



## MATHS

### BOOKS - MODERN PUBLICATION

### MOCK TEST-3

#### Exercise

1. Write the element  $a_{12}$  of the matrix  $A = [a_{ij}]_{2 \times 2}$ ,

whose elements are given by :  $a_{ij} = e^{2ix} \sin jx$



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2. Given :  $\int e^x (\tan x + 1) \sec x dx = e^x f(x) + c$ , write  $f(x)$  satisfying the above.

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3. Determine order and degree (if defined) of differential equation:  $\frac{d^4 y}{dx^4} + \sin(y''') = 0$

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4. Find  $|\vec{x}|$ , if for a unit vector  $\vec{a}$ ,  $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 15$

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5. By using elementary transformations, find the

inverse of the matrix  $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$ .



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6. Using determinant , find the area of triangle with

verticles

$(1,0)$  ,  $(6,0)$  , $(4,3)$



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7. Find  $\frac{dy}{dx}$  if  $x = a(\theta + \sin \theta)$ ,  $y = a(1 - \cos \theta)$



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8. Prove that the function  $f$  given by  $f(x) = \log \sin x$  is strictly increasing on  $\left(0, \frac{\pi}{2}\right)$  and strictly decreasing on  $\left(\frac{\pi}{2}, \pi\right)$ .



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9. Find the equation of the tangent to the curve  $x^2 + 3y = 3$ , which is parallel to the line  $y - 4x + 5 = 0$



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10. Evaluate :  $\int \left[ \log(\log x) + \frac{1}{(\log x)^2} \right] dx.$

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11. Solve :  $xyy' = 1 + x + y + xy.$

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12. If  $\vec{a}$  is any vector in space, show that :

$$\vec{a} = (\vec{a} \cdot \hat{i})\hat{i} + (\vec{a} \cdot \hat{j})\hat{j} + (\vec{a} \cdot \hat{k})\hat{k}.$$

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13. Prove that :

$$2 \tan^{-1} \left( \frac{1}{2} \right) + \tan^{-1} \left( \frac{1}{7} \right) = \tan^{-1} \left( \frac{31}{17} \right)$$

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14. Solve:  $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$

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15. If  $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ , show that  $A^2 - 4A + 3I = 0$ .

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16. If  $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ , prove that :

$$A^{-1} = A^2 - 6A + 11I$$

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17. Without expanding, prove the following

$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$

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**18.** If the function defined by :

$$f(x) = \begin{cases} 2x - 1 & x < 2 \\ a & x = 2 \\ x + 1 & x > 2 \end{cases} \text{ is continuous at } x = 2, \text{ find}$$

the value of 'a'. Also discuss the continuity of  $f(x)$  at  $x = 3$ .

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**19.** Differentiate the following w.r.t.x.

$$x^{\cot x} + (\sin x)^x.$$

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20. Evaluate :  $\int \frac{5x + 3}{\sqrt{x^2 + 4x + 10}} dx$

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21. Evaluate :  $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx.$

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22. Evaluate :  $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x + 4 \sin^2 x} dx$

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**23.** For the differential equation, find the particular solution satisfying the given condition:

$$(x + y)dy + (x - y)dx = 0, y = 1 \text{ when } x = 1$$



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**24.** Find the vector equation of the line parallel to the line :  $\frac{x - 1}{5} = \frac{3 - y}{2} = \frac{z + 1}{4}$  and passing through  $(3,0,-4)$ . Also find the distance between these two lines.



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**25.** Find the equation of the plane passing through the point  $(1, -1, 2)$  and perpendicular to the planes  $2x + 3y - 2z = 5$  and  $x + 2y - 3z = 8$ .



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**26.** A speaks truth in 60% of the cases, while B in 90% of the cases. In what percentage are they likely to contradict each other in stating the same fact? In the case of contradiction do you think, the statement of B will carry more weight as he speaks truth in more number of case than A?



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27. Consider  $f: \mathbb{R} \rightarrow [-5, \infty)$  given by  $f(x) = 9x^2 + 6x - 5$ . Show that  $f$  is invertible. Find the inverse of  $f$ .



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28. Consider the binary operations  $*$  :  $\mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$  and  $\circ$  :  $\mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$  defined as  $a * b = |a-b|$  and  $a \circ b = a$  for all  $a, b$  in  $\mathbb{R}$ . Show that  $*$  is commutative but not associative,  $\circ$  is associative but not commutative.



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**29.** Show that a cylinder of given volume, open at the top, has minimum total surface area if its height is equal to radius of the base.

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**30.** Using integration find the area of region bounded by the triangle where vertices are :  $(1,3)$ ,  $(2,5)$  and  $(3,4)$

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**31.** Show that the points :

$$A(-2\hat{i} + 3\hat{j} + 5\hat{k}), B(\hat{i} + 2\hat{j} + 3\hat{k}) \text{ and } C(7\hat{i} - \hat{k})$$

are collinear.



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32. Show that the lines

$$\frac{x - a + d}{\alpha - \delta} = \frac{y - a}{\alpha} = \frac{z - a - d}{\alpha + \delta} \quad \text{and}$$
$$\frac{x - b + c}{\beta - \gamma} = \frac{y - b}{\beta} = \frac{z - b - c}{\beta + \gamma} \quad \text{are coplanar.}$$



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33. Two tailors A and B earn Rs 150 and Rs 200 per day respectively. A can stitch 6 shirts and 4 pants while B can stitch 10 shirts and 4 pants per day. How many days shall each work if it is desired to produce

(at least) 60 shirts and 32 pants at a minimum labour cost? Solve the problem graphically.



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**34.** A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed 3 times, find the probability distribution of number of tails.



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**35.** There are three coins, one is a two-headed coin (having head on both the faces), another is a biased coin that comes up heads 75% of the time and the

third is an unbiased coin. One of the three coins is chosen at random and tossed. If it shows head, what is the probability that it was the two-headed coin?



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