MATHS

BOOKS - ACCURATE PUBLICATION

SAMPLE QUESTION PAPER-V

Section A

1. Show that the relation R in the set {1,2,3} given by R = {(1,2),(2,1)} is symmetric but neither reflexive nor transitive.

A. Symmetric

B. Reflective

C. Transitive

D. Symmetric and transitive

Answer: A

2. Choose the correct option in the question :

$\sin(\tan^{-1} x), |x| < 1$ is equal to

A. $\dfrac{x}{\sqrt{1 - x^2}}$

B. $\dfrac{1}{\sqrt{1 - x^2}}$

C. $\dfrac{1}{\sqrt{1 + x^2}}$

D. $\dfrac{x}{\sqrt{1 + x^2}}$

Answer: D

3. Choose the correct option in the question :

If $\begin{bmatrix} x + y + z \\ x + z \\ y + z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$, then $(x + y)$ equals

A. 4

B. 5

C. 6

D. 7

Answer: C

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4. Choose the correct option in the question :

If $A = \begin{pmatrix} 3 & 2 & 1 \end{pmatrix}$ then A A' is equal to

A. $\begin{pmatrix} 9 & 4 & 1 \end{pmatrix}$

B. $\begin{pmatrix} 9 \\ 4 \\ 1 \end{pmatrix}$

C. -14

D. -6

Answer: C

5. Choose the correct option in the question :

If $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 7 & 11 \\ k & 23 \end{pmatrix}$, then k equals

A. 15

B. 16

C. 17

D. 18

Answer: C

6. Find k, so that $f(x) = \begin{cases} \frac{\sin kx}{x}, & x \neq 0 \\ 4 + x, & x = 0 \end{cases}$ is continous at $x = 0$

A. 4

B. 5

C. 6

D. 7

Answer: A

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7. Choose the correct option in the question :

The derivative of $f(x) = |x| at x = 3$ equals

A. 1

B. -1

C. 0

D. 2

Answer: A

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8. Choose the correct option in the question :

Derivative of $\sin^{-1}(e^x)$ w.r.t. x is

A. $\dfrac{e^x}{\sqrt{1+e^2x}}$

B. $\dfrac{e^x}{\sqrt{1-e^{2x}}}$

C. $\dfrac{1}{\sqrt{1+e^{2x}}}$

D. $\dfrac{1}{\sqrt{1-e^{2x}}}$

Answer: B

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9. Choose the correct option in the question :

$\displaystyle\int \dfrac{dx}{e^x + e^{-x}}$ is equal to

A. $\tan^{-1}(e^x) + C$

B. $\tan^{-1}(e^{-x}) + C$

C. $\log(e^x - e^{-x}) + C$

D. $\log(e^x + e^{-x}) + C$

Answer: A

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10. $\int_0^1 \frac{2x}{5x^2 + 1} dx = \log 6$

A. $\frac{1}{2}\log 6$

B. $\frac{1}{3}\log 6$

C. $\frac{1}{4}\log 6$

D. $\frac{1}{5}\log 6$

Answer: D

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11. A homogeneous differential equation of the form $\dfrac{dx}{dy} = h\left(\dfrac{x}{y}\right)$ can

be solved by making the substitution.

    A. $y = ux$

    B. $u = yx$

    C. $x = uv$

    D. $x = v$

Answer: C

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12. Choose the correct option in the question :

If of $\dfrac{dy}{dx} + \dfrac{y}{x} = x^2$ s

    A. x

    B. 2x

    C. log x

D. log 2x

Answer: A

13. Choose the correct option in the question :

The projection of $\vec{a} = 2\hat{i} - 2\hat{j} - \hat{k}$ on $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ is equal to

A. $\dfrac{5\sqrt{6}}{3}$

B. $\dfrac{6}{\sqrt{14}}$

C. $\dfrac{\sqrt{6}}{14}$

D. $\dfrac{5}{\sqrt{6}}$

Answer: B

14. If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then show that the vectors $\left(\vec{a} + \vec{b}\right)$ and $\left(\vec{a} - \vec{b}\right)$ are perpendicular.

A. $30°$

B. $45°$

C. $60°$

D. $90°$

Answer: D

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15. Distance between the planes $2x + 3y + 4z = 4$ and $4x + 6y + 8z = 12$ is :

A. 2 units

B. 4 units

C. 8 units

D. $\dfrac{2}{\sqrt{29}}$ units

Answer: D

16. If $P(A) = \dfrac{6}{11}, P(B) = \dfrac{5}{11}$ and $P(A \cup B) = \dfrac{7}{11}$ find: $P(A \mid B)$

A. $\dfrac{1}{5}$

B. $\dfrac{2}{5}$

C. $\dfrac{3}{5}$

D. $\dfrac{4}{5}$

Answer: D

17. Fill in the blank from the given option

0.32, symmetric ,

$\frac{1}{2}\sec^2\frac{x}{2}, \; x + y - 3 = 0, \; x + y - \;\; = 2, \; \frac{1}{2}\log\left|2x^2 + 6x + 5\right| + C, \{-1, 1\}$

Range of the function $f(x) = \dfrac{|x - 3|}{x - 3}$ is ....

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18. Fill in the blank from the given option

0.32, symmetric ,

$\frac{1}{2}\sec^2\frac{x}{2}, \; x + y - 3 = 0, \; x + y - \;\; = 2, \; \frac{1}{2}\log\left|2x^2 + 6x + 5\right| + C, \{-I, 1\}$

The matrix $A \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a........matrix.

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19. Fill in the blank from the given option

0.32, symmetric ,

$\frac{1}{2}\sec^2\frac{x}{2}, x+y-3=0, x+y- = 2, \frac{1}{2}\log|2x^2+6x+5|+C, \{-I,1\}$

Differentiation of : $\sqrt{\dfrac{1-\cos x}{1+\cos x}}$ w.r.t.x is ....

20. Find the equation of the normal to curve $y^2 = 4x$ at the point (1,2)

21. Fill in the blank from the given option

Find $\displaystyle\int \dfrac{2x+3}{2x^2+6x+5}\,dx = $ ....

22. Fill in the blank from the given option

0.32,   symmetric   ,

$\frac{1}{2}\sec^2\frac{x}{2}, x+y-3=0, x+y- = 2, \frac{1}{2}\log|2x^2+6x+5|+C, \{-I,1\}$

I.F. of $\dfrac{dy}{dx} + 2y = \sin x$ is ....

23. Find the Cartesian equation of the following plane:

$$\vec{r} \cdot \left(\hat{i} + \hat{j} - \hat{k}\right) = 2$$

24. Fill in the blank from the given option

0.32, symmetric ,

$\frac{1}{2}\sec^2\frac{x}{2}$, $x + y - 3 = 0$, $x + y - = 2$, $\frac{1}{2}\log|2x^2 + 6x + 5| + C$, $\{ - I, 1\}$

If $P(A) = 0.8\, P(B) = 0.5$, $P(B \mid A) = 0.4$ then $P(A \cap B)$ is ....

25. State true of false for the statement

$$\sin^{-1}\left(2\sin^{-1}x\right) = 2x\sqrt{1 - x^2}$$

26. State true or false for the statement

The matrix $\begin{bmatrix} 3 - 2x & x + 1 \\ 2 & 4 \end{bmatrix}$ is singular for x =2.

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27. Differentiate the following w.r.t.x.

$\sin^{-1}(3x - 4x^3), \quad -\dfrac{1}{2} < x < \dfrac{1}{2}$

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28. Solve the following integration

$\displaystyle\int \dfrac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} \cdot dx, \text{ here } (\sin x + \cos x) > 0$

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29. State true of false for the statement

The value of p if $\left(2\hat{i} + 6\hat{j} + 27\hat{k}\right) \times \left(\hat{i} + 3\hat{j} + p\hat{k}\right) = \overrightarrow{0}$ is $\dfrac{11}{2}$

30. State true of false for the statement

The equation of a plane parallel to $2x - 3y + z + 9 = 0$ and passing through origin is $2x - 3y + z = 0$.

31. State true of false for the statement

If a leap year is selected at random, the chance that it will contains 53 Tuesdays is $\frac{2}{7}$.

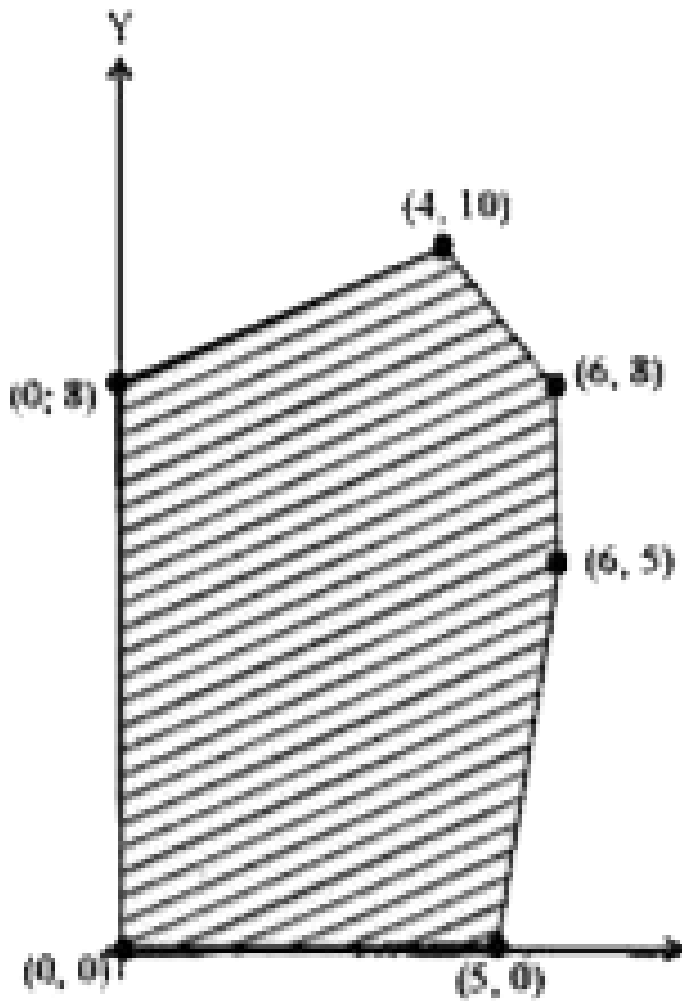**32.** State true of false for the statement

The feasible solution for LPP is shown in Let. $Z = 3x - 4y$ be objective

function. (Maximum value of Z+Minimum vlaue of Z) is equal - 13

1. If $3A - B = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, then find the matrix A.

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2. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, show that $A^{-1} = \dfrac{1}{19} A$

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3. Find the slope of the tangent to the cuve $y = 3x^4 - 4x \, at \, x = 1$

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4. Prove that the function $f(x) = x^3 - 6x^2 + 12x - 18$ is increasing on R.

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5. Evaluate $\int \dfrac{1}{x + x \log x}\,dx$.

6. Find the area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$.

7. The two vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represent the two sides AB and AC respectively of a $\triangle ABC$. Find the lengthof the median through A

8. If a unit vector $\overrightarrow{a}$ makes angle $\dfrac{\pi}{4}$ with $\hat{i}$, $\dfrac{\pi}{3}$ with $\hat{j}$ and acute angle θ with $\hat{k}$ then find the component of $\overrightarrow{a}$ ang the angle θ.

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Section C

1. Show that : $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\sin^{-1}x, \quad -\dfrac{1}{\sqrt{2}} \le x \le \dfrac{1}{\sqrt{2}}$

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2. Prove that $\cos^{-1}\dfrac{12}{13} + \sin^{-1}\dfrac{3}{5} = \sin^{-1}\dfrac{56}{65}$

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3. If $x^{16}.\, y^9 = \left(x^2 + y\right)^{17}$, prove that $\dfrac{dy}{dx} = \dfrac{2y}{x}.$

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4. Evaluate $\displaystyle\int \dfrac{2}{(1-x)(1+x^2)}\,dx$

5. Show that: $\int_0^{\frac{3}{2}} |x \cos \pi x| dx = \dfrac{5\pi - 2}{2\pi^2}$

6. Solve the followng differential equation $(y^2 - x^2) dy = 3xy dx$.

7. A and B throw a pair of dice alternately. A wins the game if he gets a total of 7 and B wins if he gets a total of 10. If A starts the game, find the probability that B wins.

8. A bag contains (2n +1) coins. It is known that n of these coins have a head on both side whereas the rest of the coins are fair A coin is plaked up at random from the bag and is tossed. If the probability that the toss results in a head is $\dfrac{31}{42}$, deter8mine the value of n .

Section D

1. If x,y,z are different and $\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, show that xyz=-1

2. Using matrix method , solve the equations

$$x - y + z = 4$$

$$2x + y - 3z = 0$$

$$x + y + z = 2$$

3. Find the value of equation of a line passing through the point (2,3,2) and parallel to the line $\vec{r} = \left(-2\hat{i} + 3\hat{j} \right) + \lambda\left(2\hat{i} - 3\hat{j} + 6\hat{k} \right)$. Also , find the distance between these two lines

4. Find the shortest distance between the lines $\vec{r} = \left(\hat{i} + 2\hat{j} + \hat{k} \right) + \lambda\left(\hat{i} - \hat{j} + \hat{k} \right)$ and $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\left(2\hat{i} + \hat{j} + 2\hat{k} \right)$

5. Maximum and minimise $z = 3x + 2y - 3$ subject to the constaints

$x + y \geq 4, x + y \leq 12, x \leq \ , y \leq 9, x, y \geq 0$

6. Graphically minimize and maximize $z = 7x + 5y$ subject to the constraints: $x + y \leq 50$, $x + 2y \leq 100$, $x - y \geq 10x$, $y \geq 0$.

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1. Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4,4), (1, 3), (3, 3), (3, 2)}. Choose the correct answer.

    A. R is reflexive and symmetric but not transitive.

    B. R is reflexive and transitive but not symmetric.

    C. R is symmetric and transitive but not reflexive.

    D. R is an equivalence relation.

Answer: B

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2. Find the principal value of $\cot^{-1}\left(\sqrt{3}\right)$

A. $\sqrt{2}$

B. $\sqrt{3}$

C. $\sqrt{5}$

D. 1

Answer: B

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3. If the matrix A is both symmetric and skew symmetric, then :

A. A is a diagonal matrix

B. A is zero matrix

C. A is a square matrix

D. None of these

Answer: B

4. If $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, then $A^2 - 3$ is:

A. 2A

B. A is zero matrix

C. 3A

D. 4A

Answer: A

5. The value of the determinant $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$ is

A. 1

B. 2

C. 0

D. 4

Answer: C

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6. The value of k so that the function $f(x) = \begin{cases} \frac{\sin 2x}{x} & \text{if } x \neq 0 \\ k & \text{if } x = 0 \end{cases}$ is continous at x=0 is:

A. 0

B. 1

C. 3

D. 2

Answer: D

7. The derivative of $\sqrt{\log(\sin x)}$ w.r.t. x is

A. $\dfrac{1}{2\sqrt{\cot x}}$

B. $\dfrac{1}{2}\sec^2 x$

C. $\dfrac{\sec x}{2\sin x \sqrt{\log(\tan x)}}$

D. None of these

Answer: C

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8. If $x = 4at, y = at^4$, then $\dfrac{dy}{dx}$ is equal to :

A. 4

B. 4a

C. 4at

D. t^3

Answer: D

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9. $\int \dfrac{dx}{x^2 + 2x + 2}$ equals :

A. $x \tan^{-1}(x + 1) + C$

B. $\tan^{-1}(x + 1) + C$

C. $(x + 1)\tan^{-1} + C$

D. $\tan^{-1} x + C$

Answer: B

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10. $\int_{-3}^{3} (x + \sin x)dx$ is equal to :

A. 0

B. 1

C. 2

D. 3

Answer: A

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11. Find the $\dfrac{dy}{dx} + y \cot x = 2 \cos x$

A. cos x

B. sin x

C. $- \cos x$

D. $- \sin x$

Answer: B

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12. The Integrating Factor of the differentiate equation $\dfrac{dy}{dx} - 2y = 3x$ is :

A. e^{2x}

B. e^{-2x}

C. e^{x}

D. $2x$

Answer: B

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13. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a} + \vec{b}$ is a unit vector if :

A. $\theta = \dfrac{\pi}{4}$

B. $\theta = \dfrac{\pi}{3}$

C. $\theta = \dfrac{\pi}{2}$

D. $\theta = \dfrac{2\pi}{3}$

Answer: D

14. Find $\left| \vec{a} \ X \ \vec{b} \right|$,if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

A. $7\sqrt{2}$

B. $10\sqrt{2}$

C. $19\sqrt{2}$

D. $21\sqrt{2}$

Answer: C

15. Distance between plane defined by 3y + 4z + 10 = 0 and the point (7, 5, 0) is :

    A. 3 units

    B. 4 units

    C. 5 units

    D. 6 units

Answer: D

16. Probability that A speaks truth is $\dfrac{4}{5}$. A coin is tossed. A reports that a head appears. The probability that actually there was head is

    A. $\dfrac{4}{5}$

    B. $\dfrac{1}{2}$

    C. $\dfrac{1}{5}$

D. $\dfrac{2}{5}$

Answer: A

Section A Fill In The Blanks

1. Fill in the blanks from the given options:

0.15, $\dfrac{x+2}{3} = \dfrac{y-4}{5} = \dfrac{z+8}{6}$, $\dfrac{1}{4\sqrt{a+\sqrt{a+x}}\sqrt{a+x}}$, $-\dfrac{1}{3}, \dfrac{1}{2}(e-1)$

, Symemetric, $\dfrac{19}{2}$ sq. units,

Let L be the set of all lines in a plane and R be the relation in L defined as

$R = \{(L_1, L_2) : L_1$ is perpendicular to $L_2\}$. Show that R is..............

2. Fill in the blanks from the given options:

0.15, $\dfrac{x+2}{3} = \dfrac{y-4}{5} = \dfrac{z+8}{6}$, $\dfrac{1}{4\sqrt{a+\sqrt{a+x}}\sqrt{a+x}}$, $-\dfrac{1}{3}, \dfrac{1}{2}(e-1)$

, Symemetric, $\dfrac{19}{2}$ sq. units,

The area of the triangle, whose vertices are (3,1), (4, 3) and (-5,4) is ....

3. If $y = \sqrt{a + \sqrt{a + x}}$, then $\dfrac{dy}{dx}$ is.............

4. Fill in the blanks from the given options:

$0.15,\ \dfrac{x+2}{3} = \dfrac{y-4}{5} = \dfrac{z+8}{6},\ \dfrac{1}{4\sqrt{a+\sqrt{a+x}}\sqrt{a+x}},\ -\dfrac{1}{3}, \dfrac{1}{2}(e-1)$

, Symemetric, $\dfrac{19}{2}$ sq. units,

The slope of the normal to the curves = $2x^2 + 3\sin x$ at x=0 is..............

5. Fill in the blanks from the given options:

$0.15,\ \dfrac{x+2}{3} = \dfrac{y-4}{5} = \dfrac{z+8}{6},\ \dfrac{1}{4\sqrt{a+\sqrt{a+x}}\sqrt{a+x}},\ -\dfrac{1}{3}, \dfrac{1}{2}(e-1)$

, Symemetric, $\dfrac{19}{2}$ sq. units,

$\displaystyle\int_0^1 xe^{x^2}dx$ equals to...........

6. Fill in the blanks from the given options:

0.15, $\dfrac{x+2}{3}=\dfrac{y-4}{5}=\dfrac{z+8}{6}$, $\dfrac{1}{4\sqrt{a+\sqrt{a+x}}\sqrt{a+x}}$, $-\dfrac{1}{3},\dfrac{1}{2}(e-1)$

, Symemetric, $\dfrac{19}{2}$ sq. units, 1

The degree of the differential equation $5x\left(\dfrac{dy}{dx}\right)^2-\dfrac{d^2y}{dx^2}-6y=\log x$

is.........

7. Find the cartesian equation of the line which passes through the point

$(-2,4,-5)$ and parallel to the line given by $\dfrac{x+3}{3}=\dfrac{y-4}{5}=\dfrac{z+8}{6}$

8. Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the values x, has the following form, where k is some unknown constant.:

$$P(X = x) = \begin{cases} 0.1 & \text{if} & x = 0 \\ kx & \text{if} & x = 1 \text{ or } 2 \\ k(5 - x) & \text{if} & x3 \text{ or } 4 \\ 0 & & otherwise \end{cases}$$

Find the value of k.

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Section A True Or False

1. Write in the simplest form : `tan^-1 (sin x/(1 + cos x))

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2. Prove inverse of a matrix, if it exists, is unique.

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3. Differentiate the following w.r.t.x.

$$\tan^{-1}\left(\frac{\sqrt{1-\cos 2x}}{\sqrt{1+\cos 2x}}\right)$$

4. $\int e^x (\sin x + \cos x)\,dx$ is equal to :

5. Find λ and μ if : $\left(2\hat{i} + 6\hat{j} + 27\hat{k}\right) \times \left(\hat{i} + \lambda\hat{j} + \mu\hat{k}\right) = \overrightarrow{0}$.

6. Find the direction cosines of the unit vector perpendicular to the plane $\overrightarrow{r} \cdot \left(6\hat{i} - 3\hat{j} - 2\hat{k}\right) + 1 = 0$ passing through the origin.
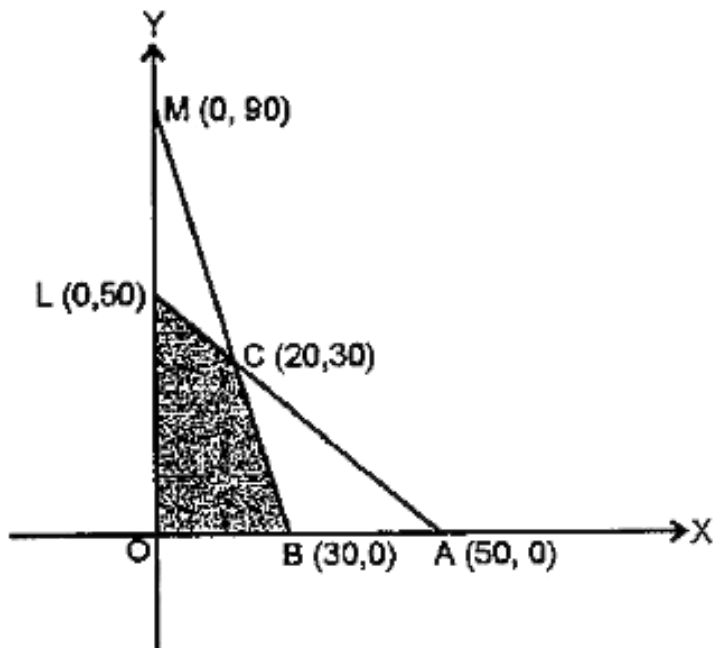
7. If $P(A) = \dfrac{3}{5}, P(B) = \dfrac{1}{5}$ find $P(A \cap B)$ if A and B are independent events.

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8. The feasible solution for a LPP is shown in Fig. Let $Z = 4x + y$ be the objective function. Maximum value of Z is 130.

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1. Find a matrix A such that $2A - 3B + 5C = O$, where $B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$

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2. For what value of x, the matrix $\begin{bmatrix} 1+x & 7 \\ 3-x & 8 \end{bmatrix}$ is a singular matrix?

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3. Find the equations of the tangent and normal to the curve given by :

$x = a\sin^3\theta, y = a\cos^3\theta$ at a point, where $\theta = \dfrac{\pi}{4}$

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4. Prove that $f(x) = 4x^3 - 6x^2 + 3x + 12$ is strictly increasing function on R.

5. Evaluate, $\int \sin^3 x \cos^2 x \, dx$

6. Using integration, find the area of the circle $x^2 + y^2 = 4$

7. X and Y are two points with position vectors $3\overrightarrow{a} + \overrightarrow{b}$ and $\overrightarrow{a} - 3\overrightarrow{b}$ respectively. Write the position vector of point Z which divides the line segment XY in the ratio $2 : 1$ externally.

8. Find λ if $\left(2\hat{i} + 6\hat{j} + 14\hat{k}\right) \times \left(\hat{i} - \lambda\hat{j} + 7\hat{k}\right) = \vec{0}$.

9. Find x and y, if $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

10. If $A = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$, find a matrix C such that 3 A + 5B + 2C is a null matrix.

11. Show that the normal at any point θ to the curve $x = a\cos\theta + a\theta\sin\theta, y = a\sin\theta - a\theta\cos\theta$ is at a constant distance from the origin.

12. Prove that the function / defined by $f(x) = x^2 - x + 1$ is neither increasing nor decreasing in (-1, 1). Hence, find the intervals in which f(x) is:

(i) strictly increasing , (ii) strictly decreasing.

13. Evaluate: $\int \dfrac{\cos(x + a)}{\sin(x + b)} dx.$

14. Using definite integrals, find the area of the circle $x^2 + y^2 = 9$

15. If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}, \ \vec{b} = 3\hat{i} - \hat{j} + 2\hat{k},$ show that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular to each other.

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16. Find a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ if $\vec{a} = 4\hat{i} + 3\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 5\hat{j} - 3\hat{k}$

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17. If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA,$ then write the value of 'k'.

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18. Find the adjoint of the following matrices :

$\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

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19. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is parallel to the line $2x + y + 9 = 0$.

20. Evaluate $\displaystyle\int \frac{dx}{\sqrt{5 - 4x - 2x^2}}$

21. Integrate the following functions : $\dfrac{(x - 3)e^x}{(x - 1)^3}$.

22. Using definite integrals, find the area of the circle $x^2 + y^2 = 16$.

23. For what values of λ are the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} - \hat{k}$ and $\vec{b} = 4\hat{i} - 2\hat{j} - 2\hat{k}$ perpendicular to each other ?

24. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively and when $\left|\vec{a} \times \vec{b}\right| = \sqrt{3}$.

Section C

1. Simplify: $\tan^{-1}\left(\dfrac{\sqrt{1+x^2} - 1}{x}\right)$

2.   Show   that:

$$\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right] = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2, \ -1 < x < 1$$

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3. Find $\dfrac{dy}{dx}$, if $y^x + x^y + x^x = a^b$.

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4. Find $\displaystyle\int \frac{x^2\,dx}{(x\sin x + \cos x)^2}$

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5. Evaluate: $\displaystyle\int_1^4 [|x-1| + |x-2||x-4|]\,dx$.

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6. Solve $xdy - ydx = \sqrt{x^2 + y^2}dx$, given that $y = 0$ when x=1.

7. Solve the differential equation:

$$\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right]\frac{dx}{dy} = 1, (x \neq 0)$$

8. A, B and C throw a pair of dice in that order alternately till one of them gets a total of 9 and wins the game. Find their respective probabilities of winning, if A starts first.

9. In a class, 5% of the boys and 10% of the girls have an IQ of more than 150. In the class 60% of the students are boys and rest girls. If a student

is selected at random and found tohave an IQ of more than 150, find the probability that the student is a boy.

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10. Compute $(AB)^{-1}$, where

$$A\begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}, B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

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11. Use matrix method to solve the following system of equations :

$$x - y + 2z = 7, 3x + 4y - 5z = -5, 2x - y + 3z = 12$$

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12. Find the direction ratios of the normal to the plane, which passes through the points (1, 0, 0) and (0, 1, 0) and makes angle $\dfrac{\pi}{4}$ with the

plane $x + y = 3$. Also find the equation of the plane.

13. Show that the lines :

$$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} \text{ and } \frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} \text{ intersect}$$

each other. Also find their point of intersection.

14. Maximise $z = 22x + 44y$ subject to the constraints

$$x + y \geq 3, \ 3x + 8y \leq 24, \ x - y \geq 0, \ x, y \geq 0$$

15. Solve the following linear programming problem graphically: Minimise

$$Z = 200x + 500y \quad \text{subject} \quad \text{to} \quad \text{the} \quad \text{constraints:}$$

$$x + 2y \geq 10, \ 3x + 4y \leq 24, \ x \geq 0, \ y \geq 0$$

1. Let $R = \{(a, a^3) : a$ is a prime numberless than 5$\}$ be a relation. Then the range of R is

   A. {8,27}

   B. {9,27}

   C. {5,18}

   D. {6,19}

Answer: A

2. The principal value of $\cos^{-1}[\cos(680°)]$ is:

A. 20°

B. 40°

C. 45°

D. None of these

Answer: B

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3. Matrices A and B will be inverse of each other only if

A. AB = BA

B. AB = BA =0

C. AB=0, BA =I

D. AB = BA = I

Answer: D

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4. If $\begin{bmatrix} x - y & 2y \\ 2y + z & x + y \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 9 & 5 \end{bmatrix}$, then the value of $(x + y + z)$ is:

A. 5

B. 8

C. 10

D. 12

Answer: C

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5. For what value of x, is the matrix $A = \begin{bmatrix} 0 & 1 & 4 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skey-symmetric matrix?

A. 2

B. 4

C. 3

D. 5

Answer: B

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6. The value of the constant k so that the function

$$f(x) = \begin{cases} \frac{1 - \cos 2x}{2x^2} & x \neq 0 \\ k & x = 0 \end{cases}$$ is continous at x=0 is:

A. 2

B. 3

C. 4

D. 1

Answer: D

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7. If $y = x^a$, $a \in R$ then $\dfrac{dy}{dx}$ is equal to :

A. $x^a \log x$

B. $x^a \log a$

C. $a^x \log a$

D. ax^{a-1}

Answer: D

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8. When $x = at^2$, $y = 3at$, then $\dfrac{dy}{dx}$ is:

A. $\dfrac{1}{t}$

B. $\dfrac{3}{t}$

C. $\dfrac{2}{t}$

D. None of these

Answer: A

9. Choose the correct answer: $\int \dfrac{e^x(1+x)}{\cos^2(xe^x)}dx$ is equal to:

A. $-\cot(ex^x) + C$

B. $\tan(xe^x) + C$

C. $\tan(e^x) + C$

D. $\cot(e^x) + C$

Answer: B

10. $\int_0^{\frac{\pi}{2}} \sqrt{1 - \sin 2x}dx$ is equal to

A. $\sqrt{2}1$

B. $\sqrt{2} - 1$

C. $\sqrt{2} + 3$

D. $\sqrt{2} - 3$

Answer: B

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11. Which of the following is a homogeneous differential equation?

A. $(4x + 6y + 5)dy - (3y + 2x + 4)dx = 0$

B. $(xy)dx - (x^3 + y^3)dy = 0$

C. $(x^3 + 2y^2)dx + 2xydy = 0$

D. $y^2dx + (x^2 - xy - y^2)dy = 0$

Answer: D

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12. Find the particular solution of the differential equation $\log\left(\dfrac{dy}{dx}\right) = 3x + 4y$ given that $y = 0$ when $x = 0$

A. $\dfrac{1}{3}e^{-4y} = \dfrac{1}{4}e^{3x} - \dfrac{7}{12}$

B. $\dfrac{1}{4}e^{-4y} = \dfrac{1}{3}e^{3x} - \dfrac{7}{12}$

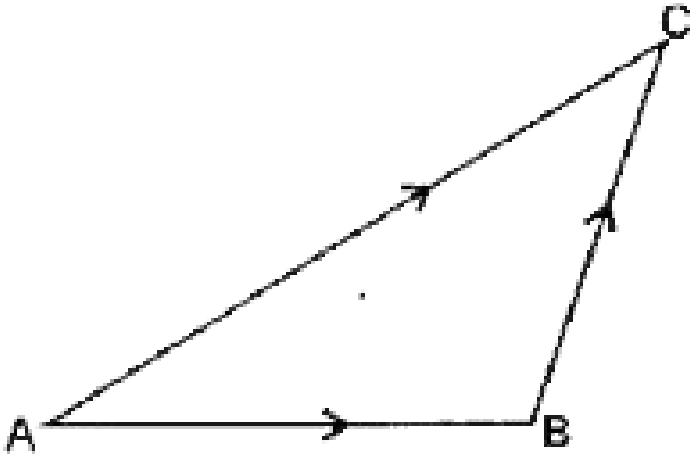C. $\dfrac{1}{4}e^{-4y} = \dfrac{1}{5}e^{3x} - \dfrac{7}{12}$

D. $\dfrac{1}{5}e^{-4y} = \dfrac{1}{3}e^{3x} - \dfrac{7}{12}$

Answer: B

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13. In triangle ABC, which of the following is not true :

A. $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

B. $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$

C. $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \vec{0}$

D. $\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$

Answer: C

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14. If $\vec{a}$ and $\vec{b}$ are unit vectors and $\left(\sqrt{3a} - \vec{b}\right)$ is a unit vector, then the angle between $\vec{a}$ and $\vec{b}$ is:

A. $30°$

B. $40°$

C. $45°$

D. $60°$

Answer: A

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15. Distance between plane defined by $3x + 4z + 15 = 0$ and the point $(5, 2, 0)$ is:

A. 3 units

B. 4 units

C. 5 units

D. 6 units

Answer: B

16. If A and B are two events such that $A \subset B$ and $P(B) \neq 0$, then which of the following is correct?

A. $P(A \mid B) = \dfrac{P(B)}{P(A)}$

B. $P(A \mid B) < P(A)$

C. $P(A \mid B) \geq P(A)$

D. None of these

Answer: C

17. Let A = {0, 1, 2, 3} and define a relation R on A as follows :

$R = \{(0, 0), (0, 1), (0, 3), (1, 0), (1, 1), (2, 2), (3, 0), (3, 3)\}$

Is R reflexive ? symmetric ? transitive ?

    A. Reflexive

    B. Symmetric

    C. Transitive

    D. None of these

Answer: C

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18. Principal value of $\sin^{-1}\left(\dfrac{1}{2}\right) + \cos^{-1}\left(-\dfrac{1}{2}\right)$ is

A. $\dfrac{\pi}{2}$

B. $-\dfrac{\pi}{2}$

C. $\dfrac{3\pi}{2}$

D. None of these

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19. If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then

a. $1 + \alpha^2 + \beta\gamma = 0$

b. $1 - \alpha^2 + \beta\gamma = 0$

c. $1 - \alpha^2 - \beta\gamma = 0$

d. $1 + \alpha^2 - \beta\gamma = 0$

A. $1 + \alpha^2 + \beta\gamma = 0$

B. $1 - \alpha^2 + \beta\gamma = 0$

C. $1 - \alpha^2 - \beta\gamma = 0$

D. $1 + \alpha^2 - \beta\gamma = 0$

Answer: C

20. Solve the matrix equation : $(1 \quad x \quad 1)\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = (0)$

A. $-\dfrac{9}{8}$

B. $-\dfrac{5}{8}$

C. $-\dfrac{4}{9}$

D. $-\dfrac{5}{7}$

Answer: A

21. The matrix $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is

A. a unit matrix

B. a diagonal matrix

C. a symmetric matrix

D. a skew-symmetric matrix

Answer: D

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22. The value of k so that the function $f(x) = \begin{cases} kx^2 & , & x \geq 1 \\ 4 & , & x < 1 \end{cases}$ is continuous at x = 1 is

A. 1

B. 2

C. 3

D. 4

Answer: D

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23. Derivative of $\cos^{-1}(\sin x)$ w.r.t. x equals

A. -1

B. 1

C. cos x

D. sin x

Answer: A

---

24. Derivative of $\tan^{-1}\left(\dfrac{1-\cos x}{\sin x}\right)$ w.r.t. x is

A. $\dfrac{1}{2}$

B. $\dfrac{1}{3}$

C. $\dfrac{1}{4}$

D. $\dfrac{1}{5}$

Answer: A

25. $\displaystyle\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$ is equal to

A. $\dfrac{-1}{\sin x + \cos x} + C$

B. $\log|\sin x + \cos x| + C$

C. $\log|\sin x - \cos x| + C$

D. $\dfrac{1}{(\sin x + \cos x)^2}$

Answer: B

26. $\displaystyle\int_1^2 \left(\log x^2\right) dx$ equals

A. $4\log 2 - 2$

B. $4 \log 2 - 3$

C. $3 \log 2 - 2$

D. $3 \log 2 - 4$

Answer: A

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27. Solution of $x^5 \dfrac{dy}{dx} = -y^5$ is

A. $x^{-2} + y^{-2} = c$

B. $x^{-4} + y^{-4} = c$

C. $x^{-3} + y^{-3} = c$

D. $x^{-5} + y^{-5} = c$

Answer: B

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28. The general solution of differential equation : $ydy + xdx = 0$ is :

A. $x^2 + y^2 = c$

B. $y^2 = x^2 + c$

C. $y^2 = cx^2$

D. $x^2y^2 = c^2$

Answer: A

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29. Vector in the direction of vector $2\hat{i} - 3\hat{j} + 6\hat{k}$ which has magnitude 21 units is

A. $2\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$

B. $4\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$

C. $3\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$

D. $5\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$

Answer: C

30. If $\sqrt{3}\left|\vec{a} \cdot \vec{b}\right| = \left|\vec{a} \times \vec{b}\right|$, then angle between $\vec{a}$ and $\vec{b}$ is

A. $\dfrac{\pi}{3}$

B. $\dfrac{\pi}{6}$

C. $\dfrac{\pi}{4}$

D. $\dfrac{\pi}{2}$

Answer: A

31. The distance of the point (3, 4, 5) from the plane

$\vec{r} \cdot \left(2\hat{i} - 5\hat{j} + 3\hat{k}\right) = 13$

A. $\dfrac{8}{\sqrt{38}}$

B. $\dfrac{10}{\sqrt{38}}$

C. $\dfrac{12}{\sqrt{38}}$

D. $\dfrac{14}{\sqrt{38}}$

Answer: C

32. If $P\left(\dfrac{A}{B}\right) > P(A)$, then which of the following is correct : :

A. $P(B \mid A) < P(B)$

B. $P(A \cap B) < P(A). P(B)$

C. $P(B \mid A) > P(B)$

D. $P(B \mid A) = P(B)$

Answer: C

33. Range of $f(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \text{ is.................} \\ -1 & \text{if } x < 0 \end{cases}$

34. The matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a .... matrix.

35. Derivative of $\tan^{-1}\left(\dfrac{\cos x}{1 + \sin x}\right)$ w.r.t. x is.........................

36. The interval in which $y = x^2 e^{-x}$ is strictly increasing is

37. $\displaystyle\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$ = ...

38. I.F. of $\dfrac{dy}{dx} + \dfrac{y}{x} = e^x, (x > 0)$ is..........................

39. Find the equation of the plane with intercept 3 on the y-axis and parallel to ZOX plane.

40. If $2P(A) = P(B) = \dfrac{7}{15}$ and $P(A/B) = \dfrac{2}{7}$ then find $P(A \cup B)$.

41. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to $-\dfrac{\pi}{3}$.

42. Value of $\begin{vmatrix} 42 & 1 & 6 \\ 28 & 7 & 4 \\ 14 & 3 & 2 \end{vmatrix}$ is 1?

43. If $y = \sin^{-1} x$, then prove that

$$\frac{d^2 y}{dx^2} = \frac{x}{(1 - x^2)^{\frac{3}{2}}}.$$

44. $\displaystyle\int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + c, n \neq -1$

45. If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = -\hat{i} + 3\hat{j} + \hat{k}$, then $\vec{a} \cdot (\vec{c} \times \vec{a})$ is

2.

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46. Show that the D.C.'s of the perpendicular from origin to the plane $\vec{r} \cdot (-2\hat{i} - 3\hat{j} + 6\hat{k}) + 14 = 0$ are $\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}$.

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47. Let E and F be two events associated with the same random experiment, then E and F are said to be independent if $P(E \cap F) = P(E) \cdot P(F)$.

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48. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 8) and (0, 5).

Let F = 4x + 6y be the objective function.

The Maximum of F - Minimum of F = 60

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Section A Fill In The Blanks

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1. Fill in the blanks from the given options:

$t^2, 2, -2, 0, \dfrac{2}{9}, x\sin x, \dfrac{1}{11}\tan^{11}x + \dfrac{1}{9}\tan^9 x + C, 1, 2, 4$

Let A = {1, 2, 3}. Then number of equivalence relations containing (1,2) is

....

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2. Fill in the blanks from the given options:

$t^2, 2, -2, 0, \dfrac{2}{9}, x\sin x, \dfrac{1}{11}\tan^{11}x + \dfrac{1}{9}\tan^9 x + C, 1, 2, 4$

Value of $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$ is............

3. If $x = 3at$, $y = at^3$, then $\dfrac{dy}{dx}$ is.......

4. The slope of the tangent to the curve $y = 3 - x^2$ at x = 1 is........

5. Fill in the blanks from the given options:

$t^2, 2, -2, 0, \dfrac{2}{9}, x\sin x, \dfrac{1}{11}\tan^{11} x + \dfrac{1}{9}\tan^9 x + C, 1, 2, 4$

$\displaystyle\int \tan^8 x \sec^4 x\, dx$............

6. I.F of $x\dfrac{dy}{dx} + y - xy\cot x = 0 (x \neq 0)$ is..........

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7. If the equations of a line AB are $\dfrac{3-x}{1} = \dfrac{y+2}{-2} = \dfrac{z-5}{4}$, then the direction ratios of a line parallel to AB are ....

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8. If $P(A/B) = \dfrac{2}{5}, 2P(A) = P(B) = \dfrac{5}{9}$, then $P(A \cap B)$ is..............

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Section A True Or False

1. $2\tan^{-1} x = \sin^{-1}\dfrac{2x}{1+x^2}, |x| \leq 1.$

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2. prove for matrices A, B and C, (ABC)' = C'B'A'

3. Derivative of $\sin^{-1}(\cos x)$ w.r.t. x is 1

4. Prove $\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\dfrac{x}{a}\right) + c, a > 0.$

5. The value of p for which $\overrightarrow{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\overrightarrow{b} = \hat{i} + p\hat{j} + 3\hat{k}$ are parallel is .

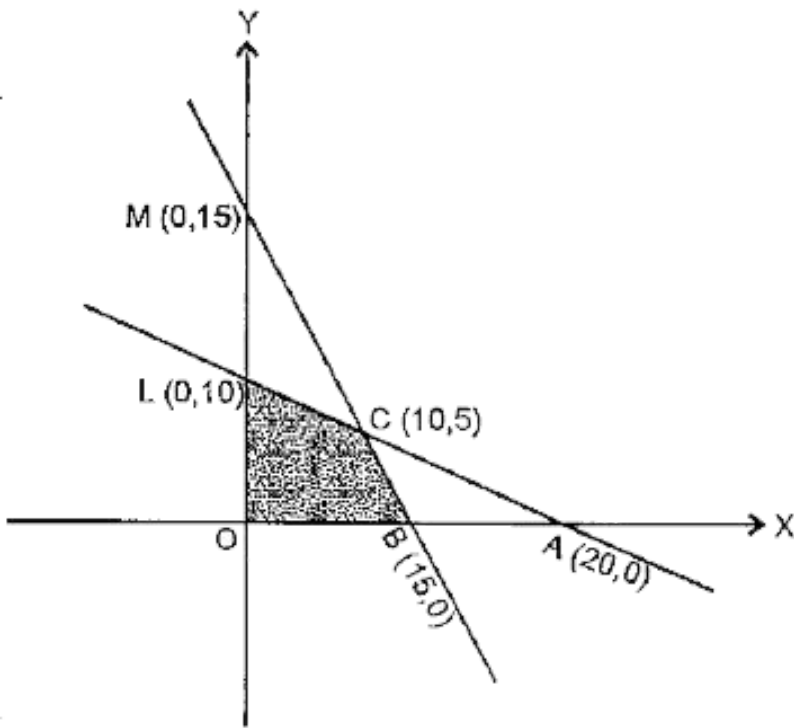**6.** The Cartesian equations of a line are $\dfrac{x-5}{3} = \dfrac{y+4}{7} = \dfrac{z-6}{2}$.

Vector equation for the line is

7. If P(not A) = 0.4, P(A U B) = 0.75 and A, B are given to be independent events, then the value of $P(B)$ is $\dfrac{3}{8}$.

8. The feasible solution for a LPP is shown in Fig. Let. $Z = 4x + 7y$ be the objective function. Maximum value of Z is 160:

Section C

1. Prove that

$$\cos^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{3} = \tan^{-1}\frac{56}{33}$$

2. If $(x = a\sin(2t)(1 + \cos(2t))$ and $(y = b\cos(2t)(1 - \cos(2t))$, then

show that $\left(\left(\dfrac{dy}{dx} \right)_{t=\frac{\pi}{4}} = \dfrac{b}{a} \right)$.

3. If $p^2 = a^2\cos^2\theta + b^2\sin^2\theta$, prove that $p + \left(\dfrac{d^2p}{d\theta^2} \right) = \dfrac{a^2b^2}{p^3}$

4. Integrate the function: $\dfrac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}, (x \in [0, 1]$

5. Using properties of definite integrals, show that

$$\int_0^{\pi/2} \dfrac{\sin^2 x}{\sin x + \cos x} dx = \dfrac{1}{\sqrt{2}}\log(\sqrt{2} + 1)$$

6. Solve the following differential equation

$\left(x^2 - y^2\right)dx + 2xy\,dy = 0$ given that y=1 when x=1

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7. An urn contains 2 white and 2 black balls. A ball is drawn at random. If it is white, it is not replace into urn, otherwise it is replaced along with another ball of the same colour. The proccess is repeated, find the probability that the third ball drawn is black.

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8. Three events A, B and C have probabilities $\dfrac{2}{5}$, $\dfrac{1}{3}$ and $\dfrac{1}{2}$, respectively. Given that $P(A \cap C) = \dfrac{1}{5}$ and $P(B \cap C) = \dfrac{1}{4}$, find the values of P(C | B) and $P(A' \cap C')$

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9. Prove that

$$\sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5} = \sin^{-1}\frac{63}{65}$$

10. Write the principle value of $\tan^{-1}(1) + \cos^{-1}\left(\frac{1}{2}\right)$.

11. Find $\dfrac{dy}{dx}$ when :

$$y = x^{\sin x - \cos x} + \frac{x^2 - 1}{x^2 + 1}$$

12. if $y = \sin^{-1}\left(\dfrac{5x + 12\sqrt{1 - x^2}}{13}\right)$ then find $\dfrac{dy}{dx}$

13. Evaluate: $\displaystyle\int e^{-\frac{x}{2}}\frac{\sqrt{1-\sin x}}{1+\cos x}\,dx$

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14. Prove that : $\displaystyle\int_0^1 x(1-x)^n\,dx = \frac{1}{(n+1)(n+2)}$

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15. Solve the following differential equations

$$xdy - (y + 2x^2)\,dx = 0$$

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16. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

1. Solve the following system of linear equations by matrix method :

$$4x + 3y + z = 10, \; 3x - y + 2z = 8, \; x - 2y - 3z - 10.$$

2. Prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)\left(a^2 + b^2 + c^2\right).$$

3. The plane $ax + by = 0$ is rotated through an angle α about its line of

intersection with the plane $z = 0$. Show that the equation to the plane in

new position is $ax + by \pm z\sqrt{a^2 + b^2} \tan\alpha = 0$.

4. Find the position vector of the foot of perpendicular and the perpendicular distance from the point P with position vector $2\hat{i} + 3\hat{j} + 4\hat{k}$ to the plane $\overrightarrow{r} \cdot \left(2\hat{i} + \hat{j} + 3\hat{k}\right) - 26 = 0$. Also find the image of P in the plane.

5. Maximise and minimise $Z = 15x + 3y$ subject to constraints.

$x + y \leq 8$

$2x + y \geq 8$

$x - 2y \geq 0$

$x, y \geq 0$

6. Maximize $z = 4x + 7y$ such that

$x + 2y \le 20, x + y \le 15, x \ge 0, y \ge 0.$

7. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of

equations $x - y + 2z = 1, 2y - 3z = 1, 3x - 2y + 4z = 2$

8. A school wants to award its students for regularity and hard work with a total cash award of Rs. 6,000. If three times the award money for hard work added to that given for regularity amounts to Rs. 11,000, represent the above situation algebraically and find the award money for each value, using matrix method.

9. Find the equation of plane passing through the points A(2, -1, 1), B(4, 3, 2) and C(6, 5, -2). Also prove that point $\left(5, \; -1, \; -\dfrac{25}{2}\right)$ lies on the plane given by points A, B and C.

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10. Find the shortest distance between the lines :
$$\dfrac{x+1}{4} = \dfrac{y-3}{-6} = \dfrac{z+1}{1} \quad \text{and} \quad \dfrac{x+3}{3} = \dfrac{y-5}{2} = \dfrac{z-7}{6}.$$

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11. Maximize $z = 15x + 5y$, subject to the constraints

$$2x + 3y \leq 12$$

$$3x + 2y \leq 12$$

$$x \geq 0, y \geq 0$$

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12. Solve the following problem graphically :

Minimise and Maximise Z = 3x + 9y

subject to the constraints :

$x + 3y \leq 60$

$x + y \geq 10$

$x \leq y$

$x \geq 0, y \geq 0$