MATHS

BOOKS - MODERN PUBLICATION

APPLICATION OF DERIVATIVES

Example

1. The side of an equilateral triangle is increasing at the rate of 2cm/s. At what rate is its area increasing when the side of the triangle is 20 cm?

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2. The radius of a spherical soap bubble is increasing at the rate of 0.3cm/s. Find the rate of change of its: Volume when the radius is 8 cm.

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3. The radius of a spherical soap bubble is increasing at the rate of 0.3 cm/s. Find the rate of change of its (ii) surface area, when the radius is 8 cm.

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4. The length 'x' of a rectangle is decreasing at the rate of 3cm/m and width 'y' is increasing at the rate of 2cm/m. When x = 10 cm and y = 6 cm, find the rate of change of : the perimeter

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5. The length 'x' of a rectangle is decreasing at the rate of 3cm/m and width 'y' is increasing at the rate of 2cm/m. When x = 10 cm and y = 6 cm, find the rate of change of : the area of the rectangle.

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6. The radius of a cylinder is increasing at the rate of 2m/s. and its altitude is decreasing at the rate of 3m/s. Find the rate of change of volume when radius is 3 metres and altitude is 5 metres.

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7. A particle moves along the curve $y = \left(\frac{2}{3}\right)x^3 + 1$. Find the points on the curve at which y-coordinate is changing twice as fast as the x-coordinate.

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8. A water tank has the shape of an inverted right circular cone, whose semi-vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rate of 5 cu m/min. Then, the rate (in m/min) at which the level of water is rising at the instant when the depth of water in the tank is 10 m. Water is a natural resource. What is the importance of water in our daily life?

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9. Water is leaking form a conical funnel at the rate of $5cm^3/s$, If the radius of the base of funnel is 5 cm and height 10 cm, find the rate at which the water level is dropping when it is 2.5 cm from the top. Is leaking of water leads to wastage of water? Should we do everything to ssave this natural resource?

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10. A stone is dropped into a quiet lake and waves move in circles at a speed of 4cm per second. At the instant, when the radius of the circular wave is 10 cm, how fast is the enclosed area increasing?

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11. A man 2m high walks at a uniform speed of 6k/h away from a lamp ost 6 metres high. Find the rate at which the length of his shadow increases.

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12. A man is walking at the rate of 6.5km/h towards the foot of a tower 120m high. At what rate is he approaching the top of tower when he is 50 m away from the tower ?

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13. A man is moving away from a tower 41.6 m high at the rate of 2 m/sec. Find the rate at which the angle of elevation of the top of tower is changing, when he is at a distance of 30m from the foot of the tower. Assume that the eye level of the man is 1.6m from the ground.

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14. Two equal sides of an isosceles triangle with fixed base 'a' are decreasing at the rate of 9 cm/second. How fast is the area of the riangle decreasing, when the two sides are equal to 'a' ?

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15. The amount of pollution content added in air in a city due to x-d i e s e l vehicles is given by$P(x) = 0.005x^3 + 0.02x^2 + 30x\cdot$ Find the arginal increase in pollution content when 3 diesel vehicles are added and write which value is indicated in the above question.

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16. The money to be spend for the welfare of the employees of a firm is proportional to the rate of change of its total revenue (Marginal revenue). If the total revenue (in rupees) received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$, find the marginal revenue, when x=5, and write which value does the question indicate.

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17. The contentment obtained after eating X-units of a new dish at a trial function is given by the function $f(x) = x^3 + 6x^2 + 5x + 3$. If the marginal contentment is defined as the rate of change f(X) with respect to the number of units consumed at an instant, then find the marginal contentment when three units of dish are consumed.

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18. Show that the function given by $f(x) = 7x - 3$, is increasing on R.

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19. The income I of a doctor is given by: $I = x^3 - 3x^2 + 5x$ Can an insurance agent ensure the growth of his income?

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20. Determine for which values of 'x' the function $f(x) = x^3 - 24x + 7$ is strictly increasing or strictly decreasing.

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21. Find the intervals in which the function f(x)= $2x^2 - 3x$ is strictly increasing.

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22. Find the intervals in which the function f(x)= $2x^2 - 3x$ is strictly decreasing.

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23. Find the intervals in which the function: $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is strictly decreasing.

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24. Find the intervals in which the function: $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is strictly decreasing.

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25. Find the intervals in which the function: $f(x) = \sin^4 x + \cos^4 x, 0 \leq x \leq \frac{\pi}{2}$ is strictly increasing or decreasing.

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26. Find the intervals in which : $f(x)\sin 3x - \cos 3x, 0 < x < \pi,$ is strictly increasing or strictly decreasign.

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27. Find the value of 'x' for which $f(x) = x^x$, x > 0 is strictly increasing or decreasing.

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28. If a,b,c are real numbers, then find the intervals in which $f(x) = \begin{vmatrix} x + a^2 & ab & ac \\ ab & x + b^2 & bc \\ ac & bc & x + C^2 \end{vmatrix}$ is striclty increasing or decreasing.

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29. prove that $\frac{x}{1 + x} < \log(1 + x) < x$, for all $x > 0$

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30. Find the slope of the tangent to the curve: $x = at^2, y = 2at. att = 2$

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31. Find the point at which the tangent to the curve $y = \sqrt{4x - 3 - 1}$ has its slope $\frac{2}{3}$.

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32. Find the equation of all lines having slope 2 and being tangent to the curve $y + \frac{2}{x - 3} = 0$

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33. Find the equation of the tangent to the curve $y = \frac{x - 7}{(x - 2)(x - 3)}\left(y = \frac{x - 7}{x^2 - 5x + 6}\right)$ at the point, where it cuts the x=axis.

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34. Find the equation of the normal to the curve $y = x^3$ at the point (2,1).

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35. Find the equations of the tangent and normal to the curve given by :

$x = a\sin^3\theta, y = a\cos^3\theta$ at a point, where $\theta = \frac{\pi}{4}$

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36. Find the equations of the tangent and the normal to the curve $x = 1 - \cos\theta, y = \theta - \sin\theta at\theta = \frac{\pi}{4}$

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37. Find the equation of the tangent to the curve $x^2 + 3y = 3$, which is parallel to the line y - 4x + 5 = 0

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38. Determine the points on the curve $x^2 + y^2 = 13$, where the tangents are perpendicular to the line 3x - 2y = 0

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39. Show that the equation of normal at any point 't' on the curve: $x = 3\cos t - \cos^3 t$ and $y = 3\sin t - \sin^3 tis: 4(y\cos^3 t - x\sin^3 t) = 3\sin$

.

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40. If $\frac{x}{a} + \frac{y}{b} = 2$ touches the curve $\frac{x^n}{a^n} + \frac{y^n}{b^n} = 2$ at the point (α, β), then

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41. Find the point on the curve $y = 2x^3 - 15x^2 + 36x - 21$ at which the tangent is parallel to x-axis. Also, find the equation of tangents.

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42. Show that the curve $xy = a^2$ and $x^2 + y^2 = 2a^2$ touch each other.

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43. Find the angle of intersection of the following curves: $xy = 6$ and $x^2y = 12$.

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44. If the curves $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ intersect orthogonally, prove that: $\frac{1}{a} - \frac{1}{a'} = \frac{1}{b} - \frac{1}{b'}$

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45. Find the values of 'x' for which $f(x) = [x(x-2)]^2$ is an increasing function. Also find the points on the curtve, where the tangent is parallel to x-axis.

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46. Find the maximum and the minimum values, if any, of the function f given by $f(x) = x^2, x \in R$

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47. Find the maximum and minimum values of f , if any, of the function given by $f(x) = |x|, x \in R$

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48. Find the maximum and minimum values, if any, of the following functions without using derivates:

$f(x) = (2x - 1)^2 + 3$

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49. Find the maximum and minimum values, if any, of the following functions without using derivates:

$f(x) = 16x^2 - 16x + 28$

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50. Find the maximum and minimum values, if any, of the following functions without using derivates:

$f(x) = |x + 1| + 3$

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51. Find the maximum and minimum values, if any, of the following functions given by: $h(x) = \sin(2x) + 5$

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52. Find the maximum and minimum values, if any, of the following functions without using derivates:

f(x) = sin(sinx)

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53. Find the points of absolute maximum and minimum of : $y = (x-1)^{1/2}(x-2), 1 \leq x \leq 9$

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54. Determine the absolute maximum and absolute minimum values of each of the following in the stated domains:

$y = \frac{1}{2}x^2 + 5x + \frac{3}{2}, \ -6 \leq x \leq \ -2$

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55. Determine the absolute maximum and absolute minimum values of each of the following in the stated domains: $f(x) = (x+1)^{2/3}, 0 \leq x \leq 8$

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56. Calculate the absolute maximum and absolute minimum value of the function $f(x) = \frac{x+1}{\sqrt{x^2+1}} 0 \leq x \leq 2$

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57. Find the absolute maximum and the absolute minimum value of the function given by: $f(x) = \sin^2 x - \cos x, x \in [0, \pi]$

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58. Find the points of local maxima and ocal minima, if any, of the function: $f(x) = (x-1)(x+2)^2$ Find also the local maximum and local minimum values.

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59. Find the points of local maxima and local minima, if any, of the following function: $f(x) = \sin x + \frac{1}{2}\cos 2x, 0 < x, \frac{\pi}{2}$

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60. Find all the points of local maxima and local minima of the function f given by $f(x) = 2x^3 - 6x^2 + 6x + 5$

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61. Find two positive numbers whose sum is 24 and their sum of squares is minimum.

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62. Show that of all rectangles with given perimeter square has maximum area

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63. Two sides of a triangle are a and b. Find the angle between them such that area shall be maximum

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64. Show that rectangle of maximum perimeter, which can be inscribed in a circle of a radius 'a' is a square of side $\sqrt{2}a$.

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65. Show that a triangle of maximum area that can be inscribed in a circle of radius a is an equilateral triangle.

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66. An open box with a square base is to be made out of a given iron sheet of area 27sq.m. Show that the maximum volume of the box is 13.5 cu. Cm.

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67. The given quantity of metal is to be cost into a half cylinder with a rectangular base and semicircular ends. Show that in order that the total surface area may be minimum, the ratio of the length of the cylinder to the diameter of its semi-circular ends is $\pi : (\pi + 2)$.

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68. A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, find the dimensions of the rectangle that will produce the largest area of the window.

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69. Show that the height of the circular cylinder of maximum volume that can be inscribed in a given right-circular cone of height h is $\frac{1}{3}$h

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70. Let AP and BQ be two vertical poles at points A and B respectively. If AP=16 m , BQ=22 m and $AB = 20m, thenf \in dthedis \tan aceofap \oint$R on AB from the point A such that $RP^2 + RQ^2$is minimum.

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71. If the length of three sides of a trapezium other than base are equal to 10 cm, then find the area of trapezium when it is maximum.

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72. A square piece of tin of side 24 cm is to be made into a box without top by cutting a square from each comer and folding up the flaps to form a box. What should be the side of square to be cut off so that the volume of box is maximum also find the volume ?

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73. Show that the height of the cylinder, open at the top of given surface area and greatest volume is equal to the radius of its base.

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74. Find the point on the curve $y^2 = 4x$, which is nearest to the point (2, 1).

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75. An Apache helicopter of enemy is flying along the curve given by $y = x^2 + 7$. A soldier, placed at (3, 7), wants to shoot down the helicopter when it is nearest to him. Find the nearest distance.

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76. The cost of fuel for running a bus is proportional to the square of the speed generated in km/h. It cost Rs 48 per hour when the bus is moving with a speed of 20 km/h. What is the most economical speed if the fixed charges are Rs 108 for one hour, over and above the running charges?

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77. Manufacturer can sell x items at a price of rupees $Rs\left(5-\left(\frac{x}{100}\right)\right)$ each. The cost price of x items is $Rs\left(\left(\frac{x}{5}\right)+500\right)$. Find the number of items he should sell to earn maximum profit.

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78. Find the rate of change of the area of a circle with respect to its radius r when : r = 3 cm

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79. Find the rate of change of the area of a circle with respect to its radius r when : r = 8 cm

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80. The volume of a cube is increasing at the rate of $8c\frac{m^3}{s}$. How fast is the surface area increasing when the length of an edge is 12 cm?

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81. The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

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82. An edge of a variable cube is increasing at the rate of 3 cm/s. How fast is the volume of the cube increasing when the edge is 10 cm long?

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83. A stone is dropped into a quiet lake and waves move in circles at the speed of 5 cm/s. At the instant when the radius of the circular wave is 8 cm, how fast is the enclosed area increasing?

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84. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference ?

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85. The length 'x' of a rectangle is decreasing at the rate of 5 cm per minute and the width 'y' is increasing at the rate of 4 cm per minute, when x = 8 cm and y = 6 cm, find the rate of change of the perimeter of the rectangle.

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86. The length 'x' of a rectangle is decreasing at the rate of 5 cm per minute and the width 'y' is increasing at the rate of 4 cm per minute, when x = 8 cm and y = 6 cm, find the rate of change of the area of the rectangle.

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87. A balloon, which always remains spherical on inflation, is being inflated by pumping in $900cm^3$ Of gas per sec. Find the rate at which the radius of the balloon increases when the radius is 15 cm.

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88. A balloon which always remain spherical has a variable radius. Find the rate at which its volume is increasing with the radius when the later is 10 cm.

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89. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ?

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90. The radius of an air bubble is increasing at the rate of $\frac{1}{2}c\frac{m}{s}$. At what rate is the volume of the bubble increasing when the radius is 1 cm?

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91. A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$. Find the rate of change of its volume with respect to x.

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92. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

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93. Sand is pouring from a pipe at the rate of 12 cubic cm./sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. At which rate is the height of the sand-cone increasing when the height is 4 cm. ?

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94. The total cost $C(x)$ in Rupees associated with the production of x units of an item is given by $C(x) = 0.007x^3 + 0.003x^2 + 15x + 4000$ Find the marginal cost when 17 units are produced.

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95. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 13x^2 + 26x + 15$ Find the marginal revenue when x = 7.

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96. Find the rate of change of the area of a circle with respect to its radius r at r = 6 cm

A. 10π

B. 12π

C. 8π

D. 11π

Answer:

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97. The total revenue received from the sale of 'x' units of a product is given by $R(x) = 3x^2 + 36x + 5$. Find the marginal revenue when x = 5.

A. 116

B. 96

C. 90

D. 126

Answer:

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98. Show that the function given by $f(x) = 3x + 17$ is increasing on R.

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99. Show that the function given by $f(x) = e^2x$ is increasing on R.

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100. Find the intervals in which the function f given by $f(x) = 2x^3 - 3x + 5$ is strictly increasing

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101. Find the intervals in which the function f given by $f(x) = 2x^2 - 3x$ is decreasing.

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102. Find the intervals in which the function : $f(x) = 2x^3 - 3x^2 - 36x + 7$ is Strictly increasing

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103. Find the intervals in which the function : $f(x) = 2x^3 - 3x^2 - 36x + 7$ is Strictly decreasing

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104. Find the intervals in which the following functions are strictly increasing or decreasing: $x^2 + 2x - 5$

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105. Find the intervals in which the following functions are strictly increasing or decreasing: $10 - 6x - 2x^2$

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106. Find the intervals in which the following functions are strictly increasing or decreasing: $-2x^3 - 9x^2 - 12x + 1$

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107. Determine the intervals in which the following functions are strictly increasing or striclty decreasing.

$f(x) = 6 - 9x - x^2$

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108. Determine the intervals in which the following functions are strictly increasing or stricIty decreasing.

$f(x) = (x+1)^3(x-3)^3$

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109. Show that $y = \log(1+x) - 2\frac{x}{2+x}, x \succ 1,$ is an increasing function of x. throughout its domain.

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110. Find the values of x for which $y = [x(x-2)]^2$ is an increasing function.

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111. Prove that $y = 4\frac{\sin\theta}{2+\cos\theta} - \theta$, is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$.

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112. Prove that the logarithmic function is increasing on $(0, \infty)$.

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113. Prove that the function f given by $f(x) = x^2 - x + 1$ is neither strictly increasing nor decreasing on $(-1, 1)$.

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114. Which of the following functions are strictly decreasing on $\left(0, \frac{\pi}{2}\right)$?

A. cos x

B. cos 2 x

C. cos 3x

D. tan x

Answer:

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115. On which of the following intervals is the function f given by $f(x) = x^{100} + \sin x - 1$decreasing ?

A. (0,1)

B. $\left(\frac{\pi}{2}, \pi\right)$

C. $\left(0, \frac{\pi}{2}\right)$

D. None of these

Answer:

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116. For what values of a the function f given by $f(x) = x^2 + ax + 1$ is increasing on $[1, 2]$?

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117. Let I be any interval disjoint from $[-1, 1]$ Prove that the function f given by $f(x) = x + \frac{1}{x}$ is increasing on I.

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118. Prove that the function f given by f(x) = log sin x is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ and strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$.

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119. Prove that $f(x) = \log\cos x$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$ and strictly increasing on $\left(\frac{\pi}{2}, \pi\right)$

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120. Prove that the function given by $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing in R.

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121. The interval in which $y = x^2 e^{-x}$ is increasing is:

A. $(-\infty, \infty)$

B. (-2,0)

C. $(2, \infty)$

D. (0,2)

Answer:

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122. Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at x = 4 .

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123. Find the slope of the tangent to the curve $y = \frac{x-1}{x-2}$ at x = 10.

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124. Find the slope of the tangent to curve $y = x^3 - x + 1$ at the point whose x-coordinate is 2.

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125. find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose x-coordinate is 2.

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126. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose x-coordinate is 3.

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127. Find the slope of the normal to the curve $x = a\cos^3\theta, y = a\sin^3\theta at\theta = \frac{\pi}{4}$

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128. Find the slope of the normal to the curve $x = 1 - a\sin\theta, y = b\cos^2\theta at\theta = \frac{\pi}{2}$

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129. Find points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to the x-axis.

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130. Find a point on the curve $y = (x - 2)^2$ at which the tangent is parallel to the chord joining the points (2,0) and (4,4)

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131. Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$

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132. Find the equation of all lines having slope – 1 that are tangents to the curve $y = \frac{1}{x - 1}, x \neq -1$

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133. Find the equation of all lines having slope 2 which are tangent to the curve $y = \frac{1}{x-3}, x \neq 3.$

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134. Find the equations of all lines having slope 0 which are tangent to the curve $y = \frac{1}{x2 - 2x + 3}$

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135. Find points on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are parallel to x-axis

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136. Find points on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are parallel to y-axis

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137. Find the equations of the tangent and normal to the given curves at the indicated points: $y = x^4 - 6x^3 + 13x^2 - 10x + 5at(0, 5)$

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138. Find the equations of the tangent and normal to the given curves at the indicated points: $y = x^4 - 6x^3 + 13x^2 - 10x + 5at(1, 3)$

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139. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$, which is parallel to the line $2x - y + 9 = 0$.

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140. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is perpendicular to the line $5y - 15x = 13$

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141. Show that the tangents to the curve y= $7x^3 + 11$ at the points x = 2 and x =-2 are parallel.

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142. Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to the y-coordinate of the point.

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143. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

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144. Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangents are parallel to the x-axis.

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145. Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$

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146. Find the equation of the normals to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$

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147. Find the equations of the tangent and normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$

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148. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1.$

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149. Find the equations of the tangent and normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_0, y_0)

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150. Find the equation of the tangent to the curve $y = \sqrt{3x - 2}$ which is parallel to the line $4x - 2y + 5 = 0$.

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151. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is:

A. 3

B. $\frac{1}{3}$

C. -3

D. $-\frac{1}{3}$

Answer:

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152. The line $y = x + 1$, is a tangent to the curve $y^2 = 4x$ at the point.

A. (1,2)

B. (2,1)

C. (1,-2)

D. (-1,2)

Answer:

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153. Using differentials, find the approximate value of each of the following up to 3 places of decimal: $\sqrt{25.3}$

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154. Using differentials, find the approximate value of each of the following up to 3 places of decimal: $\sqrt{49.5}$

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155. Using differentials, find the approximate value of each of the following up to 3 places of decimal: $\sqrt{0.6}$

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156. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(0.009)^{1/3}$

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157. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(0.999)^{1/10}$

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158. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(15)^{1/4}$

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159. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(26)^{1/3}$

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160. Using differentials, find the approximate value of each of the following up to 3 places of decimal: $\frac{(255)^1}{4}$

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161. Using differentials, find the approximate value of each of the following up to 3 places of decimal: $\frac{(82)^1}{4}$

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162. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(401)^{1/2}$

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163. Using differentials, find the approximate value of each of the following up to 3 places of decimal: $(0.0037)^{\frac{1}{2}}$

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164. Using differentials, find the approximate value of each of the following up to 3 places of decimal: $(26.57)^{\frac{1}{3}}$

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165. Using differentials, find the approximate value of each of the following up to 3 places of decimal: $(81.5)^{\frac{1}{4}}$

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166. Using differentials, find the approximate value of each of the following up to 3 places of decimal: `(3.968)^ 3/2

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167. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(32.15)^{1/5}$

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168. Find the approximate value of f(2.01) where $f(x) = 4x^2 + 5x + 2$.

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169. Find the approximate value of $f(5 \cdot 001)$, where $f(x) = x^3 - 7x^2 + 15$.

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170. Find the approximate change in the volume V of a cube of side x metres caused by increasing the side by 1%.

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171. Find the approximate change in the surface area of a cube of side x metres caused by decreasing the side by 1% .

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172. If the radius of a sphere is measured as 7 m with an error of 0.02 m, then find the approximate error in calculating its volume.

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173. If the radius of a sphere is measured as 7 m with an error of 0.02 m, then find the approximate error in calculating its volume.

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174. If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating its surface area.

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175. If $f(x) = 3x^2 + 15x + 5$, then the approximate value of f (3.02) is :

A. 47.66

B. 57.66

C. 67.66

D. 77.66

Answer:

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176. The approximate change in the volume of a cube of side x metres caused by increasing the side by 3% is:

A. $0.06x^3m^3$

B. $0.6x^3m^3$

C. $0.09x^3m^3$

D. $0.9x^3m^3$

Answer:

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177. Find the maximum and minimum values, if any, of the following functions given by: $f(x) = (2x - 1)^2 + 3$

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178. Find the maximum and minimum values, if any, of the following functions given by: $g(x) = -|x + 1| + 3$

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179. Find the maximum and minimum values, if any, of the following functions given by: $h(x) = \sin(2x) + 5$

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180. Find the maximum and minimum values, if any, of the following functions given by: $f(x) = |\sin(4x) + 3|$

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181. Find the maximum and minimum values, if any, of the following functions given by: $h(x) = x + 1, x \in (-1, 1)$

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182. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximm and local minimum

values:

$f(x) = x^2$

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183. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximm and local minimum values:

$f(x) = x^3 - 3x$

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184. Find the local maxima and local minima of the following functions. Find also the local maximum and the local minimum values, as the case may be: `h(x) = sinx - cosx','0

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185. Find the local maxima and local minima of the following functions. Find also the local maximum and the local minimum values, as the case may be: `h(x) = sinx - cosx','0

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186. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be: $f(x) = x^3 - 6x^2 + 9x + 15$

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187. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be: $f(x) = \frac{x}{2} + \frac{2}{x}, x > 0$

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188. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be: $f(x) = \frac{1}{x^2 + 2}$

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189. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be: $f(x) = x\sqrt{1 - x}$

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190. Prove that the following functions do not have maxima or minima: $f(x) = e^x$

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191. Prove that the following functions do not have maxima or minima: $g(x) = \log x$

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192. Prove that the following functions do not have maxima or minima: $h(x) = x^3 + x^2 + x + 1$

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193. Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals: $f(x) = x^3, x \in [-2, 2]$

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194. Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals:

$f(x) = \sin x + \cos x, x \in [0, \pi]$

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195. Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals: $f(x) = 4x - \frac{1}{2}x^2, x \in \left[-2, \frac{9}{2}\right]$

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196. Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals: $f(x) = (x-1)^2 + 3, x \in [-3, 1]$

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197. Find the maximum profit that a company can make, if the profit function is given by $p(x) = 41 - 72x - 18x^2$

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198. Find both the maximum value and the minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval $[0, 3]$

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199. The value of x for which function sin 2x attains its maximum is :

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200. What is the maximum value of the function $\sin x + \cos x$?

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201. Find the maximum value of $2x^3 - 24x + 107$ in the interval $[1, 3]$. Find the maximum value of the same function in $[-3, -1]$.

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202. It is given that at $x = 1$, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value, on the interval $[0, 2]$. Find the value of a.

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203. Find the maximum and minimum values of $x + \sin 2x$ on $[0, 2\pi]$

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204. . Find two numbers whose sum is 24 and whose product is as large as possible.

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205. Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

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206. Find two positive numbers x and y such that their sum is 35 and the product x^2y^5 is a maximum.

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207. Find two positive numbers whose sum is 16 and whose sum of cubes is minimum.

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208. A square piece of tin of side 18 cm is to be made into a box without top by cutting a square from each comer and folding up the flaps to form a box. What should be the side of square to be cut off so that the volume of box is maximum and also find the volume of box ?

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209. A rectangular sheet of tin 45 cm x 24 cm is to be made into a box without top, by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is the maximum possible.

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210. Show that of all rectangles inscribed in a given circle the square has maximum area.

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211. Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

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212. Of all the closed cylindrical cans (right circular), of a given volume of $100cm^3$, find the dimensions of the can which has the minimum surface area?

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213. A wire of length 28 m is to be cut into two pieces, one of the pieces is to be made into a square and the other into a circle. What should be the length ·of two pieces so that the combined area of the square and the circle is minimum ?

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214. Prove that volume of largest cone, which can be inscribed in a sphere, is $\left(\frac{8}{27}\right)^{th}$ part of volume of sphere.

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215. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ time the radius of the base.

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216. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

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217. Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

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218. The point on the curve $x^2 = 2y$ which is nearest to the point $(0, 5)$ is:

A. $(2\sqrt{2}, 4)$

B. $(2\sqrt{2}, 0)$

C. (0,0)

D. (2,2)

Answer:

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219. For all real values of x, the minimum value of $\frac{1 - x + x^2}{1 + x + x^2}$ is:

A. 0

B. 1

C. 3

D. $\frac{1}{3}$

Answer:

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220. The maximum value of $[x(x-1)+1]^{\frac{1}{3}}, 0 \leq x \leq 1$ is:

A. $\left(\frac{1}{3}\right)^{1/3}$

B. $\frac{1}{2}$

C. 1

D. 0

Answer:

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221. Using differentials, find the approximate value of the following: $\left(\frac{17}{81}\right)^{\frac{1}{4}}$

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222. Using differentials, find the approximate value of the following: $33^{-\frac{1}{5}}$

Watch Video Solution

223. Show that the function given by $f(x) = \frac{\log x}{x}$ has maximum at $x = e$

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224. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base ?

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225. Find the equation of the normal to curve $y^2 = 4x$ at the point (1,2)

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226. Show that the normal at any point θ to the curve $x = a\cos\theta + a\theta\sin\theta, y = a\sin\theta - a\theta\cos\theta$ is at a constant distance from the origin.

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227. Find the intervals in which the function f given by $f(x) = \dfrac{4\sin x - 2x - x\cos x}{2 + \cos x}$ is increasing.

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228. Find the intervals in which the function f given by $f(x) = \frac{4\sin x - 2x - x\cos x}{2 + \cos x}$ is increasing.

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229. Find the intervals in which the function f given by $f(x) = x^3 + \left(\frac{1}{x^3}\right), x \neq 0$ is increasing.

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230. Find the intervals in which the function f given by $f(x) = x^3 + \left(\frac{1}{x^3}\right), x \neq 0$ is decreasing.

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231. Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex at one end of the major axis.

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232. A tank with rectangular base and rectangular sides open at the top is to be constructed so that its depth is 3 m and volume is 75 m3. If building of tank costs Rs. 100 per square metre for the base and Rs. 50 per square metres for the sides, find the cost of least expensive tank.

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233. The sum of the perimeter of a circle and square is k, where k is some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle.

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234. A window is in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

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235. A point on the hypotenuse of a triangle is at distance a and b from the sides of the triangle.Show that the minimum length of the hypotenuse is $\left(a^{\frac{2}{3}}+b^{\frac{2}{3}}\right)^{\frac{2}{3}}$

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236. Find the points at which the function f given by $f(x)=(x-2)^4(x+1)^3$ has local maxima.

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237. Find the points at which the function f given by $f(x)=(x-2)^4(x+1)^3$ has local minima.

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238. Find the points at which the function f given by $f(x) = (x-2)^4(x+1)^3$ has point of inflexion.

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239. Find the absolute maximum and minimum values of the function f given by $f(x) = \cos^2 x + \sin x, x \in [0, \pi]$

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240. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $4\frac{r}{3}$.

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241. Let f be a function defined on $[a, b]$ such that $f'(x) > 0$ for all $x \in (a, b)$. Then prove that f is an increasing function on (a, b).

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242. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $2\frac{R}{\sqrt{3}}$. Also find the maximum volume.

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243. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle α is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 \tan^2 \alpha$

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244. A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic metre per hour. Then the depth of the wheat is increasing at the rate of:

A. $1m^3/h$

B. $0.1m^3/h$

C. $1.1m^3/h$

D. $0.5m^3/h$

Answer:

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245. The slope of the tangent to the curve $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$at the point (2,-1) is:

A. $\frac{22}{7}$

B. $\frac{6}{7}$

C. $\frac{7}{6}$

D. $-\frac{6}{7}$

Answer:

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246. The line $y = mx + 1$, is a tangent to the curve $y^2 = 4x$ if the value of m is:

A. 1

B. 2

C. 3

D. $\frac{1}{2}$

Answer:

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247. The normal at the point (1,1) on the curve $2y + x^2 = 3$ is:

A. x+y=0

B. x-y=0

C. x+y+1=0

D. x-y=0

Answer:

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248. The normal to the curve $x^2 = 4y$ passing (1,2) is:

A. x+y=3

B. x-y=3

C. x+y=1

D. x-y=1

Answer:

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249. The points on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts with the axes are:

A. $\left(4, \frac{+8}{3}\right)$

B. $\left(4, \frac{-8}{3}\right)$

C. $\left(4 \pm \frac{3}{8}\right)$

D. $\left(\pm 4, \frac{3}{8}\right)$

Answer:

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250. For the curve $y = 5x - 2x^3$, if x increases at te rate of 2units/s, then how fast is the slope of curve changing when x =3?

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251. Prove that the function f(x) = tanx-4x is strictly decreasing on $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$

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252. Show that the function: $f(x) = 4x^3 - 18x^2 + 27x - 7$ has neither maximum nor minimum.

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253. Find the condition for the curves: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, xy = c^2$ to intersect orthogonally.

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254. Find the difference between the greatest and least values of the function f(x) = sin2x - x, on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

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255. Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with its vertex at one end of the major axis.

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256. An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius 'a' . Show that the area of triangle is maximum when $\theta=\frac{\pi}{6}$

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257. A spherical ball of salt is dissovlging water in such a manner that the rate of decrease of the volume at any instant is proportional to the surface. Prove that the radis is decreasing at a constant rate.

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258. A kite is movgin horizontally at the height of 151.5 metres. If the speed of the kite is 10m/s, how fast is the string being let out, when the kite is 250 m away from the key who is flying the kite ? The height of the boy is 1.5 m.

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259. The volume of a cube is increasing at a constant rate. Prove that the increase in its surface area varies inversely as the length of the side.

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260. If x and y are the sides of two squares such that $y = x - x^2$, find the rate of the change of the area of the second square with respect to the first square.

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261. Show that for a ≥ 1 $f(x)=\sqrt{3}\sin x-\cos x-2ax+b$ is decreasing in R.

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262. Show that the function f given by $f(x)=\tan^{-1}(\sin x+\cos x), x>0$ is always an increasing function in $f, \left(0, \frac{\pi}{4}\right)$

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263. Find for which values of 'x', the functions: $y=x^4-\frac{4x^3}{3}$ is increasing and for which values, it is decreasing.

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264. Show that $f(x) = 2x + \cot^{-1} x + \log\left(\sqrt{1+x^2} - x\right)$ is increasing in R.

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265. Find the angle of intersection of the curves $y = x^2$ and $y = 4 - x^2$.

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266. Prove that the curves xy = 4 and $x^2 + y^2$ =8 touch each other.

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267. The slope of the tangent to the curve $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$at the point (2,-1) is:

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268. Using differentials, find the approximate value of $\sqrt{0.082}$

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269. Find the approximate value of $(1.999)^5$

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270. Find the approximate volume of metal in a hollow spherical shel, whose internal and external radii are 3 cm and 3.0005 cm respectively.

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271. At what point, the slope of the crve : $y = -x^3 + 3x^2 + 9x - 27$ is maximum? Also, find the maximum slope.

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272. A car starts from a point P at time $t=0$ seconds and stops at point Q. The distance x, in metres, covered by it, in t seconds is given by $x=t^2\left(2-\left(\frac{t}{3}\right)\right)$ Find the time taken by it to reach Q and also find distance between P and Q.

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273. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of $5\frac{m^3}{h}$. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is $4m$.

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274. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base ?

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275. The bottom of a rectangular swimming tank is 25 m by 40m. Water is pumped into the tank at the rate of 500 cubic metres per minute. Find the rate at which the level of water in the tank is rising.

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276. A ladder 13m long leans against a wall. The foot of the ladder is pulled along the ground away from the wall, at the rate of 1.5m/sec. How fast is the angle theta between the ladder and the ground is changing when the foot of the ladder is 12m away from the wall.

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277. The radius of a cylinder is increasing at the rate of 2 cm/sec and the height is decreasing at the rate of 3 cm/sec. The rate of change of volume when the radius is 3 cm and height is 5 cm.

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278. A kite is 120 m high and 130 m of string is out. If the kite is moving away horizontally at the rate of 52m/s, find the rate at which the string is being paid out.

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279. Show that the function f given by $f(x) = \tan^{-1}(\sin x + \cos x), x > 0$ is always an increasing function in $f, \left(0, \frac{\pi}{4}\right)$

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280. Find the equation of tangents to the curve $y = \cos(x + y), \ -2\pi \leq x \leq 2\pi$ that are parallel to the line $x + 2y = 0$.

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281. The common tangent to the parabola $y^2 = 4ax$ and $x^2 = 4ay$ is

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282. Tangents are drawn from the origin to the curve y = sin x . Prove that their points of contact lie on the curve $x^2y^2 = (x^2 - y^2)$.

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283. Show that $\frac{x}{a} + \frac{y}{b} = 1$ touches the curve $y = be^{-x/a}$ at the point, where the curve crosses the y-axis.

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284. Show that the line $x\cos\alpha + y\sin\alpha + p$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $a^2\cos^2\alpha = b^2\sin^2\alpha = p^2$ and the point of contact is

$\left(a^2 \frac{\cos \alpha}{p}\right), \left(b^2 \frac{\sin \alpha}{p}\right).$

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285. If the straight line $x \cos \alpha + y \sin \alpha = p$ touches the curve $x^m y^n = a^{m+n}$, prove that $p^{m+n}. m^m. n^n = (m+n)^{m+n} a^{m+n} \cos^m \alpha \sin^n \alpha$

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286. Find the points on the curve $y = 3x^2 - 9x + 8$ at which the tangents are equally inclined to the axes.

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287. The equation of the tangents at(2,3) on the curve $y^2 = ax^3 + b$ is y = 4x-5. Find the values of 'a' and 'b'

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288. A circular disc of radius $3cm$ is being heated. Due to expansion, its radius increases at the rate of $0.05c\frac{m}{s}$. Find the rate at which its area is increasing when radius is $3.2cm$.

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289. The radius of a sphere shrinks from 10 to 9.8 cm. Find approximately the decrease in its volume.

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290. If the error committed in measuring the radius of a circle is 0.01%, find the corresponding error in calculating the area.

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291. The area S of a triangle is calculated by measuring b,c and A. If the be an error ΔA in the measurement of A, show that the relative error in area is given by $\frac{\Delta S}{s} = \cot A . \Delta A$

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292. The presure 'p' and volume 'v' of a gas are connected by the relation $pv^{\frac{1}{4}} =$ constant. Find the percentage error in 'p' corresponding to decrease of $\frac{1}{2}\%$ in V.

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293. Show that the function given by $f(x) = \frac{\log x}{x}$ has maximum at $x = e$

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294. The sum of the perimeter of a circle and square is k, where k is some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle.

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295. Find the points at which the function f given by $f(x) = (x-2)^4(x+1)^3$ has local maxima.

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296. Find the points at which the function f given by $f(x) = (x-2)^4(x+1)^3$ has local minima.

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297. Find the points at which the function f given by $f(x) = (x-2)^4(x+1)^3$ has point of inflexion.

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298. Show that $\sin^p\theta\cos^q\theta$attains a maximum when $\theta = \tan^{-1}\sqrt{\left(\frac{p}{q}\right)}$

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299. Which fraction exceeds its pth power by the greatest possible number?

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300. If the sum of the lengths of hypotenuse and a side of a right-angled triangle is given. Show that the area is maximum, when the angle between then is 60°

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301. Divide 4 into two positive numbers such that the sum of the square of one and cube of the other is a minimum.

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302. A cylinderical can is to be made to hold 1 litre of oil. Find the dimensions which will minimize the cost of the metal to make the can.

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303. Find the shortest distance of the point $(0, c)$ from the parabola $y = x^2$ where $0 \leq c \leq 5$

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304. A beam of length'l' is suported at one end. If W is the uniform load per unit length, the bending moment M at a distance 'x' from the end is given by $M = \frac{1}{2}lx - \frac{1}{2}Wx^2$ Find the point on the beam at which the bending moment has maximum value.

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305. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $4\frac{r}{3}$.

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306. Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex at one end of the major axis.

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307. Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

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308. A windows of fixed perimeter (including the base of the arc) is in the form of a rectangle surmounted by a semi- circle. The semi- circular portion is fitted with coloured glass while the rectangular portion is fitted with clear glass.

The clear glass transmits three times as much light per square metre as the coloured glass does. what is the ratio of the sides of the rectangle so that the window transmits the maximum light?

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309. The radius of a spherical soap bubble is increasing at the rate of 0.2 cm/s. Find the rate of change of its (ii) surface area, when the radius is 4 cm.

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310. Is the function $f(x) = x^2, \xi n R$ increasing?

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311. The function $f(x) = x^2 - 6x + 9$ is increasing for x > 3

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312. Find the slope of the tangent to the curve $y = 3x^2 - 4x$ at the point, whose x-co-ordinats is 2.

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313. Find the equation of the tangent to the curve $y = 3x^2 at(1, 1)$

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314. The function $f(x) = x^2, x \in R$ has no maximum value.

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315. Find the absolute minimum value of $y = x^2 - 3x \in 0 \leq x \leq 2.$

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316. What are the maximum and minimum values, if any, of $f(x) = x, x \in (0, 1)$?

Watch Video Solution

317. Prove that x^x has minimum value at x =$\frac{1}{e}$

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318. Find two positive numbers whose product is 49 and their sum is minimum.

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Exercise

1. Find the rate of change of the area of a circle with respect to its radius 'r' when r = 2 cm.

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2. Find the rate of change of the area of a circle with respect to its radius 'r' when r = 2 cm.

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3. An edge of a variable cube is increasing at the rate of 3 cm/s. How fast is the volume of the cube increasing when the edge is 10 cm long?

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4. The radius of a soap-bubble is increasing at the rate of 0.2cm/s. Find the rate of increase of its volume when the radius is 5 cm.

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5. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference ?

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6. The radius of a circle is increasing uniformly at the rate of 4 cm per second. Find the rate at which the area of the circle is increasing when the radius is 8 cm.

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7. If the area of a circle increases uniformly, then show that the rate of increment of its circumference is inversely proportional to its radius.

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8. The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

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9. The radius of an air bubble is increasing at the rate of $\frac{1}{2}c\frac{m}{s}$. At what rate is the volume of the bubble increasing when the radius is 1 cm?

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10. The radius of spherical balloon is increasing at the rate 5 cm per second. At what rate is the surface of the balloon increasing, when the radius is 10 cm?

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11. The radius of a spherical soap bubble is increasing at the rate of 0.3cm/s. Find the rate of change of its: Volume when the radius is 8 cm.

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12. A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$. Find the rate of change of its volume with respect to x.

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13. A balloon, which always remains spherical on inflation, is being inflated by pumping in $900cm^3$Of gas per sec. Find the rate at which the radius of the balloon increases when the radius is 15 cm.

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14. The volume of a cube is increasing at the rate of 9 cm^3/sec. How fast is surface area increasing when the length of an edge is 10 cm ?

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15. The volume of a cube is increasing at the rate of $8c\frac{m^3}{s}$. How fast is the surface area increasing when the length of an edge is 12 cm?

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16. The volume of a cube is increasing at the rate of 7 cubic metre per second. How fast is the surface area increasing when the length of an edge is 24 cm?

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17. A particle moves along the curve $y = \frac{4}{3}x^3 + 5$. Find the points on the curve at which the y-coordinate changes as fast as the x-coordinate.

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18. A particle move along the curve $6y = x^3 + 2$. Find the points on the crve at which the y-coordinate is changing 8 times as fast as x-coordinate.

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19. The radius of a cylinder increases at the rate of 1 cm/s and its height decreases at the rate of 1 cm/s. Find the rate of change of its volume when the radius is 5 cm and the height is 15 cm. If the volume should not change even when the radius and height are changed, what is the relation between the radius and height?

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20. The contentment obtained after eating X-units of a new dish at a trial function is given by the function $f(x) = x^3 + 6x^2 + 5x + 4$. If the marginal contentment is defined as the rate of change f(X) with respect to the number of units consumed at an instant, then find the marginal contentment when three units of dish are consumed.

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21. The cost function of a firm is given by:

$C(x) = 300x - 10x^2 + \frac{1}{3}x^3$ where 'x' is the output. If the marginal cost is defined as the rate of change of C(x) with respect to 'x', then find the the marginal cost when 5 units are produced.

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22. The total cost C(x) associated with product of 'x' units of an item is given by: $C(x) = 0.005x^3 - 0.02x^2 + 30x + 500$. Find the marginal cost when 3 units are produced.

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23. The total revenue in rupees received form the sale of 'x' units of a product is givne by : $R(x) = 7x^2 + 50x + 119$. Find the marginal revenue when 10 units of the product are sold.

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24. Total revenue from the sale of 'x' units of the product is given by: $R(x) = 40 + 40x - \frac{x^2}{2}$ Find the marginal revenue when x = 6 and interpret it.

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25. A man 160 cm tall, walks away from a source of light situated at the top of a pole 6 m high, at the rate of 1.1m/sec. How fast is the length of his shadow increasing when he is 1 m away from the pole ?

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26. A man of height $2m$ walks at a uniform speed of $5k\frac{m}{h}$ away from a lamp post which is $6m$ high. Find the rate at which the length of his shadow increases.

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27. The side of a square sheet is increasing at 3 cm per minute. At what rate is the area increasing when the side is 10 cm long?

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28. The side of a square is increasing at the rate of 0.2 cm/s. Find the rate of increase of te perimeter of the square.

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29. The length 'x' of a rectnagle is decreasing at the rate of 3cm/m and thw width 'y' is increasing at the rate of 2 cm/m. Find the rates of change of: the perimeter

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30. The length 'x' of rectangle is decreasing at the rate of 2 cm/min and width 'y' is increasing at the rate of 3 cm/min. When $x = 8$ and $y = 6$ cm, find the rate of change of the area of rectangle

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31. An edge of a variable cube is increasing at the rate of 3 cm second. How fast is the volume of the cube increasing when the edge is 10 cm long?

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32. Water is dripping out form a conical funnel at the uniform rate of $2cm^3/s$ through a tiny hole at the vertex at the bottom. When the slant height of the water is 5 cm, find the rate of decrease of the slant height of the water.

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33. An inverted conical vessel whose height is 10 cm and the radisu of whse base is 5 cm is being filled with water at the uniform rate of 1.5 cu cm/m. Find the rate at which the level of water in the vessel is rising when the depth is 4 cm.

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34. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ?

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35. The radius of a circular soap bubble is increasing at the rate of 0.2 cm/s. Find the rate of change of its: Volume when the radius is 4 cm.

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36. The radius of a circular soap bubble is increasing at the rate of 0.2 cm/s. Find the rate of change of its: Surface area when the radius is 4 cm.

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37. In a competition, a brave child tries to inflate a huge spherical balloon bearing slogans against child labour at the rate of 900 cubic centimeter of gas per second. Find the rate at which the radius of the balloon is increasing when its radius is 15 cm. Also write any three value/life skill reflected in this question.

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38. Water is running into a conical vessel, 15 cm deep and 5 cm in radius, at the rate of 0.1 cm^2/sec . When the water is 6 cm deep, find at what rate is the water level rising?

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39. Water is running into a conical vessel, 15 cm deep and 5 cm in radius, at the rate of 0.1 cm^2/sec . When the water is 6 cm deep, find at what rate is the water level rising?

Watch Video Solution

40. Water is running into a conical vessel, 15 cm deep and 5 cm in radius, at the rate of 0.1 cm^2/sec . When the water is 6 cm deep, find at what rate is the water level rising?

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41. Show that the following functions are strictly increasing on R: f(x) = 3x + 17

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42. Show that the following functions are strictly increasing on R : $f(x) = e^{x}$

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43. Show that the following functions are strictly increasing on R: $f(x) = e^{2x}$

Watch Video Solution

44. Without using the dervative, show that f(x) = |x| is strictly increasing in $(0, \infty)$

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45. Without using the dervative, show that f(x) = |x| is strictly decreasing in $(- \infty, 0)$

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46. Show that the function f given by, $f(x) = x^3 - 3x^2 + 4x, x \in R$ is increasing on R.

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47. Show that the function $f(x) = x^3 - 6x^2 + 15x + 4$ is strictly increasing in R.

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48. Prove that : $f(x) = x^2$ is a decreasing function for x < 0, where x $\in$ R

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49. Prove that $f(x) = \frac{3}{x} + 7$ is strictly decreasing for $x \in R - \{0\}$.

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50. Prove that the function given by $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing in R.

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51. Prove that $f(x) = 4x^3 - 6x^2 + 3x + 12$ is strictly increasing function on R.

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52. f(x) = cos x is strictly decreasing in $(0, \pi)$

Watch Video Solution

53. f(x) = cos x is strictly increasing in $(\pi, 2\pi)$

Watch Video Solution

Watch Video Solution

54. f(x) = cos x is neither increasing nor decreasing in $(0, 2\pi)$

Watch Video Solution

55. f(x) = sin x is a strictly increasing in $\left(0, \frac{\pi}{2}\right)$

Watch Video Solution

56. f(x) = sin x is a strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$

Watch Video Solution

57. Show that the function given by $f(x) = \sin x$ is neither increasing nor decreasing in $(0, \pi)$.

Watch Video Solution

58. Prove that $f(x) = 2\sin x + 1$ is an increasing function $on\left[0, \frac{\pi}{2}\right]$

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59. $f(x) = \tan^{-1}(\sin x + \cos x)$ is strictly decreasing function on $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

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60. Prove that the logarithmic function is increasing on $(0, \infty)$.

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61. Prove that the function f given by f(x) = log sin x is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ and strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$.

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62. Prove that $f(x) = \log \cos x$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$ and strictly increasing on $\left(\frac{\pi}{2}, \pi\right)$

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63. Find the intervals in which the following functions are increasing: $x^3 - 3x$

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64. Find the intervals in which the following functions are strictly increasing : $f(x) = 10 - 6x - 2x^2$

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65. Find the intervals in which the functions: $f(x) = x^3 + 2x^2 - 1$is strictly increasing and decreasing

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66. Find the intervals in which the function $f(x) = 30 - 24x + 15x^2 - 2x^3$ is strictly decreasing.

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67. Find the values of 'a for which the function : $f(x) = x^2 - 2ax + 6$ is increasing when x > 0

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68. Find the value of 'a' for which f(x) = sinx - ax + b is decreasing function on R.

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69. Find the values of x for which $y = [x(x-2)]^2$ is an increasing function.

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70. Prove that f(x) = ax + b, where 'a and b' are constants and a> 0 is a strictly increasing function for all real values of x, without using the derivative.

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71. Determine for which values of x, the following function are increasing or decreasing:

$f(x) = -3x^2 + 12x + 8$

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72. Determine for which values of x, the following function are increasing or decreasing:

$f(x) = x^3 - 12x$

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73. Determine for which values of x, the following function are increasing or decreasing:

$f(x) = 2x^3 - 24x + 107$

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74. Find the intervals in which the following functions are strictly increasing or strictly decreasing

$x^4 - 2x^2$

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75. Determine for which values of x, the following function are increasing or decreasing:

$f(x) = x^3 + \frac{1}{x^3}, x \neq 0$

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76. For what value of 'x' are the following funtions increasing or decreasing? $y = x + \frac{1}{x}, x \neq 0$

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77. For what value of 'x' are the following funtions increasing or decreasing? $y = 5x^{3/2} - 3x^{5/2}, x \geq 0$

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78. Determine the intervals in which the following functions are strictly increasing or stricIty decreasing.

$f(x) = x^2 + 2x - 5$

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79. Find the interval in which the function $f(x) = 10 - 6x - 2x^2$ is strictly increasing and strictly decreasing.

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80. Determine the intervals in which the following functions are strictly increasing or striclty decreasing.

$f(x) = 6 - 9x - x^2$

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81. Determine the intervals in which the following functions $f(x) = x^2 - 4x + 6$ are strictly increasing or strictly decreasing.

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82. Determine the intervals in which the following functions are strictly increasing or stricIty decreasing.

$$f(x) = \frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{5}{2}x^2 - 6x + 7$$

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83. Determine the intervals in which the following functions are strictly increasing or stricIty decreasing.

$$f(x) = 20 - 9x + 6x^2 - x^3$$

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84. Determine the intervals in which the following functions are strictly increasing or stricIty decreasing.

$$f(x) = 2x^3 - 15x^2 + 36x + 17$$

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85. Determine the intervals in which the following functions are strictly increasing or stricIty decreasing.

$f(x) = 2x^3 - 9x^2 + 12x + 15$

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86. Determine the intervals in which the following functions are strictly increasing or stricIty decreasing.

$f(x) = 2x^3 - 3x^2 - 36x + 7$

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87. Find the intervals in which the following functions are strictly increasing or strictly decreasing

$x^3 - 6x^2 + 9x + 15$

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88. Find the intervals in which the following functions are strictly increasing or strictly decreasing

$4x^3 - 6x^2 - 72x + 30$

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89. Determine the intervals in which the following functions are strictly increasing or striclty decreasing.

$f(x) = 2x^3 - 12x^2 + 18x + 5$

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90. Determine the intervals in which the following functions are strictly increasing or striclty decreasing.

$$f(x) = \frac{4x^2 + 1}{x}$$

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91. Determine the intervals in which the following functions are strictly increasing or striclty decreasing.

$f(x) = -2x^3 - 9x^2 - 12x + 1$

Watch Video Solution

92. Determine the intervals in which the following functions are strictly increasing or striclty decreasing.

$f\Big(x = (x-1)(x-2)^3$

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93. Determine the intervals in which the following functions are strictly increasing or striclty decreasing.

$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$

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94. Determine the intervals in which the following functions are strictly increasing or striclty decreasing.

$f(x) = x^4 - 4x^3 + 4x^2 + 15$

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95. Determine the intervals in which the following functions are strictly increasing or striclty decreasing.

$f(x) = (x+1)^3(x-3)^3$

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96. Determine the intervals in which the following functions are strictly increasing or striclty decreasing.

$f(x) = x^8 + 6x^2$

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97. On which of the following intervals is the function 'f' given by $f(x) = x^{100} + \sin x - 1$ strictly increasing?

A. (-1,1)

B. (0,1)

C. $\left(\frac{\pi}{2}, \pi\right)$

D. $\left(0, \frac{\pi}{2}\right)$

Answer:

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98. Find the intervals in which f(x) = sin x - cos x , where $0 < x < 2\pi$, is strictly increasing or decreasing.

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99. Find the intervals in which the function given by : $f(x) = \sin x + \cos x, \ 0 \le x \le 2\pi.$ is strictly increasing and strictly decreasing.

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100. Find the intervals in which the function 'f' given by: $f(x) = \sin x - \cos x, 0 \le x \le 2\pi$ is strictly increasing or decreasing.

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101. Find the intervals in which the function $f(x) = 2x^3 - 9x^2 + 12x + 29$ is monotonic increasing

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102. Find the intervals in which the function $f(x) = 2x^3 - 9x^2 + 12x + 29$ is monoonic decreasing.

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103. Find the intervals in which the function given by $f(x) = \sin 3x$, x in [$0, \frac{\pi}{2}$] is increasing.

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104. Find the intervals in which the function given by $f(x) = \sin 3x$, x in [$0, \frac{\pi}{2}$] is decreasing.

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105. Which of the following functions are strictly decreasing on $\left(0, \frac{\pi}{2}\right)$?

A. cos x

B. cos 2 x

C. cos 3x

D. tan x

Answer:

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106. If $y = 4x - \frac{x^2}{2}$, when 'x' denotes the number of hours worked and y denotes the amount (in Rs) earned. Then find the value of 'x' (in interval) for which the income remains increasing ? Explain the importance of earning in life?

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107. If x> - 1 , show that $\frac{x}{\sqrt{1+x}}$-log(1+x) + 9 is an increasing function of x.

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108. Find the slopes of the tangents to the following curves: $y = 3x^2 atx = 1$

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109. Find the slope of the tangent to the curve $y = x^3 - x$ at $x = 2$

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110. Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at x = 4 .

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111. Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at x = 4 .

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112. Find the slope of the tangent to the curve $y = \frac{x-1}{x-2}$ at x = 10.

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113. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose x-coordinate is 3.

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114. Find the slope of the tangent to the curve $y = x^3 - x^2 + 1$ at the point whose x-coordinate is 2 .

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115. Find the slope of the tangent to the curve: $y = 3x^2 - 6x$ at the point on it, whose x-co-ordinate is 2.

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116. Find the slope of the tangent to the curve: $y = x^3 - 2x + 8$ at the point (1,7).

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117. Find the slope of the normal to the curve: $y = 2x^2 - 1at(1, 1)$

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118. Find the slope of the tangent to the curve $y = x^3 - x^2 + 1$ at the point whose x-coordinate is 2 .

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119. Find the slope of the tangent to the curve: $y = \tan^2 x + \sec xatx = \frac{\pi}{4}$

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120. Find the slope of the tangent to the curve: $x = a(\theta - \sin\theta), y = a(1 - \cos\theta) at \theta = \frac{\pi}{2}$

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121. Find the slope of the tangent to the curve: $x = 1 - a\sin\theta, y = b\cos^2\theta at \theta = \frac{\pi}{2}$

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122. Find the slope of the tangent to the curve: $x = a\cos^3\theta, y = a\sin^3\theta at \theta = \frac{\pi}{4}$

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123. Find the equations of the tangent line to the curve: $y = 2x^2 + 3y^2 = 5$ at the point (1,1)

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124. Find the equations of the tangent line to the curve: $y = x^3 - 3x + 5$ at the point (2,7)

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125. Find the equations of the tangent line to the curve: $y = \sin x at x = \frac{\pi}{4}$

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126. Find the equations of the tangent line to the curve: $y = \cot^2 x - 2\cot x + 2 at x = \frac{\pi}{4}$

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127. Find the equations of the tangent line to the curve: $y = \sec^4 x - \tan^4 xatx = \frac{\pi}{3}$

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128. Find the equations of the tangent and normal lines to the following curves: $y = x^2 at(0, 0)$

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129. Find the equations of the tangent and normal lines to the following curves: $y = x^3 at(1, 2)$

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130. Find the equations of the tangents and normal lines to the $y = 2x^2 - 3x - 1at(1, -2)$

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131. Find the equations of the tangent and normal to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$ at (1, 1).

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132. Find the equations of the tangents and normal lines to the $y = x^4 - 6x^3 + 13x^2 - 10x + 5at(1, 3)$

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133. Find the equations of the tangent to the given curves at the indicated points: $y = x^4 - 6x^3 \_ 13x^2 - 10x + 5$ at $(0, 5)$

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134. Find the equations of the tangent to the given curves at the indicated points: $x = \cos t, y = \sin t$ at $t = \frac{\pi}{4}$

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135. Find the equation of the tangent to the curve $x = a\cos^3\theta, y = a\sin^3\theta at\theta = \frac{\pi}{4}$

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136. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is:

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137. Find the equations of the tangent and normal lines to the following curves: $y(x-2)(x-3)-x+7=0$ at the point, where it meets x=axis.

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138. Find the equation of the tangent and normal to the given curve at the indicated points :

$y=\sin^2 x$ at $x=\frac{\pi}{2}$

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139. Find the equations of the tangent and normal lines to the following curves: $y=\frac{1+\sin x}{\cos x} at x=\frac{\pi}{4}$

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140. Find the equations of the tangent to the given curves at the indicated points: $x = \cos t, y = \sin t$ at $t = \frac{\pi}{4}$

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141. Find the length of tangent, subtangent normal and subnormal to $y^2 = 4ax$ at $(at^2, 2at)$

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142. Find the equation of the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1at(x_1y_1)$

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143. Find the equations of the tangent and the normal to the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(\sqrt{2}a, b)$

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144. Find the equation of the normal to the curve $ay^2 = x^3 at(am^2, am^3)$

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145. Find the equation of the tangent to the curve $2x^2 - y = -7$, which is parallel to the line $4x - y + 3 = 0$.

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146. Find the equation of the tangent to the curve $y = \sqrt{3x - 2}$ which is parallel to the line $4x - 2y + 5 = 0$.

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147. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$, which is parallel to the line $2x - y + 9 = 0$.

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148. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is perpendicular to the line $5y - 15x = 13$

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149. Find the equations of the tangents to the curve: $y = x^3 + 2x - 4$. Which are perpendicular to line x + 14y + 3 = 0

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150. Find the equation of the normals to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$

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151. Find the equations of the normals to the curve: $3x^2 - y^2 = 8$, which are parallel to the line x + 3y = 6

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152. If the normal at any point on the curve $x^{2/3} + y^{2/3} = a^{2/3}$ makes an angle ϕ with the x-axis, then prove that the equation of the normal is $y \cos \phi - x \sin \phi = a \cos 2\phi$

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153. Find the equation of the normal to curve $x^2 = 4y$ which passes through the point (1, 2).

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154. Find the equation of the normal to the curve: $y^2 = 4x$, which passes through the point (1,2).

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155. Find the equation of the tangent to the curve: $y = x^2 - 2x + 9$, which is parallel to the line: 2x + y + 7 = 0

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156. Find the equation of tangent to the curve $y = x^3 - 2x^2 - 2x$ at which tangent line is parallel to line $y = 2x - 3$.

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157. Find the equation of the tangent to the curve $y = \sqrt{5x - 3}$, which is parallel to the line 4x - 2y + 3 = 0

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158. Find the equation of the tangents to the function $y = 4x^3 - 3x + 5$, which are perpendicular to the line $9y + x + 3 = 0$.

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159. Find the equation of the normals to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$

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160. Find the equations of the normal to the curve: $y = x^3 + 5x^2 - 10x + 10$, where the normal is parallel to the line x - 2y + 10 = 0

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161. Find the equation of tangent to the curve given by $x = a\sin^3 t, y = b\cos^3 t$ at a point where $t = \frac{\pi}{2}$.

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162. Find the equation of the tangent at $t = \frac{\pi}{4}$ to the curve: x = sin 3t, y = cos 2t.

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163. Find the point(s) on the curve: $y = 3x^2 - 12x + 6$ at which the tangent is parallel to x-axis.

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164. Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangents are parallel to the x-axis.

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165. Find the point(s) on the curve: $y = \frac{1}{4}(x^2)$, where the slope of the tangent is $\frac{16}{3}$

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166. Find the point(s) on the curve: $y = x^2 + 1$, at w2hich the slope of the taget is equal to x-coordinate

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167. Find the point(s) on the curve: $y = x^2 + 1$, at w2hich the slope of the taget is equal to y-coordinate

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168. Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$

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169. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

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170. Find the points on the following curve at which the tangents are parallel to x-axis. $y = x^3 - 3x^2 - 9x + 7$.

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171. At what point on the curve $x^2 + y^2 - 2x - 4y + 1 = 0$ is the tangent paralle to x-axis?

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172. Find points on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1$at which the tangents are parallel to x-axis.

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173. Find points on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1$ at which the tangents are parallel to y-axis.

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174. Show that the tangents to the curve y= $7x^3 + 11$ at the points x = 2 and x =-2 are parallel.

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175. Find the equations of all lines having slope 0 which are tangent to the curve $y = \frac{1}{x2 - 2x + 3}$

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176. Find the equation of all lines having slope – 1 that are tangents to the curve $y = \frac{1}{x - 1}, x \neq \; - 1$

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177. Find the equation of all lines having slope 2 which are tangent to the curve $y = \frac{1}{x - 3}, x \neq 3.$

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178. Find the point of intersection of the tangetn lines to the curve $y = 2x^2$ at the points (1,2) and (-1,2)

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179. Prove that the tangents to the curve $y = x^2 - 5x + 6$ at the points (2,0) and (3,0) are at right-angles.

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180. Find the angle of intersection of the curves: $y^2 = 4x \text{ and } x^2 = 4y$

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181. Find the angle of intersection of the curves: $x^2 + y^2 - 4x - 1 = 0 \text{ and } x^2 + y^2 - 2y - 9 = 0$

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182. Show that the following curves cut each other orthogonally: $y = x^3$ and $6y = 7 - x^2$

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183. Show that the following curves cut each other orthogonally: $x^2 + 4y^2 = 8$ and $x^2 - 2y^2 = 4$

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184. If the curves: $\alpha x^2 + \beta y^2 = 1$ and $\alpha' x^2 + \beta' y^2 = 1$ intersect orthogonally, prove that $(\alpha - \alpha')\beta\beta' = (\beta - \beta')\alpha\alpha'$

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185. Prove that the curves $4x = y^2$ and $4xy = k$ cut at right angles if $k^2 = 512$.

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186. Show that the curves $2x = y^2$ and $2xy = k$ cut at right angles if $k^2 = 8.$

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187. Prove that the curves $y^2 = 4ax$ and $xy = c^2$ cut at right angles if $c^4 = 32a^4$

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188. Show that the normal at any point θ to the curve $x = a\cos\theta + a\theta\sin\theta, y = a\sin\theta - a\theta\cos\theta$ is at a constant distance from the origin.

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189. Find the equations of the tangent and normal to the curve $16x^2 + 9y^2 = 145at(x_1, y_1)$, where $x_1 = 2$ and $y_1 > 0$. Also find the points of intersection where both tangent and normal cut the x-axis.

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190. Find a point on the curve $y = x^3$, where the tangent to the curve is parallel to the chord joining the points (1, 1) and (3, 27).

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191. Find a point on the parabola $y = (x - 3)^2$, where the tangent is parallel to the line joining (3,0) and (4,1).

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192. Show that the area of the triangle formed by the tangent and the normal at the point (a,a) on the curve $y^2(2a - x) = x^3$ and the line

$x = 2ais\frac{5a^2}{4}$ sq. units.

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193. Find the equations of the normal at a point on the curve $x^2 = 4y$, which passes through the point (1,2). Also find the equation of the corresponding tangent.

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194. Find the equations of the tangents to the curve $3x^2 - y^2 = 8$, which pass thorugh the point $\left(\frac{4}{3}, 0\right)$

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195. Determine the values of 'x' for which the function $f(x) = x^2 + 2x - 3$ is an increasing. Also find the co-ordinates of the points on thecurve $y = x^2 + 2x - 3$, where the normal is parallel to the line x - 4y + 7 = 0

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196. Determine the intervals in which the function $f(x) = (x-1)(x+1)^2$ is increasing or decreasing. Find also the points at which the tangents to the curve are parallel to x-axis.

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197. Using differentials, find the approximate value of $\sqrt{26}$

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198. Using differentials, find the approximate the value of $\sqrt[3]{0.026}$ upto three places of decimals.

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199. Find the approximate change in the volume V of a cube of side x metres caused by increasing the side by 1%.

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200. If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating its surface area.

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201. If $y = x^4 + 10$ and x changes from 2 to 1.99, find the approximate change in y.

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202. Use differentials to calculating approximate value of $\log_e(9.01)$

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203. Use differentials to find the approximate value of $\tan 46^{\circ}$, if it is being given that $1^{\circ} = 0.01745$ radian.

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204. If in a ΔABC, the side c and the angle C remain constant, while the remaining elements are changed slightly. Using differential, show that $\frac{da}{\cos A} + \frac{db}{\cos B} = 0.$

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205. The time t of a complete oscillation of a simple pendulum of length l is given by the equation $T = 2\pi\sqrt{\frac{1}{g}}$ where g is constant. What is the percentage error in T when l is increased by 1%? Comment the above result.

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206. In the following, find the approximate values, using differentials: $\sqrt{37}$

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207. In the following, find the approximate values, using differentials: $\sqrt{50}$

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208. In the $\sqrt{401}$, find the approximate values, using differentials.

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209. In the $\sqrt{0.0037}$, find the approximate values, using differentials.

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210. Use differentials to find the approximate values of the following

$\sqrt{0.037}$

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211. In the following, find the approximate values, using differentials:

$\sqrt{25.2}$

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212. Find the approximate value of $\sqrt{49.5}$ using differentials

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213. Use differentials to find the approximate values of the following

$\sqrt{36.6}$

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214. In the following, find the approximate values, using differentials: $\sqrt{16.3}$

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215. Use differentials to approximate

$\sqrt{0.60}$

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216. In the following, find the approximate values, using differentials: $\sqrt{0.17}$

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217. In the following, find the approximate values, using differentials: $\sqrt{0.26}$

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218. In the following, find the approximate values, using differentials: $\sqrt{0.82}$

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219. In the following, find the approximate values, using differentials: $\sqrt{0.24}$

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220. In the following, find the approximate values, using differentials: $\sqrt{0.50}$

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221. Using differential find approximate value of $\sqrt[3]{26}$

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222. In the following, find the approximate values, using differentials: $(28)^{1/3}$

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223. In the $25^{\frac{1}{3}}$, find the approximate values, using differentials.

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224. In the following, find the approximate values, using differentials: $(26.57)^{1/3}$

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225. Using differentials, find the approximate value of $(0.731)^1 / 3$.

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226. In the following, find the approximate values, using differentials:

$\sqrt[3]{0.009}$

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227. In the following, find the approximate values, using differentials:

$\sqrt[3]{0.007}$

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228. In the following, find the approximate values, using differentials:

$(15)^{1/4}$

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229. In the following, find the approximate values, using differentials: $(82)^{1/4}$

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230. In the following, find the approximate values, using differentials: $(255)^{1/4}$

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231. In the following, find the approximate values, using differentials: $(81.5)^{1/4}$

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232. Using differentials, find the approximate value of the following: $\left(\frac{17}{81}\right)^{\frac{1}{4}}$

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233. In the following, find the approximate values, using differentials: $(32.15)^{1/5}$

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234. Using differential, find the approximate value of $(0.999)^{\frac{1}{10}}$

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235. In the following, find the approximate values, using differentials: $(3.968)^{3/2}$

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236. Using differentials, find the approximate value of the following: $33^{-\frac{1}{5}}$

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237. Find the approximate value of : f(3,02), where $f(x) = 3x^2 + 15x + 3$

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238. Find the approximate value of $f(5 \cdot 001)$, where $f(x) = x^3 - 7x^2 + 15.$

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239. Find the approximate change in the volume V of a cube of side 'x' metres caused by increasng the side by 2%.

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240. Find the approximate change in the surface area of a cube of side x metres caused by decreasing the side by 1% .

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241. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its volume.

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242. Use differentials to find $\cos 61^{\circ}$, it being given that $\sin 60^{\circ} = 0.86603$ and $1^{\circ} = 0.01745$ radian.

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243. If $y = x^4 - 10$ and if x changes form 2 to 1.99, what is approximate change in y?

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244. Find the approximate value of f(2.01) where $f(x) = 4x^2 + 5x + 2$.

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245. Use differenttials, find the approximate value of the following: $\sin\left(\frac{22}{14}\right)$

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246. Use differenttials, find the approximate value of the following: $\frac{\cos(11\pi)}{36}$

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247. If y = sin x and x change form $\frac{\pi}{2} \rightarrow \frac{22}{14}$, what is the approximate change in y?

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248. A circular metal plate expands under heating so that its radius increases by 2%. Find the approximate increase in the area of he plate if the radius of the plate before heating is 10 cm.

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249. Find the percentage error in calculating the surface area of a cubical box if an error of 1% is made in increasing the lengths of edges of the cube.

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250. The radius of a spherical diamond is measured as 6 cm with an error of 0.04 cm. Obtain the approximate error in calculating its volume. If the cost of $1cm^3$ diamond is Rs 1600, what is the loss to the buyer of the diamond?

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251. Find the maximum or minimum values, if any, of the following functions without using the derivatives:

$f(x) = -(x-1)^2 + 2$

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252. Find the maximum and minimum values, if any, of the following functions given by: $f(x) = -(x-1)^2 + 10$

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253. Find the maximum and minimum values, if any, of the following functions without using derivates:

$f(x) = (2x - 1)^2 + 3$

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254. Find the maximum or minimum values, if any, of the following functions without using the derivatives:

$f(x) = 9x^2 + 12x + 2$

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255. Find the maximum or minimum values, if any, of the following functions without using the derivatives:

$f(x) = x + 1, x \in (- 1, 1)$

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256. Find the maximum or minimum values, if any, of the following functions without using the derivatives:

$g(x) = x^3 + 1$

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257. Find the maximum or minimum values, if any, of the following functions without using the derivatives:

$f(x) = |x + 2| - 1$

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258. Find the maximum or minimum values, if any, of the following functions without using the derivatives:

$g(x) = -|x - 1| + 3$

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259. Find the maximum or minimum values, if any, of the following functions without using the derivatives:

`f(x) = sin 2x + 5

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260. Find the maximum or minimum values, if any, of the following functions without using the derivatives:

f(x)=|sin4x+3|

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261. The function $f(x) = x^2, x \in R$ has no maximum value.

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262. Find the points of absolute maximum and minimum of each of the following: $y = x(1 + 10x - x^2), 3 \leq x \leq 9$

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263. Find the points of absolute maximum and minimum of each of the following: $y = \frac{1}{3}x^{3/2} - 4x, 0 \leq x \leq 64$

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264. Find the points of absolute maximum and minimum of each of the following: $y = \sqrt{5}\left(\sin x + \frac{1}{2}\cos 2x\right), 0 \leq x \leq \frac{\pi}{2}$

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265. Find the maximum and the minimum values, if any, of the function given by $f(x) = x, x \in (0, 1)$.

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266. Find the aboslute minimum value of $y = \lambda - 3x$ in $0 \leq x \leq 2$

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267. At what points in the interval $[0, 2\pi]$, does the function sin 2x attain its maximum value?

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268. Find the maximum and minimum values of the function : $f(x) = 2x^3 - 15x^2 + 36x + 11.$

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269. Find local minimum value of the function f given by $f(x) = 3 + |x|, x \in R$

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270. Find the absolute maximum and the absolute minimum value of the function given by: $f(x) = x^{50} - x^{20}, [0, 1]$

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271. Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals: $f(x) = 4x - \frac{1}{2}x^2, x \in \left[-2, \frac{9}{2}\right]$

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272. Find absolute maximum and minimum values of a function f given by $f(x) = 12x^{\frac{4}{3}} - 6x^{\frac{1}{3}}, x \in [-1, 1]$.

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273. Find the absolute maximum and the absolute minimum value of the function given by: $f(x) = x^3 - \frac{5}{2}x^2 - 2x + 1, 0 \leq x \leq 3$

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274. Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals: $f(x) = (x-1)^2 + 3, x \in [-3, 1]$

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275. Find the absolute maximum and the absolute minimum value of the function given by: $f(x) = 2x^3 - 15x^2 + 36x + 1$in [1,5]

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276. Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals: $f(x) = \sin x + \cos x, x \in [0, \pi]$

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277. Find the absolute maximum and minimum values of the function f given by $f(x) = \cos^2 x + \sin x, x \in [0, \pi]$

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278. Find the absolute maximum and the absolute minimum value of the function given by: $y = x + \sin 2x$ in $[0, 2\pi]$

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279. Find the absolute maximum and the absolute minimum value of the function given by: $y = 2\cos 2x - \cos 4x, 0 \le x \le \pi$

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280. Find both the maximum value and the minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval $[0, 3]$

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281. Find the maximum value of $2x^3 - 24x + 107$ in the interval $[1, 3]$. Find the maximum value of the same function in $[-3, -1]$.

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282. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximm and local minimum values:

The constant functionα

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283. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximm and local minimum values:

$f(x) = x^2$

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284. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximm and local minimum

values:

$f(x) = x^3 - 3x$

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285. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximm and local minimum values:

f(x) = cosx, 0 < x < pi`

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286. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximm and local minimum values:

$f(x) = \sin x + \cos x, 0 < x < \frac{\pi}{2}$

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287. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximm and local minimum values:

$f(x) = \sin x - \cos x, 0 < x < 2\pi$

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288. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximm and local minimum values:

$g(x) = \frac{x}{2} + \frac{2}{x}, x \neq 0$

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289. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be: $f(x) = \frac{1}{x^2 + 2}$

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290. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximm and local minimum values:

$f(x) = x\sqrt{1-x}, x > 0$

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291. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximm and local minimum values:

$f(x) = x^3 - 12x^2 + 36x - 4$

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292. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximm and local minimum

values:

$$f(x) = x^3 - 6x^2 + 9x + 15, 0 \le x \le 6$$

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293. Find the points of local maxima and local minima if any. Also find the local maximum and local minimum value :

$$f(x) = x^3 - 3x + 3$$

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294. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximm and local minimum values:

$$f(X) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105$$

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295. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximm and local minimum values:

$f(x) = 3x^4 + 4x^3 - 12x^2 + 12$

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296. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximm and local minimum values:

$f(x) = x\sqrt{1 - x}, x > 0$

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297. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximm and local minimum values:

$f(x) = -x + 2\sin x, 0 \leq x \leq 2\pi$

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298. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximm and local minimum values:

$f(x) = \sin^4 x + \cos^4 x, 0 < x < \frac{\pi}{2}$

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299. Prove that x^x has minimum value at x $=\frac{1}{e}$

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300. The curve $y = \frac{x^2 + ax + b}{x - 10}$ has a stationary point at (4,1). Find the values of 'a' and 'b' and also show that y is maximum at this point.

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301. $y = \frac{ax - b}{(x-1)(x-4)}$ has a turning point P (2,-1). Find the values of 'a' and 'b' and show that y is maximum at P.

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302. Find two positive numbers whose sum is 14 and product is maximum.

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303. Find two positive numbers whose sum is 16 and product is maximum.

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304. Amongst all pairs of positive numbers with product 256 find those whose sum is least

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305. Amongst all pairs of positive numbers with product 64 find those whose sum is least

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306. Find two numbers whose sum is 15 and the square of one multiplied by the cube of the other is maximum.

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307. Find two positive numbers whose sum is 15 and the sum of whose squares is minimum.

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308. Find two positive numbers 'x' and 'y' such that their sum is 35 and product x^2y^5 is maximum.

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309. Find two positive numbers whose sum is 16 and whose sum of cubes is minimum.

Watch Video Solution

310. Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

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311. How should we choose two numbers, each greater than or equal to -2 whose sum is $\frac{1}{2}$ so that the sum of square of the first and cube of the second is minimum?

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312. Find the maximum slope of the curve: $y = -x^3 + 3x^2 + 2x - 27$

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313. If the sum of the lengths of hypotenuse and a side of a right-angled triangle is given. Show that the area is maximum, when the angle between then is 60°

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314. A wire of length 36 m is to be cut into two pieces. One of the pieces is to be made into a square and other into a circle. What could be the lengths of the two pieces so that the combined area of the square and the circle is minimum ?

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315. A wire of length 36 cm is cut into two pieces . One of the pieces is to be made into a square and the other into a equilaterel triangle. Find the length of each piece so that the sum of the areas of the square and the triangle is minimum.

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316. Prove that the perimeter of a right-angled triangle of given hypotenuse equal to 5 cm is maximum when the triangle is isosceles.

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317. Prove that the area of right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.

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318. Prove that the least perimeter of an isosceles triangle in which a circle of radius 'r' can be inscribed is $6r\sqrt{3}$

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319. Show that, of all the rectangles with a given area, the square has the smallest perimeter.

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320. Show that rectangle of maximum perimeter, which can be inscribed in a circle of a radius 'a' is a square of side $\sqrt{2}a$.

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321. Show that of all rectangles inscribed in a given circle the square has maximum area.

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322. A rectangle is inscribed in a semi-circle of radius 'r' with one of its sides on diameter of semi-circle. Find the dimensions of the rectangle so that its area is maximum. Find also the area.

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323. Of all rectangles , each of which has perimeter: 40 cm . Find the one having maximum area. Also find that area.

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324. Of all rectangles , each of which has perimeter: 60 cm . Find the one having maximum area. Also find that area.

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325. An open box with a square base is to be made out of a given iron sheet of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

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326. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

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327. Show that the semi-vertical angle of the right-circular cone of maximum volume and of given slant height is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

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328. Prove that the semi-vertical angle of the right circular cone of given volume and least curved surface area is $\cot^{-1}\sqrt{2}$

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329. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle α is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 \tan^2 \alpha$

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330. Show that the volume of the greatest cylinder, which can be inscribed in a cone of height 'h' and semi-vertical angle $30^\circ is \frac{4}{81}\pi h^3$

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331. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $4\frac{r}{3}$.

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332. Prove that volume of largest cone, which can be inscribed in a sphere, is $\left(\frac{8}{27}\right)^{th}$ part of volume of sphere.

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333. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ time the radius of the base.

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334. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $2\frac{R}{\sqrt{3}}$. Also find the maximum volume.

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335. Show that the radius of right circular cylinder of maximum volume, that can be inscribed in a sphere of radius 18 cm, is $6\sqrt{6}$ cm.

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336. Prove that the radius of the right-circlar cylinder of greatest curved surface, which can be inscribed in a given cone, is half of that of the cone.

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337. Of all the closed cylindrical cans (right-circular). Which enclose a given volume of: 1228π cubic centimeters.. Find the dimensions of the can, which has the minimum surface area.

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338. Show that the surface area of a closed cuboid with surface base and given volume is minimum when it is cube.

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339. A figure consists of a semi-circle with a rectangle on its diameter. Given perimeter of the figure, find the dimensions in order that the area may be maximum.

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340. A window is in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

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341. A window consists if a semi-circle with a rectangel on its diameter. If the perimeter of the window is 30 meters, find the dimensions of the window in order that its area may be maximum.

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342. Show that the height of the cylinder, open at the top of given surface area and greatest volume is equal to the radius of its base.

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343. Show that the height of the cylinder, open at the top of given surface area and greatest volume is equal to the radius of its base.

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344. Show that the height of a closed right-circular cylinder of given volume and least surface area is equal to its diameter.

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345. Given the sum of the perimiter of a square and a circle, show that the sum of their areas is least when the side of the square is equal to radius of the circle.

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346. A square piece of tin of side 24 cm is to be made into a box without top by cutting a square from each comer and folding up the flaps to form a box. What should be the side of square to be cut off so that the volume of box is maximum also find the volume ?

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347. A square-based tank of capacity 250 cu m has to bedug out. The cost of land is Rs 50 per sq m. The cost of digging increases with the depth and for the whole tank the cost is Rs $400 \times (depth)^2$. Find the dimensions of the tank for the least total cost.

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348. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is $\frac{8}{m^3}$. If building of tank costs Rs 70 per sq metres for the base and Rs 45 per square metre for sides. What is the cost of least expensive tank?

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349. A rectangular sheet of tin 45 cm x 24 cm is to be made into a box without top, by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is the maximum possible.

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350. An open box with a square base is to be made out of a given iron sheet of area 27sq.m. Show that the maximum volume of the box is 13.5 cu. Cm.

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351. A farmer wants to construct a circular well and a square garden in his field. He wants to keep sum of their permeters fixed. The prove that the sum of their area is least when the side of a square garden is double the radius of the circular well. Do you think good planning can save energy, time and money.

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352. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water.

Show that the cost of the material will be least when the depth of the tank is half of its width.

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353. A helicopter is flying along the curve $y = x^2 + 2$. A solider is placed at the point (3,2). Find the nearest distance between the soldier and the helicopter.

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354. Find the rate of change of the area of a circle with respect to its radius r at r = 6 cm

A. 10π

B. 12π

C. 8π

D. 11π

Answer:

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355. The total revenue in Rupees received from its sale of x units of a product is given by $R(X) = 3x^2 + 36x + 5$. Find the marginal revenue, when x = 15

A. 116

B. 96

C. 90

D. 126

Answer:

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356. The interval in which $y = x^2 e^{-x}$ is increasing is:

A. $(-\infty, \infty)$

B. (-2,0)

C. $(2, \infty)$

D. (0,2)

Answer:

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357. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is:

A. 3

B. $\frac{1}{3}$

C. -3

D. $-\frac{1}{3}$

Answer:

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358. The line $y = x + 1$, is a tangent to the curve $y^2 = 4x$ at the point.

A. (1,2)

B. (2,1)

C. (1,-2)

D. (-1,2)

Answer:

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359. If $f(x) = 3x^2 + 15x + 5$, then the approximate value of f (3.02) is :

A. 47.66

B. 57.66

C. 67.66

D. 77.66

Answer:

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360. The approximate change in the volume of a cube of side x metres caused by increasing the side by 3% is:

A. $0.06x^3m^3$

B. $0.6x^3m^3$

C. $0.09x^3m^3$

D. $0.9x^3m^3$

Answer:

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361. The point on the curve $x^2 = 2y$ which is nearest to the point $(0, 5)$ is:

A. $(2\sqrt{2}, 4)$

B. $(2\sqrt{2}, 0)$

C. (0,0)

D. (2,2)

Answer:

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362. For all real values of x, the minimum value of $\dfrac{1 - x + x^2}{1 + x + x^2}$ is:

A. 0

B. 1

C. 3

D. $\frac{1}{3}$

Answer:

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363. The maximum value of $[x(x-1)+1]^{\frac{1}{3}}, 0 \leq x \leq 1$ is:

A. $\left(\frac{1}{3}\right)^{\frac{1}{3}}$

B. $\frac{1}{2}$

C. 1

D. 0

Answer:

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364. A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic metre per hour. Then the depth of the wheat is increasing at the rate of:

A. $1m^3 / \min ute$

B. $0.1m^3 / \min ute$

C. $1.1m^3 / \min ute$

D. $0.5m^3 / \min ute$

Answer:

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365. The slope of the tangent to the curve $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$at the point (2,-1) is:

A. $\frac{22}{7}$

B. $\frac{6}{7}$

C. $\frac{7}{6}$

D. $\frac{-6}{7}$

Answer:

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366. The line $y = mx + 1$, is a tangent to the curve $y^2 = 4x$ if the value of m is:

A. 1

B. 2

C. 3

D. $\frac{1}{2}$

Answer:

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367. The normal at the point (1,1) on the curve $2y + x^2 = 3$ is:

A. x+y=0

B. x-y=0

C. x+y+1 =0

D. x-y+1 =0

Answer:

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368. The normal to the curve $x^2 = 4y$ passing (1,2) is:

A. x+y=3

B. x-y=3

C. x+y=1

D. x-y=1

Answer:

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369. The points on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts with the axes are:

A. $\left(4, \pm\frac{8}{3}\right)$

B. $\left(4, -\left(\frac{8}{3}\right)\right)$

C. $\left(4, \pm\frac{3}{8}\right)$

D. $\pm 4, \frac{3}{8}\right)$

Answer:

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370. The abscissa of the point on the curve $3y = 6x - 5x^3$, the normal at which passes through origin is

A. 1

B. $\frac{1}{3}$

C. 2

D. $\frac{1}{2}$

Answer:

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371. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 = 2$

A. touch each other

B. cut at right angle

C. cut at an angle $\frac{\pi}{3}$

D. cut at an angle $\frac{\pi}{4}$

Answer:

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372. The tangent to the curve given by: $x = e^t \cos t, y = e^t \sin tatt = \frac{\pi}{4}$

makes with x-axis an angle

A. 0

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer:

Watch Video Solution

373. The equation of the normal to the curve y = sin x at (0,0) is

A. x=0

B. y=0

C. x+y=0

D. x-y=0

Answer:

Watch Video Solution

374. The point on the curve $y^2 = x$, where the tangent makes an angle of $\frac{\pi}{4}$ with x-axis is

A. $\left(\frac{1}{2}, \frac{1}{4}\right)$

B. $\left(\frac{1}{4}, \frac{1}{2}\right)$

C. (4,2)

D. (1,1)

Answer:

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375. The volume of a cube is increasing at the rate of 9 cm^3/sec. How fast is surface area increasing when the length of an edge is 10 cm ?

A. $1.8cm^2/s$

B. $2.7cm^2/s$

C. $3.6cm^2/s$

D. None of these

Answer:

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376. The length 'x' of a rectnagle is decreasing at the rate of 3cm/m and thw width 'y' is increasing at the rate of 2 cm/m. Find the rates of change of: the perimeter

A. 3cm/min

B. 2cm/min

C. 1 cm/min

D. 4 cm/min

Answer:

Watch Video Solution

377. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference ?

A. $1.4\pi cm / s$

B. 2.4 cm/s

C. 0.4 cm/s

D. $-0.4 cm / s$

Answer:

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378. The radius of an air bubble is increasing at the rate of $\frac{1}{2}c\frac{m}{s}$. At what rate is the volume of the bubble increasing when the radius is 1 cm?

A. $2\pi cm^3 / s$

B. $3\pi cm^3 / s$

C. $\frac{3}{2}\pi cm^3 / s$

D. None of these

Answer:

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379. Find the slope of tangent to the curve $y = 2x^2 - 3$ at $x = \frac{1}{4}$

A. -1

B. 1

C. $\frac{1}{2}$

D. None of these

Answer:

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380. The interval for which the function $f(x) = x^2 - 6x + 3$, is strictly increasing is :

A. $(1, \infty)$

B. (1,2)

C. $(3, \infty)$

D. None of these

Answer:

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381. The absolute maximum value of the function $f(x) = x^2 - 3x$ on [0,2] is

A. -2

B. 0

C. $-\frac{9}{4}$

D. None of these

Answer:

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382. The slope of the tangent to the curve given by: $x = 1 - \cos\theta, y = \theta - \sin\theta at\theta = \frac{\pi}{2}$ is

A. 0

B. -1

C. 1

D. Not defined.

Answer:

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383. Which of the following has neither local maxima nor local minima?

A. $f(x) = x^2 + x$

B. f(x) = logx

C. $f(x) = x^3 - 3x + 3$

D. f(x) = 3+|x|

Answer:

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384. Edge of a cube is increasing at the rate of 2 cm/s. The rate of change of its volume when the edge is 3 cm is :

A. $8cm^3 / s$

B. $54cm^3 / s$

C. $6cm^3 / s$

D. None of these

Answer:

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385. Radius of a sphere is increasing at the rate of 2 cm/s The rate of change of its volume, when radius is 6 cm, is

A. $288\pi cm^3 / s$

B. $8cm^3 / s$

C. $12\pi cm^3 / s$

D. None of these

Answer:

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386. In which of the following interval x^2e^x is increasing ?

A. $(-\infty, -2) \cup (0, \infty)$

B. (-2,0)

C. $(-\infty, \infty)$

D. None of these

Answer:

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387. For what value of x, slope of the tangent to the curve $y = x^3 + x + 1$ is 10.

A. 3

B. -3

C. $\sqrt{3}$

D. None of these

Answer:

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388. The value of x for which function sin 2x attains its maximum is :

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{2}$

Answer:

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389. Which of the following functions are strictly decreasing on $\left(0, \frac{\pi}{2}\right)$?

A. sin x

B. cos 3x

C. cos 2x

D. tan x

Answer:

Watch Video Solution

390. The radius of an air bubble is increasing at the rate of 0.2 cm/s. The rate of increase of its volume when the radius is 5 cm is

A. 1) $5\pi cm^3 / s$

B. 2) $\pi cm^3 / sc$

C. 3) $20\pi c\frac{m^3}{s}$

D. 4) $12.5cm^3 / s$

Answer:

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391. In $\left[0, \frac{\pi}{2}\right]$ the function f(x) = log sin x is

A. 1) strictly increasing

B. 2) increasing

C. 3) strictly decreasing

D. 4) decreasing

Answer:

Watch Video Solution

392. Which of the following function is always increasing?

A. 1) x+sin2x

B. 2) x-sin2x

C. 3) 2x+sin 3x

D. 4) 2x-sinx

Answer:

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393. The slope of the normal to the curve $y = 3x^2 + 2\sin x$ at x =0 is

A. 2

B. -2

C. $-\frac{1}{2}$

D. None of these

Answer:

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394. The radius of a circle is increasing at the rate of 0.14 cm/sec. The rate of change of its area at r = 7 cm is

A. 1.96π

B. 0.98π

C. 14π

D. None of these

Answer:

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395. Which of the following function is strictly decreasing in $\left(0, \frac{\pi}{2}\right)$?

A. cos x

B. cos 2x

C. cos 3x

D. tanx

Answer:

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396. The function f(x) = sin x + cos x has maxima or minima at x =

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer:

Watch Video Solution

397. Find the rate of change of the area of a circle per second with respect to its radius r when r = 5 cm.

A. $8\pi cm^2 / s$

B. $10\pi cm^2 / s$

C. $11\pi cm^2 / s$

D. None of these

Answer:

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398. Find the rate of change of the area of a circle with respect to its radius r at r = 6 cm

A. 10π

B. 12π

C. 8π

D. 11π

Answer:

Watch Video Solution

399. Which of the following function is increasing for all values of x in its domain ?

A. sin x

B. log x

C. x^2

D. |x|

Answer:

Watch Video Solution

400. The interval in which te function $f(x) = x^2 - 6x + 5$ increasing is

A. 1) $(-\infty, 1)$

B. 2) $(1, \infty)$

C. 3) [3,6]

D. 4) None of these

Answer:

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401. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 = 2$

A. touch each other

B. cut at right angle

C. cut at an angle $\frac{\pi}{3}$

D. cut at an angle $\frac{\pi}{4}$

Answer:

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402. If x is real, the maximum value of $x^2 - 8x + 17$ is

A. 1

B. 2

C. 3

D. 4

Answer:

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403. A straight line parallel to the line 2x-y+5 =0 is also a tangent to the curve $y^2 = 4x + 5$. Then the point of contact is

A. (2,1)

B. (-1,1)

C. (1,3)

D. (3,4)

Answer:

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404. The function $f(x) = 2x^3 - 15x^2 + 36x + 6$ is stictly decreasing in the interval

A. (2,3)

B. $(-\infty, 2)$

C. (3,4)

D. $(-\infty, 3) \cup (4, \infty)$

Answer:

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405. The slope of the tangent to the curve $y^2 e^{xy} = 9e^{-3}x^2$ at (-1,3) is

A. $\dfrac{-15}{2}$

B. $\dfrac{-9}{2}$

C. 15

D. $\frac{15}{2}$

Answer:

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406. The tangent to the curve y $= x^3 + 1$ at (1,2) makes an angle θ with y-axis, then the value of tanθ is

A. $\left(-\frac{1}{3}\right)$

B. 3

C. -3

D. $\frac{1}{3}$

Answer:

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407. A stone is dropped into a quiet lake and wave move in circles at a speed of 5cm per second. At the instant when the radius of the circular wave is 8 cm, how fast is the enclosed area increasing?

A. $6\pi cm^2 / s$

B. $8\pi cm^2 / s$

C. $\frac{8}{3}\pi cm^2 / s$

D. $80\pi cm^2 / s$

Answer:

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408. If 'f' is a real valued differentiable function such that `f(x) f'(x) < 0 for all real x, then

A. f(x) must be an increasing function

B. f(x) must be a decreasing function

C. |f(x)| must be an increasing function

D. |f(x)| must be a decreasing function.

Answer:

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409. Maximum value of the function $f(x) =$ x/8+2/x` on the interval [1,6] is

A. 1

B. $\frac{9}{8}$

C. $\frac{13}{12}$

D. $\frac{17}{8}$

Answer:

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410. $f(x) = \frac{\log x}{x}$ is increasing in the interval

A. (1,2e)

B. (0,e)

C. (2,2e)

D. $\left(\frac{1}{e}, 2e\right)$

Answer:

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411. The total number of local maxima and local minima of the function

`f(x) = {(2+x)^3}-3

A. 0

B. 1

C. 2

D. 3

Answer:

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412. Given $p(x) = x^4 + ax^3 + bx^2 + cx + d$ such that x = 0 is the only real root of P'(x) =0. If P(-1) < P(1), then in the interval [-1,1]

A. P(-1) is the minimum and P(1) is the maximum of P

B. P(-1) is not minimum but P(1) is the maximum of P

C. P(-1) is the minimum but P(1) is not be maximum of P

D. Neither P(-1) is the minimum nor P(1) is the maximum of P.

Answer:

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413. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis is

A. y=0

B. y=1

C. y=2

D. y=3

Answer:

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414. Let $f: R \to R$ be a positive increasing function with $\lim_{x \to \infty} \frac{f(3x)}{f(x)} = 1, then \lim_{x \to \infty} \frac{f(2x)}{f(x)}$=

A. 1

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. 3

Answer:

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415. Let $F : R \to R$ be defined by: $f(x) = \begin{cases} k-2x & \text{if} \quad x \leq -1 \\ 2x+3 & \text{if} \quad x > -1 \end{cases}$ If 'f' has a local maximum at x =-1 then a possible value of k is

A. 1

B. 0

C. $-\frac{1}{2}$

D. -1

Answer:

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416. A spherical balloon is filled with 4500pi cubic metres of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72pi cubic metres per minute, then the rate (in metres per minute) at which the radius of the balloon decreases 49 minutes after the leakage begins is :

A. $\frac{9}{7}$

B. $\frac{7}{9}$

C. $\frac{2}{9}$

D. $\frac{9}{2}$

Answer:

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417. The real number k for which the equations $2x^3 + 3x + k = 0$ has two distinct real roots in [0,1]

A. lies between 2 and 3

B. lies between -1 and 0

C. does not exist

D. lies between 1 and 2

Answer:

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418. The number of points in $(-\infty, \infty)$ for which $x^2 - x \sin x - \cos x = 0$, is

A. 0

B. 4

C. 2

D. 0

Answer:

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419. The normal to the curve $x^2 + 2xy - 3y^2 = 0$ at (1, 1)

A. does not meet the curve again

B. meets the curve again in the second quadrant

C. meets the curve again in the third quardrant

D. meets the curve again in the foruth quadrant

Answer:

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420. Let f(x) be a polynomial of degree four having extreme values at x =1 and x=2. If $\lim_{x \to 0}\left[1+\frac{f(x)}{x^2}\right]$=3, then f(2) is equal to

A. -8

B. -4

C. 0

D. 4

Answer:

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421. Consider $f(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{(1-\sin x)}}\right), x \in \left(0, \frac{\pi}{2}\right)$ A normal to y = $f(x) at x = \frac{\pi}{6}$ also passes through the point

A. $\left(0, 2\frac{\pi}{3}\right)$

B. $\left(\frac{\pi}{6}, 0\right)$

C. $\left(\frac{\pi}{4}, 0\right)$

D. (0,0)

Answer:

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422. A wire of the length 2 units is cut into two parts which are bent respectively to form a square of side=x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then

A. $(4-\pi)x = \pi r$

B. x=2r

C. 2x=r

D. $2x = (\pi + 4)r$

Answer:

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423. The least value of $\alpha \in R$ for which $4\alpha x^2 + \frac{1}{x} \geq 1, \text{ for all } x > 0, \text{ is}$

A. $\frac{1}{64}$

B. $\frac{1}{32}$

C. $\frac{1}{27}$

D. $\frac{1}{25}$

Answer:

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424. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference ?

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425. Show that the function f given by, $f(x) = x^3 - 3x^2 + 4x, x \in R$ is increasing on R.

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426. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose x-coordinate is 3.

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427. A man of height $2m$ walks at a uniform speed of $5k\frac{m}{h}$ away from a lamp post which is $6m$ high. Find the rate at which the length of his shadow increases.

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428. Find the intervals in which the function given by : $f(x) = \sin x + \cos x, \ 0 \leq x \leq 2\pi.$ is strictly increasing and strictly decreasing.

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429. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1.$

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430. Evaluate $\sqrt{401}$, using differentials.

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431. It is given that at $x = 1$, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value, on the interval $[0, 2]$. Find the value of a.

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432. Find the equations of the tangents to the curve $3x^2 - y^2 = 8$, which pass thorugh the point $\left(\frac{4}{3}, 0\right)$

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433. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $2\frac{R}{\sqrt{3}}$. Also find the maximum volume.

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