MATHS

BOOKS - MODERN PUBLICATION

EXCLUSIVELY FOR JEE(ADVANCED)

Exercise

1. vector ^ i − 3 ^ j + 4 ^ k .Find the magnitude of the vector

A. There is exactly one choice for such $\vec{v}$

B. There are infinitely many choices for such $\vec{v}$

C. If $\hat{u}$ lies in the xy-plane, then $|u_1| = |u_2|$

D. If $\hat{u}$ lies in xz-plane, the $2|u_1| = |u_3|$

Answer:

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2. Let P be the image of the point $(3, 1, 7)$ with respect to the plane $x - y + z = 3$. Then, the equation of the plane passing through P and containing the straight line $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$ is

A. $x + y - 3z = 0$

B. $3x + z = 0$

C. $x - 4y + 7z = 0$

D. $2x - y = 0$

Answer:

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3. Given L, = {1,2, 3,4},M= {3,4, 5, 6} and N= {1,3,5} Find L-(M⋃N).

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4. Given that N= {1,2,3, ..., 100}. Then write (i) the subset of N whose elements are even numbers. (ii) the subset of N whose element are perfect square numbers.

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5. In R^3, consider the planes: $P_1 : y = 0$ and $P_2 : x + z = 1$. Let P_3 be a plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the point (0,1,0) from P_3 is 1 and the distance of a point (α, β, γ) from P_3 is 2, then which of the following relations is (are) true?

A. $2\alpha + \beta + 2\gamma + 2 = 0$

B. $2\alpha - \beta + 2\gamma + 4 = 0$

C. $2\alpha + \beta - 2\gamma - 10 = 0$

D. $2\alpha - \beta + 2\gamma - 8 = 0$

Answer:

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6. In R^3, let L be a straight line passing throughthe origin. Suppose that all points on L are at a constant distance from the two planes: $P_1 : x + 2y - z + 1 = 0$ and $P_2 : 2x - y - z + 1 = 0$. Let M be the locus of the feet of the perpendiculars drawn from the points on L t the plane P_1. Which of the following point(s) lie(s) on M?

A. $\left(0, -\frac{5}{6}, -\frac{2}{3}\right)$

B. $\left(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$

C. $\left(-\frac{5}{6}, 0, \frac{1}{6}\right)$

D. $\left(-\frac{1}{3}, 0, \frac{2}{3}\right)$

Answer:

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7. Let $\vec{x}$, $\vec{y}$ and $\vec{z}$ be three vectors, each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. If $\vec{a}$ is a non-zero vector perpendicular to $\vec{x}$ and $\vec{y} \times \vec{z}$ and $\vec{b}$ is non-zero vector perpendicular to $\vec{y}$ and $\vec{z} \times \vec{x}$, then :

A. $\vec{b} = \left(\vec{b} \cdot \vec{z}\right)\left(\vec{z} - \vec{x}\right)$

B. $\vec{a} = \left(\vec{a} \cdot \vec{y}\right)\left(\vec{y} - \vec{z}\right)$

C. $\vec{a} \cdot \vec{b} = -\left(\vec{a} \cdot \vec{y}\right)\left(\vec{b} \cdot \vec{z}\right)$

D. $\vec{a} = \left(\vec{a} \cdot \vec{y}\right)\left(\vec{z} - \vec{y}\right)$

Answer:

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8. From a point $P(\lambda, \lambda, \lambda)$, perpendicular PQ and PR are drawn respectively on the lines $y = x, z = 1$ and $y = -x, z = -1.$ If P is such tthat $\angle QPR$ is a right angle , then the possible value(s) of λ is (are)

A. $\sqrt{2}$

B. 1

C. -1

D. $-\sqrt{2}$

Answer:

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9. Let $\overrightarrow{A}$ be vector parallel to line of intersection of planes P_1 and P_2 through origin. P_1 is parallel to the vector $2\hat{j} + 3\hat{k}$ and $4\hat{j} - 3\hat{k}$ and P_2 is parallel to $\hat{j} - \hat{k}$ and $3\hat{i} - 3\hat{j}$, then the angle between the vectors $\overrightarrow{A}$ and $2\hat{i} + \hat{j} - 3\hat{k}$ is :

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{3\pi}{4}$

Answer:

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10. Let $\alpha = a\hat{i} + b\hat{j} + c\hat{k}, \beta = b\hat{i} + c\hat{j} + a\hat{k}$ and $\gamma = c\hat{i} + a\hat{j} + b\hat{k}$ be three coplanar vectos with $a \neq b$ and $v = \hat{i} + \hat{j} + \hat{k}$. Then v is perpendicular to

A. $\overrightarrow{\alpha}$

B. $\overrightarrow{\beta}$

C. $\overrightarrow{\gamma}$

D. $\overrightarrow{\alpha} + \overrightarrow{\beta} + \overrightarrow{\gamma}$

Answer:

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11. If $\overrightarrow{a} = x\hat{i} + y\hat{j} + z\hat{k}, \overrightarrow{b} = y\hat{i} + z\hat{j} + x\hat{k}, \overrightarrow{c} = z\hat{i} + x\hat{j} + y\hat{k},$

then $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$ is :

A. parallel to $(y - z)\hat{i} + (z - x)\hat{j} + (x - y)\hat{k}$

B. orthogonal to $\hat{i} + \hat{j} + \hat{k}$

C. orthogonal to $(y + z)\hat{i} + (z + x)\hat{j} + (x + y)\hat{k}$

D. parallel to $\hat{i} + \hat{j} + \hat{k}$

Answer:

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12. Let $\vec{a}$ and $\vec{b}$ be two non- zero perpendicular vectors. A vector $\vec{r}$ satisfying the equation $\vec{r} \times \vec{b} = \vec{a}$ can be

a $.b - \dfrac{a \times b}{|b|^2}$

b. $2b - \dfrac{a \times b}{|b|^2}$

c. $|a|b - \dfrac{a \times b}{|b|^2}$

d. $|b|b - \dfrac{a \times b}{|b|^2}$

A. $\vec{b} = \dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$

B. $2\vec{b} = \dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$

C. $\left|\vec{a}\right|\vec{b} = \dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$

D. $\left|\vec{b}\right|\vec{b} = \dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$

Answer:

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13. Let a plane pass through orogin and is parallel to the line: $\dfrac{x-1}{2} = \dfrac{y+3}{-1} = \dfrac{z+1}{-2}$ such that distance between plane and the line is $\frac{5}{3}$.Then equation of the plane is :

A. $x - 2y + 2z = 0$

B. $x - 2y - 2z = 0$

C. $2x + 2y + z = 0$

D. $x + y + z = 0$

Answer:

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14. A non-zero vectors a is parallel to the line of intersection of the plane determined by the vectors $\hat{i}, \hat{i} + \hat{j}$ and the plane determined by the vectors $\hat{i} - \hat{j}$ and $\hat{i} + \hat{k}$. Find the angle between a and $\hat{i} - 2\hat{j} + 2\hat{k}$.

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{2\pi}{3}$

D. $\frac{3\pi}{4}$

Answer:

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15. For two events A and B, let $P(A) = \frac{3}{5}, P(B) = \frac{2}{3}$, then which of the following statements is correct?

A. $P\left(A \cap \overline{B}\right) \leq \frac{1}{3}$

B. $P(A \cup B) \geq \frac{2}{3}$

C. $\frac{4}{15} \leq P(A \cap B) \leq \frac{3}{5}$

D. $\frac{2}{5} \leq P\left(\frac{A}{B}\right) \leq \frac{9}{10}$

Answer:

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16. If A and B are two events such that $P(B) \neq 1$, B^c denotes the event complementary to B, then :

A. $P\left(\frac{A}{B^c}\right) = \frac{P(A) - P(A \cap B)}{1 - P(B)}$

B. $P(A \cap B) \geq P(A) + P(B) - 1$

C.

D.

Answer:

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17. Suppose that $\vec{p}$, $\vec{q}$ and $\vec{r}$ are three non-coplanar vectors in R^3. Let the components of a vector $\vec{s}$ along $\vec{p}$, $\vec{q}$ and $\vec{r}$ be 4,3 and 5 respectively. If the componenets of ths vector $\vec{s}$ along

$$\left(-\vec{p}+\vec{q}+\vec{r}\right), \left(\vec{p}-\vec{q}+\vec{r}\right) \text{ and } \left(-\vec{p}-\vec{q}+\vec{r}\right)$$

are x, y and z respectively, then the value of $2x+y+z$ is.

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18. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-coplanar unit vectors such that the angle bet ween every pair of them is $\frac{\pi}{3}$. If $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}=p\vec{a}+q\vec{b}+r\vec{c}$, where p,q and r are scalars, then the value of $\frac{p^2+2q^2+r^2}{q^2}$ is :

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19. The square of the shortest distance between the lines : $\overrightarrow{r} = s\left(\hat{i} - \hat{j} - \hat{k}\right)$ and $\overrightarrow{r} = 3\hat{j} + t\left(\hat{i} - \hat{k}\right)$ is :

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20. Write the set A={x:x is an integer,−1≤x≤4} in roster form.

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21. If $\left|\overrightarrow{a}\right| = 3, \left|\overrightarrow{b}\right| = 1$ and $\left|\overrightarrow{a} + \overrightarrow{b}\right| = \left|\overrightarrow{a} - \overrightarrow{b}\right|$, then $\left|\overrightarrow{a} \times \overrightarrow{b}\right|$ is:

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22. If $\left|\overrightarrow{a}\right| = \left|\overrightarrow{b}\right| = \left|\overrightarrow{c}\right| = 1$ and $\overrightarrow{a} \cdot \overrightarrow{b} = \overrightarrow{b} \cdot \overrightarrow{c} = \overrightarrow{c} \cdot \overrightarrow{a} = \frac{1}{2}$, then the value of $4\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$ is:

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23. Consider the lines : $L_1 : \frac{x-7}{3} = \frac{y-7}{2} = \frac{z-3}{1}$ and $L_2 : \frac{x-1}{2} = \frac{y+1}{4} = \frac{z+1}{3}$. If a line 'L' whose direction ratios are $< 2, 2, 1 >$ intersects the lines L_1 and L_2 at A and B respectively, then the distance of $\frac{AB}{2}$ is:

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24. Value of λ so that the planes: $x - y + z + 1 = 0, \lambda x + 3y + 2z - 3 = 0, 3x + \lambda y + z - 2 = 0$ form a triangular prism is :

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25. If A and B are two events of an expeiment with $P(A) = \frac{1}{4}, P(A \cup B) = \frac{1}{2}$, then $3P\left(\frac{B}{A}\right)$ is :

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26. One function is selected at random from all functions of the set $S = \{1, 2, 3, ..., n\}$ to itself. If, probability that it is one-one is $\frac{3}{32}$, then n is :

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27. Football teams T_1 and T_2 have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of T_1 winning. Drawing and

losing a game against T_2 are $\frac{1}{2}, \frac{1}{6}$ and $\frac{1}{3}$ respectively. Each team gets 3 points for a win. 1 point for a draw and 10 pont for a loss in a game. Let X and Y denote the total points scored by teams T_1 and T_2 respectively. after two games.

$P(X = Y)$ is

A. $\frac{1}{4}$

B. $\frac{5}{12}$

C. $\frac{1}{2}$

D. $\frac{7}{12}$

Answer:

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28. Football teams T_1 and T_2 have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of T_1 winning. Drawing and losing a game against T_2 are $\frac{1}{2}, \frac{1}{6}$ and $\frac{1}{3}$ respectively. Each team gets 3 points for a win. 1 point for a draw and 10 pont for a loss in a game. Let X and Y denote the total points scored by teams T_1 and T_2 respectively. after two games.

$P(X = Y)$ is

A. $\frac{11}{36}$

B. $\frac{1}{3}$

C. $\frac{13}{36}$

D. $\frac{1}{2}$

Answer:

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29. Find the S.D. between the lines : $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$

A. $\sqrt{30}$

B. $6\sqrt{10}$

C. $3\sqrt{30}$

D. $9\sqrt{10}$

Answer:

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30. Find the S.D. between the lines : $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$

A. (1,8,2)

B. (3,8,3)

C. (-3,8,3)

D. none of these

Answer:

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31. Box 1 contains three cards bearing number 1,2,3 , box 2 contains five cards bearing numbers 1,2,3,4,5 and box 3 contains seven cards bearing numbers 1,2,3,4,5,6,7. A card is drawn from each of the boxes. Let x_i be the number on the card drawn from the ith box , $i = 1, 2, 3$.

The probability that x_1, x_2 and x_3 are in arithmetic progression , is

A. $\frac{29}{105}$

B. $\frac{53}{105}$

C. $\frac{57}{105}$

D. $\frac{1}{2}$

Answer:

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32. Box 1 contains three cards bearing number 1,2,3 , box 2 contains five cards bearing numbers 1,2,3,4,5 and box 3 contains seven cards bearing numbers 1,2,3,4,5,6,7. A card is drawn from each of the boxes. Let x_i be the number on the card drawn from the ith box , $i = 1, 2, 3.$

The probability that x_1, x_2 and x_3 are in arithmetic progression , is

A. $\frac{9}{105}$

B. $\frac{10}{105}$

C. $\frac{11}{105}$

D. $\frac{7}{105}$

Answer:

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33. A fair die is tossed repeated until a six is obtained. Let X denote the number of tosses required.

The probability that $X = 3$ is

A. $\frac{25}{216}$

B. $\frac{25}{36}$

C. $\frac{5}{36}$

D. $\frac{125}{216}$

Answer:

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34. A fair die is tossed repeatedly until a 6 is obtained. Let X denote the number of tosses rerquired.

The probability that X ≥ 3 equals

A. $\frac{125}{216}$

B. $\frac{25}{36}$

C. $\frac{5}{36}$

D. $\frac{25}{216}$

Answer:

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35. Let n_1 and n_2 be the number of red and black balls respectively, in box I. Let n_3 and n_4 be the number of red and black ball respctively, in box II.: One of the two boxes , box I andbox II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red.

If the probability that this red ball was drawn from box II is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1, n_2, n_3 and n_4 is (are):

A. $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$

B. $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$

C. $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$

D. $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$

Answer:

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36. Let n_1 and n_2 be the number of red and black balls respectively, in box I. Let n_3 and n_4 be the number of red and black ball respctively, in box II.: One of the two boxes , box I andbox II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red.

If the probability that this red ball was drawn from box II is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1, n_2, n_3 and n_4 is (are):

A. $n_1 = 4$ and $n_2 = 6$

B. $n_1 = 2$ and $n_2 = 3$

C. $n_1 = 10$ and $n_2 = 20$

D. $n_1 = 3$ and $n_2 = 6$

Answer:

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37. For all sets A and B, $(A - B) \cup (A \cap B) = A$. Is this statement true?

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38. For all sets A, B and C, if $A \subset C$ and $B \subset C$, then $A \cup B \subset C$.

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