MATHS

BOOKS - NEW JYOTHI MATHS (TAMIL ENGLISH)

PRINCIPLES OF MATHEMATICAL INDUCTION

Examples

1. Let P(n) be the statement:

"$n^3 + n$ is divisible by 3."

i. Prove that P(3) is true.

ii. Verify whether the statement is true for n = 4.

2. Prove by the principle of mathematical induction that $2 + 2^2 + 2^3 + \ldots\ldots + 2^n = 2(2^n - 1)$.

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3. Prove by the principle of mathematical induction that $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots\ldots\ldots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$.

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4. By the principle of mathematical induction, prove that, for $n \geq 1$

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

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5. Consider the statement

$P(n) : 9^n - 1$ is a multiple of 8, where n is a natural number

i. Is P(1) true?

ii. Assuming P(k) is true, show that P(k + 1) is also true.

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6. Consider the statement "$7^n - 3^n$ is divisible by 4"

i. Verify the result for n = 2.

ii. Prove the statement using mathematical induction.

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7. Prove that $1^2 + 2^2 + \ldots + n^2 > \frac{n^3}{3}, n \in N$

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8. Show that n(n + 1) (n + 2) is divisible by 6 where n is a natural number.

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9. Prove that $1^2 + 2^2 + 3^2 + \ldots . + n^2 = \frac{n(n+1)(2n+1)}{6}$

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10. Which among the following is the least number that will divide $7^{2n} - 4^{2n}$ for every positive integer n?

$[4, 7, 11, 33]$

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11. Prove by mathematical induction $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ where $i = \sqrt{-1}$.

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12.

$$1 + \frac{1}{(1+2)} + \frac{1}{(1+2+3)} + \ldots . + \frac{1}{(1+2+3+\ldots n)} = \frac{2n}{(n+1)}$$

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13.

$$1.2.3 + 2.3.4 + \dots. + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

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14. Prove that by using the principle of mathematical induction for all $n \in N$:

$$1.3 + 2.3^2 + 3.3.^3 + \dots. + n.3^n = \frac{(2n-1)3^{n+1} + 3}{4}$$

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15. By the principal of mathematic induction, prove that, for $n \geq 1$

$$1.2 + 2.3 + 3.4 + \dots + n.(n+1) = \frac{n(n+1)(n+2)}{3}$$

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16. Prove that by using the principle of mathematical induction for all $n \in N$:

$$1.3 + 3.5 + 5.7 + \ldots. + (2n-1)(2n+1) = \frac{n(4n^2+6n-1)}{3}$$

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17. Prove that by using the principle of mathematical induction for all $n \in N$:

$$1.2 + 2, 2^2 + 3.2^3 + \ldots. + n.2^n = (n-1)2^{n+1} + 2$$

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18. Using the mathematical induction, show that for any natural number n,

$$\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \ldots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}$$

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19. Using the mathematical induction, show that for any natural number n,

$$\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\ldots+\frac{1}{n.(n+1).(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$$

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20. $a+ar+ar^2+\ldots\ldots+ar^{n-1}=\frac{a(r^n-1)}{r-1}$

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21. Use the principle of mathematical induction to prove that for every natural number n.

$$\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\left(1+\frac{(2n+1)}{n^2}\right)=(n+1)^2$$

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22. $\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)......\left(1+\frac{1}{n}\right)=(n+1)$

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23. By the principle of mathematical induction, prove that, for $n \geq 1$

$$1^2+3^2+5^2+...+(2n-1)^2=\left(\frac{n(2n-1)(2n+1)}{3}\right)$$

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24. Prove that by using the principle of mathematical induction for all

$n \in N$:

$$\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+...+\frac{1}{(3n-2)(3n+1)}=\frac{n}{3n+1}$$

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25. Prove that by using the principle of mathematical induction for all

$n \in N$:

$$\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\ldots.+\frac{1}{(2n+1)(2n+3)}=\frac{n}{3(2n+3)}$$

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26. Prove that by using the principle of mathematical induction for all

$n \in N$:

$$1+2+3+\ldots..+n<\frac{1}{8}(2n+1)^2$$

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27. Prove that by using the principle of mathematical induction for all

$n \in N$:

$n(n+1)(n+5)$ is a multiple of 3

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28. Prove that by using the principle of mathematical induction for all

$n \in N$:

$10^{2n-1} + 1$ is divisible by 11

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29. Prove that by using the principle of mathematical induction for all

$n \in N$:

$x^{2n} - y^{2n}$ is divisible by x+y

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30. Prove that by using the principle of mathematical induction for all

$n \in N$:

$41^n - 14^n$ is multiple of 27

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31. Prove that by using the principle of mathematical induction for all

$n \in N$:

$(2n + 7) < (n + 3)^2$

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Exercise

1. If $n \in N, 10^{2n-1} + 1$ is divisible by

A. 11

B. 99

C. 101

D. 111

Answer: A

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2. If $n \in N, \text{then } 3^{2n+2} - 8n - 9$ is divisible by

A. 8

B. 16

C. 10

D. 64

Answer: A

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3. $41^n - 14^n, n \in N$ is divisible by

A. 27

B. 81

C. 17

D. 23^2

Answer: A

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4. The smallest natural number 'n' for which $n! < \left(\frac{n+1}{2}\right)^n$ holds is

A. 1

B. 2

C. 3

D. 4

Answer: B

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5. The inequality $n! > 2^{n-1}$ is true for

A. $n \in N$

B. $n > 1, n \in N$

C. $n > 2, n \in N$

D. for no $n \in N$

Answer: C

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6. $49^n + 16n - 1$ is divisible by

A. 3

B. 64

C. 19

D. 29

Answer: B

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7. If $n \in N$, then $2.4^{2n+1} + 3^{3n+1}$ is divisible by

A. 2

B. 9

C. 11

D. 27

Answer: C

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8. The greatest positive integer which divides $(n+1)(n+2)(n+3)\ldots(n+r)$ for all $n \in N$ is

A. r

B. r!

C. n + r

D. (r + 1)!

Answer: B

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9. The greatest positive integer which divides $n(n+1)(n+2)(n+3)$ for all $n \in N$ is

A. 2

B. 6

C. 24

D. 120

Answer: C

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10. $49^n + 16n - 1$ is divisible by

A. 3

B. 64

C. 11

D. 27

Answer: B

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11. Using Mathematical Induction, the number a_n's are defined by

$a_0 = 1, a_{n+1} = 3n^2 + n + a_n (n \leq 0) \text{ then } a_n =$

A. $n^3 + n^2 + 1$

B. $n^3 - n^2 + 1$

C. $n^3 - n^2$

D. $n^3 + n^2$

Answer: B

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12. For all $n \in N$ which of the following is divisible by 9?

A. $8^n + 1$

B. $4^n - 3n - 1$

C. $3^{2n} + 3n + 1$

D. $10^n + 1$

Answer: B

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13. If $n \in N$ then $n^3 + 2n$ is divisible by

A. 3

B. 2

C. 6

D. 4

Answer: A

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14. Let P(n) denote the statement that $n^2 + n$ is odd. It is seen that P(n) = P(n + 1). P(n) is true for all.

A. $n > 1$

B. n

C. $n > 2$

D. None of these

Answer: D

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15. The smallest natural number 'n' for which $n! < \left(\frac{n+1}{2}\right)^n$ holds is

A. 1

B. 2

C. 3

D. 4

Answer: B

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Questions From Competitive Exams

1. $10^n + 3(4^{n+2}) + 5$ is divisible by $(n \in N)$

A. 7

B. 5

C. 9

D. 17

Answer: C

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2. If $a_n = \sqrt{7 + \sqrt{7 + \sqrt{7 + \dots .}}}$ having n radical signs then by method of mathematical induction which is true

A. $a_n > 7, \forall n \geq 1$

B. $a_n > 3, \forall n \geq 1$

C. $a_n < 4, \forall n \geq 1$

D. $a_n < 3, \forall n \geq 1$

Answer: B

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3. Let $S(k) = 1 + 3 + 5 + \ldots + (2k - 1) = 3 + k^2$. Then which of the following is true?

A. $S(k) \Rightarrow S(k - 1)$

B. $S(k) \Rightarrow S(k + 1)$

C. S(1) is correct

D. Principle of mathematical induction can be used to prove the formula.

Answer: B

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4. Write the negation of the following statements:

r: For every real number x, either $x > 1$ or $x < 1$.

A. Statement - 1 is true, Statement - 2 is false

B. Statement - 1 is false, Statement - 2 is true

C. Statement - 1 is true, Statement - 2 is true, Statement - 2 is a correct explanation for Statement - 1

D. Statement - 1 is true, Statement - 2 is true, Statement - 2 is not a correct explanation for Statement - 1

Answer: D

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