Prove that $f: X \rightarrow Y$ is injective iff for all $\subset s A, B o f X, f(A \cap B)=f(A) \cap f(B)$.

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90. Prove that $\mathrm{f}: X \rightarrow Y$ is surjective iff for all
$A \subseteq X,(f(A))^{\prime} \subseteq f\left(A^{\prime}\right)$, where $\mathrm{A}^{\prime}$ denotes the complement of
$A$ in $X$.

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91. Show that the operation * given by $x^{*} y=x+y+-x y$ is a binary oeration on $\mathrm{Z}, \mathrm{Q}$ and R but not on N .

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92. Determine whether $a * b=2 a+3 b$ on $Z$ operations as defined by * are binary operations on the sets specified in each case. Give reasons if it is not a binary operation.

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93. 

Determine
whether
$a * b=m a-n b o n Q+$ where $m$ and $n \in N \quad$ operations as defined by * are binary operations on the sets specified in each case. Give reasons if it is not a binary operation.

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94. Determine whether $a * b=a+b(\bmod 7) \operatorname{on}\{0,1,2,3,4,5,6\}$ operations as defined by * are binary operations on the sets specified in each case. Give reasons if it is not a binary operation.
95. Determine whether $a * b=\min \{a, b\}$ on $N$ operations as defined by * are binary operations on the sets specified in each case. Give reasons if it is not a binary operation.

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96. Determine whether $a * b=\operatorname{GCD}\{a, b\}$ on $N$ operations as defined by * are binary operations on the sets specified in each case. Give reasons if it is not a binary operation.

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97. Determine whether $a * b=\operatorname{LCM}\{a, b\}$ on $N$ operations as defined by * are binary operations on the sets specified in each
case. Give reasons if it is not a binary operation.

## D Watch Video Solution

98. 

Determine
whether
$a * b=\operatorname{LCM}\{a, b\} \operatorname{on}\{0,1,2,3,4 \ldots, 10\}$ operations as
defined by * are binary operations on the sets specified in each
case. Give reasons if it is not a binary operation.

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99. Determine whether $a * b=\sqrt{a^{2}+b^{2}}$ on $Q_{+}$operations as defined by * are binary operations on the sets specified in each case. Give reasons if it is not a binary operation.

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100. Determine whether $a * b=a \times b(\bmod 5) \operatorname{on}\{0,1,2,3,4\}$ operations as defined by * are binary operations on the sets specified in each case. Give reasons if it is not a binary operation.

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101. Determine whether $a * b=a^{2}+b^{2}$ on $N$ operations as defined by * are binary operations on the sets specified in each case. Give reasons if it is not a binary operation.

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102. Determine whether $a * b=a+b-a b o n R-\{1\}$ operations as defined by * are binary operations on the sets specified in each case. Give reasons if it is not a binary operation.
103. Constract the composition table/multiplication table for the binary operation * defined on $\{0,1,2,3,4\}$ by
$a * b=a \times b(\bmod =5)$. Find the identity element if any. Also find the inverse elements of 2 and 4.
