



# MATHS

## BOOKS - OMEGA PUBLICATION

### APPLICATION OF DERIVATIVES

#### Questions

1. A balloon which always remains spherical is being inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at

which the radius of the balloon increases when the radius is 15 cm.



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2. A ladder 16 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 m/sec. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ?



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3. Sand is pouring from a pipe at the rate of  $9 \text{ cm}^3 / \text{s}$ . The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 3 cm ?



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4. A stone is dropped into a quiet lake and waves move in circles at the speed of  $5 \text{ cm/s}$ .

At the instant when the radius of the circular wave is 8 cm, how fast is the enclosed area increasing?



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5. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference ?



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6. The total revenue in Rupees received from the sale of  $x$  units of a product is given by  $R(x) = 13x^2 + 26x + 15$  Find the marginal revenue when  $x = 7$ .



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7. Find the interval in which the function  $f(x) = x^2 + 2x - 5$  is strictly increasing or decreasing.



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8. Find the intervals in which the function  $f(x) = -2x^3 - 9x^2 - 12x + 1$  is strictly increasing or decreasing.



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9. Show that  $y = \log(1 + x) - 2\frac{x}{2 + x}, x > 1,$  is an increasing function of  $x$  throughout its domain.



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10. Find points at which the tangent to the curve  $y = x^3 - 3x^2 - 9x + 7$  is parallel to the x-axis.



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11. Prove that  $y = 4\frac{\sin \theta}{2 + \cos \theta} - \theta$ , is an increasing function of  $\theta$  in  $\left[0, \frac{\pi}{2}\right]$ .



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**12.** Let  $I$  be any interval disjoint from  $[-1, 1]$

Prove that the function  $f$  given by

$$f(x) = x + \frac{1}{x} \text{ is increasing on } I.$$



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**13.** Prove that the function 'f' given by  $f(x) =$

$\log \sin x$  is strictly increasing on  $\left(0, \frac{\pi}{2}\right)$



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**14.** Prove that the function  $f$  given by  $f(x) = \log(\sin x)$  is strictly increasing on  $\left(0, \frac{\pi}{2}\right)$  strictly decreasing on  $\left(\frac{\pi}{2}, \pi\right)$



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**15.** Prove that the function  $f$  given by  $f(x) = \log(\cos x)$  is strictly decreasing on  $\left(0, \frac{\pi}{2}\right)$



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**16.** Prove that  $f(x) = \log \cos x$  is strictly decreasing on  $\left(0, \frac{\pi}{2}\right)$  and strictly increasing on  $\left(\frac{\pi}{2}, \pi\right)$



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**17.** Find the intervals in which the function given by :  $f(x) = \sin x + \cos x, 0 \leq x \leq 2\pi$ . is strictly increasing and strictly decreasing.



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**18.** Find the intervals in which  $f(x) = 6 - 9x - x^2$  is strictly increasing or decreasing.



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**19.** Find the intervals in which the function  $f$  given by  $f(x) = 4x^3 - 6x^2 - 72x + 30$  is increasing.



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**20.** Find the intervals in which the function  $f(x) = 10 - 6x - 2x^2$  is strictly increasing.



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**21.** Determine the intervals in which the following functions  $f(x) = x^2 + 2x - 5$  are strictly increasing or strictly decreasing.



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**22.** Show that the function  $f$  given by,  
 $f(x) = x^3 - 3x^2 + 4x, x \in R$  is increasing  
on  $R$ .



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**23.** Find the slope of the normal to the curve  
 $x = 1 - a \sin \theta, y = b \cos^2 \theta$  at  $\theta = \frac{\pi}{2}$



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24. Find the equations of all lines having slope

0 which are tangent to the curve

$$y = \frac{1}{x^2 - 2x + 3}$$



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25. Find the equation of the tangent line to

the curve  $y = x^2 - 2x + 7$  which is

perpendicular to the line  $5y - 15x = 13$



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**26.** Find the equation of the tangent to the curve  $y = \sqrt{3x - 2}$  which is parallel to the line  $4x - 2y + 5 = 0$ .



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**27.** For the curve  $y = 4x^3 - 2x^5$ , find all the points at which the tangent passes through the origin.



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**28.** Find the equation of the normal at the point  $(am^2, am^3)$  for the curve  $ay^2 = x^3$



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**29.** Find the equations of the tangent and normal to the parabola  $y^2 = 4ax$  at the point  $(at^2, 2at)$



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**30.** Prove that the curves  $x = y^2$  and  $xy = k$  cut at right angles if  $8k^2 = 1$ .



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**31.** Find the equations of the tangent and normal to the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  at the point  $(x_0, y_0)$



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**32.** Using differentials find the approximate value of  $(0.009)^{1/3}$



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**33.** Using differentials, find the approximate value of  $(15)^{1/4}$



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**34.** Using differentials, find the approximate value of  $(26)^{1/3}$ .



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**35.** Find the approximate value of  $f(5.001)$ , where  $f(x) = x^3 - 7x^2 + 15$ .



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**36.** Find the approximate change in the volume  $V$  of a cube of side  $x$  metres caused by increasing the side by  $1\%$ .



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**37.** If the radius of a sphere is measured as  $9$  m with an error of  $0.03$  m, then find the approximate error in calculating its surface area.



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**38.** Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values:

$$f(x) = \sin x + \cos x, 0 < x < \frac{\pi}{2}$$



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$$f(x) = \sin x - \cos x, 0 < x < 2\pi$$



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**40.** Find the local maximum and local minimum if any, of the function

$f(x) = x\sqrt{1-x}, x > 0$ . Also find the local maximum and local minimum values.



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**41.** Find the absolute maximum and the absolute minimum value of the function given

by:  $f(x) = \sin^2 x - \cos x, x \in [0, \pi]$



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**42.** Find the absolute maximum and absolute minimum value of the function

$$f(x) = 4x - \frac{1}{2}x^2 \text{ in } [-2, 4, 5]$$



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**43.** . Find two numbers whose sum is 24 and whose product is as large as possible.



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**44.** Find two positive numbers  $x$  and  $y$  such that  $x + y = 60$  and  $xy^3$  is maximum.



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**45.** Find two positive numbers whose sum is 16 and whose sum of cubes is minimum.



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**46.** A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top, by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum ?





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**47.** Show that of all rectangles inscribed in a given circle the square has maximum area.



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**48.** Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.



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**49.** If all the closed cylindrical cans (right circular), which enclose a given volume of 100 cubic centimeters. Find the dimensions of the can which has the minimum surface area.



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**50.** A wire of length 34 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What

should be the length of two pieces so that the combined area of the square and the circle is minimum ?



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51. Prove that volume of largest cone, which can be inscribed in a sphere, is  $\left(\frac{8}{27}\right)^{th}$  part of volume of sphere.



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**52.** Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius  $r$  is  $4\frac{r}{3}$ .



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**53.** Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is  $\tan^{-1} \sqrt{2}$ .



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**54.** A balloon which always remains spherical is being inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.



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**55.** A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall

decreasing when the foot of the ladder is 4 m away from the wall ?



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**56.** Sand is pouring from a pipe at the rate of 12 cubic cm./sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. At which rate is the height of the sand-cone increasing when the height is 4 cm. ?



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**57.** A stone is dropped into a quiet lake and waves move in circles at the speed of 5 cm/s. At the instant when the radius of the circular wave is 8 cm, how fast is the enclosed area increasing?



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**58.** The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of



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$R(x) = 13x^2 + 26x + 15$  Find the marginal revenue when  $x = 7$ .



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**60.** Find the interval in which the function  $f(x) = x^2 + 2x - 5$  is strictly increasing or decreasing.



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**61.** Find the intervals in which the function  $f(x) = -2x^3 - 9x^2 - 12x + 1$  is strictly increasing or decreasing.



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**62.** Show that  $y = \log(1 + x) - \frac{2x}{2 + x}$  is strictly increasing function of  $x$  for all values of  $x > -1$ .



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**63.** Find points at which the tangent to the curve  $y = x^3 - 3x^2 - 9x + 7$  is parallel to the x-axis.



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**64.** Prove that  $y = 4\frac{\sin \theta}{2 + \cos \theta} - \theta$ , is an increasing function of  $\theta$  in  $\left[0, \frac{\pi}{2}\right]$ .



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**65.** Let  $I$  be any interval disjoint from  $(-1,1)$ . Prove that the function  $f(x) = x + \frac{1}{x}$  is strictly increasing on  $I$ .



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**66.** Prove that the function 'f' given by  $f(x) = \log \sin x$  is strictly increasing on  $\left(0, \frac{\pi}{2}\right)$



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**67.** Prove that the function f given by  $f(x) = \log(\sin x)$  is strictly increasing on  $\left(0, \frac{\pi}{2}\right)$   
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**70.** Find the intervals in which the function given by :  $f(x) = \sin x + \cos x$ ,  $0 \leq x \leq 2\pi$ . is strictly increasing and strictly decreasing.



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**71.** Find the intervals in which  $f(x) = 6 - 9x - x^2$  is strictly increasing or decreasing.



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**75.** Show that the function  $f$  given by  $f(x) = x^3 - 3x^2 + 4x, x \in R$  is strictly increasing on  $R$ .



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76. Find the slope of the normal to the curve

$$x = 1 - a \sin \theta, y = b \cos^2 \theta \text{ at } \theta = \frac{\pi}{2}$$



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77. Find the equations of all lines having slope

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$$y = \frac{1}{x^2 - 2x + 3}$$



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**85.** Using differentials find the approximate value of  $(0.009)^1 / 3$



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**86.** Using differentials, find the approximate value of  $(15)^{1/4}$



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**88.** Find the approximate value of  $f(5.001)$ ,

where  $f(x) = x^3 - 7x^2 + 15$ .



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**89.** Find the approximate change in the

volume  $V$  of a cube of side  $x$  metres caused by

increasing the side by 1%.



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90. If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating its surface area.



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91. Find the local maximum and local minimum value of the function  $\sin x + \cos x$ ,  $0 < x < \frac{\pi}{2}$



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**92.** Find the local maximum and local minimum value of the function  $\sin x - \cos x$  in  $0 < x < 2\pi$



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**93.** Find the local maximum and local minimum if any, of the function  $f(x) = x\sqrt{1-x}$ ,  $x > 0$ . Also find the local maximum and local minimum values.



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**94.** Find the absolute maximum and absolute minimum value of the function

$$f(x) = \sin x + \cos x - \sin x \text{ in } [0, \pi]$$



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**95.** Find the absolute maximum and absolute minimum value of the function

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**97.** Find two positive numbers  $x$  and  $y$  such that  $x + y = 60$  and  $xy^3$  is maximum.



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**98.** Find two positive numbers whose sum is 16 and whose sum of cubes is minimum.



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**99.** A rectangular sheet of tin 45 cm x 24 cm is to be made into a box without top, by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is the maximum possible.





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**100.** Show that of all rectangles inscribed in a given circle the square has maximum area.



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**101.** Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.



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**102.** If all the closed cylindrical cans (right circular), which enclose a given volume of 100 cubic centimeters. Find the dimensions of the can which has the minimum surface area.



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**103.** A wire of length 28 cm is to be cut into two pieces. One of the pieces is to be made into a square and other into a circle. What

would be the length of the two pieces so that the combined area of the square and the circle is minimum ?



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**104.** Prove that volume of largest cone, which can be inscribed in a sphere, is  $\left(\frac{8}{27}\right)^{th}$  part of volume of sphere.



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**105.** Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius  $r$  is  $4\frac{r}{3}$ .



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**106.** Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is  $\tan^{-1} \sqrt{2}$ .



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## Important Questions From Miscellaneous Exercise

1. The two equal sides of an isosceles triangle with fixed base  $b$  are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base ?



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2. Show that the normal at any point  $\theta$  to the curve

$$x = a \cos \theta + a\theta \sin \theta, y = a \sin \theta - a\theta \cos \theta$$

is at a constant distance from the origin.



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3. Find the maximum area of an isosceles triangle inscribed in the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  with its vertex at one end of the major axis.



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4. A window is in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.



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5. An open box with a square base is to be made out of a given iron sheet of area  $c^2$

square units. Show that the maximum volume

of the box is  $\frac{c^3}{6\sqrt{3}}$  cubic units.



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6. Show that the height of the cylinder of maximum volume that can be inscribed in a

sphere of radius  $R$  is  $2\frac{R}{\sqrt{3}}$ . Also find the

maximum volume.



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7. Show that semi-vertical angle of right circular cone of given surface area and maximum volume is  $\sin^{-1}\left(\frac{1}{3}\right)$ .



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8. Using differentials, find the approximate value of  $\left(\frac{17}{81}\right)^{1/4}$



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9. Find the intervals in which the function  $f$  given by  $f(x) = x^3 + \left(\frac{1}{x^3}\right)$ ,  $x \neq 0$  is increasing.



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10. Find the intervals in which the function  $f$  given by  $f(x) = x^3 + \left(\frac{1}{x^3}\right)$ ,  $x \neq 0$  is decreasing.



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**11.** The length ' $x$ ' of a rectangle is decreasing at the rate of 5 cm per minute and the width ' $y$ ' is increasing at the rate of 4 cm per minute, when  $x = 8$  cm and  $y = 6$  cm, find the rate of change of the perimeter of the rectangle.



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**12.** The length ' $x$ ' of a rectangle is decreasing at the rate of 5 cm per minute and the width ' $y$ ' is increasing at the rate of 4 cm per minute,



when  $x = 8$  cm and  $y = 6$  cm, find the rate of change of the area of the rectangle.



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**13.** If the sum of the lengths of hypotenuse and a side of a right-angled triangle is given. Show that the area is maximum, when the angle between them is  $60^\circ$



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**14.** Manufacturer can sell  $x$  items at a price of rupees  $Rs\left(5 - \left(\frac{x}{100}\right)\right)$  each. The cost price of  $x$  items is  $Rs\left(\left(\frac{x}{5}\right) + 500\right)$ . Find the number of items he should sell to earn maximum profit.



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**15.** Show that the right circular cone of least curved surface and given volume has an

altitude equal to  $\sqrt{2}$  time the radius of the base.



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**16.** A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, find the dimensions of the rectangle that will produce the largest area of the window.



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17. The volume of a cube is increasing at the rate of  $8c \frac{m^3}{s}$ . How fast is the surface area increasing when the length of an edge is 12 cm?



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18. Find the maximum value of  $2x^3 - 24x + 107$  in the interval  $[1, 3]$ . Find the maximum value of the same function in  $[-3, -1]$ .



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**19.** The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 10 cm.



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**20.** Find the maximum profit that a company can make, if the profit function is given by

$$p(x) = 41 - 72x - 18x^2$$



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21. Find both the maximum value and the minimum value of

$3x^4 - 8x^3 + 12x^2 - 48x + 25$  on the interval  $[0, 3]$



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22. Prove that the radius of the right-circular cylinder of greatest curved surface, which can

be inscribed in a given cone, is half of that of the cone.



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**23.** Find the equation of the normals to the curve  $y = x^3 + 2x + 6$  which are parallel to the line  $x + 14y + 4 = 0$



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**24.** Find the equation of the tangent line to the curve  $y = x^2 - 2x + 7$ , which is parallel to the line  $2x - y + 9 = 0$ .



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**25.** The two equal sides of an isosceles triangle with fixed base  $b$  are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base ?







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**26.** Show that the normal at any point  $\theta$  to the curve

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**35.** The length  $x$  of a rectangle is decreasing at the rate of 5 cm/minute and the width  $y$  is increasing at the rate of 4 cm/minute. When  $x = 8$  cm and  $y = 6$  cm, find the rate of change of the perimeter



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**36.** The length  $x$  of a rectangle is decreasing at the rate of 5 cm/min and the width  $y$  is increasing at the rate of 4 cm/min. when  $x = 8$

cm and  $y = 6$  cm, find the rates of change of the area of the rectangle.



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$$p(x) = 41 - 24x - 18x^2.$$



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**45.** Find the maximum and minimum values of  $x^4 - 8x^3 + 12x^2 - 48x + 25$  on the interval  $[0,3]$



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**46.** Prove that the radius of the right-circular cylinder of greatest curved surface, which can be inscribed in a given cone, is half of that of the cone.



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**47.** Find the equation of the normals to the curve  $y = x^3 + 2x + 6$  which are parallel to the line  $x + 14y + 4 = 0$



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**48.** Find the equation of the tangent line to the curve  $y = x^2 - 2x + 7$ , which is parallel to the line  $2x - y + 9 = 0$ .



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## Multiple Choice Questions Mcqs

1. Find the rate of change of the area of a circle with respect to its radius  $r$  when

$$r = 4\text{cm}$$

A.  $6\pi\text{cm}^2 / \text{s}$

B.  $4\pi\text{cm}^2 / \text{s}$

C.  $8\pi\text{cm}^2 / \text{s}$

D. None of these

**Answer: D**



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2. Find the rate of change of the area of a circle per second with respect to its radius  $r$  when  $r = 5$  cm.

A.  $8\pi cm^2 / cm$

B.  $10\pi cm^2 / cm$

C.  $11\pi cm^2 / cm$

D. None of these

**Answer: B**



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3. Find the rate of change of the area of the circle with respect to its radius  $r$  when  $r = 6$  cm.

A.  $10\pi$

B.  $12\pi$

C.  $8\pi$

D. None of these



**Answer: B**



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4. The total revenue in Rupees received from its sale of  $x$  units of a product is given by  $R(X) = 3x^2 + 36x + 5$ . Find the marginal revenue, when  $x = 15$

A. 116

B. 96

C. 90

D. 126

**Answer: D**



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5. An edge of a variable cube is increasing at the rate of 3 cm second. How fast is the volume of the cube increasing when the edge is 10 cm long?

A.  $900\text{cm}^3 / \text{sec}$

B.  $100\text{cm}^3 / \text{sec}$

C.  $300\text{cm}^3 / \text{sec}$

D. None of these

**Answer: A**



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6. A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic metre per hour. Then the depth of the wheat is increasing at the rate of:

A.  $1m^3 / h$

B.  $0.1m^3 / h$

C.  $1.1m^3 / h$

D.  $0.5m^3 / h$

**Answer: A**



**Watch Video Solution**

7. Find the slope of the tangent to the curve

$$y = 3x^4 - 4x \text{ at } x = 4 .$$

A. 674

B. 764

C. 476

D.  $-764$

**Answer: B**



**Watch Video Solution**

**8.** The slope of the normal to the curve

$y = 2x^2 + 3 \sin x$  at  $x = 0$  is:

A. 3

B.  $\frac{1}{3}$

C.  $-3$

D.  $-\frac{1}{3}$

**Answer: D**



**Watch Video Solution**

**9.** The line  $y = x + 1$ , is a tangent to the curve  $y^2 = 4x$  at the point.

A. (1, 2)

B. (2, 1)

C. (1, - 2)

D. ( - 1, 2)

**Answer: A**



**Watch Video Solution**

**10.** The slope of the tangent to the curve

$x = t^2 + 3t - 8, y = 2t^2 - 2t - 5at$  the

point (2,-1) is:

A.  $\frac{22}{7}$

B.  $\frac{6}{7}$

C.  $\frac{7}{6}$

D.  $-\frac{6}{7}$

**Answer: B**



**Watch Video Solution**

**11.** The line  $y = mx + 1$ , is a tangent to the curve  $y^2 = 4x$  if the value of  $m$  is:



A. 1

B. 2

C. 3

D.  $\frac{1}{2}$

**Answer: A**



**Watch Video Solution**

**12.** The normal at the point (1,1) on the curve

$2y + x^2 = 3$  is:

A.  $x + y = 0$

B.  $x - y = 0$

C.  $x + y + 1 = 0$

D.  $x - y + 1 = 0$

**Answer: B**



**Watch Video Solution**

**13.** If  $f(x) = 3x^2 + 15x + 5$ , then the approximate value of  $f(3.02)$  is :

A. 47.66

B. 57.66

C. 67.66

D. 77.66

**Answer: D**



**Watch Video Solution**

**14.** The approximate change in the volume of a cube of side  $x$  metres caused by increasing the side by 3% is:

A.  $0.06x^3m^3$

B.  $0.6x^3m^3$

C.  $0.09x^3m^3$

D.  $0.9x^3m^3$

**Answer: C**



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**15.** The point on the curve  $x^2 = 2y$  which is nearest to the point  $(0, 5)$  is:

A.  $(2\sqrt{2}, 4)$

B.  $(2\sqrt{2}, 0)$

C.  $(0, 0)$

D.  $(2, 2)$

**Answer: A**



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**16.** For all real values of  $x$ , the minimum value

of  $\frac{1 - x + x^2}{1 + x + x^2}$  is:

A. 0

B. 1

C. 3

D.  $\frac{1}{3}$

**Answer: D**



**Watch Video Solution**

**17.** The maximum value of

$[x(x - 1) + 1]^{\frac{1}{3}}, 0 \leq x \leq 1$  is:

A.  $\left(\frac{1}{3}\right)^{\frac{1}{3}}$

B.  $\frac{1}{2}$

C. 1

D. 0

**Answer: C**



**Watch Video Solution**

**18.** Find the rate of change of the area of a circle with respect to its radius  $r$  when

$$r = 4\text{cm}$$

A.  $6\pi\text{cm}^2 / \text{s}$

B.  $4\pi\text{cm}^2 / \text{s}$

C.  $8\pi\text{cm}^2 / \text{s}$

D. None of these

**Answer: D**



**Watch Video Solution**



19. Find the rate of change of the area of a circle with respect to its radius  $r$  when  $r = 5$  cm, is

A.  $8\pi \text{ cm}^2 / \text{cm}$

B.  $10\pi \text{ cm}^2 / \text{cm}$

C.  $11\pi \text{ cm}^2 / \text{cm}$

D. None of these

**Answer: B**



**Watch Video Solution**

20. Find the rate of change of the area of the circle with respect to its radius  $r$  when  $r = 6$  cm.

A.  $10\pi$

B.  $12\pi$

C.  $8\pi$

D. None of these

**Answer: B**



**Watch Video Solution**

21. The total revenue in Rupees received from its sale of  $x$  units of a product is given by  $R(X) = 3x^2 + 36x + 5$ . Find the marginal revenue, when  $x = 15$

A. 116

B. 96

C. 90

D. 126

**Answer: D**



**Watch Video Solution**

22. An edge of a variable cube is increasing at the rate of 3 cm/s. How fast is the volume of the cube increasing when the edge is 10 cm long?

A.  $900\text{cm}^3 / \text{sec}$

B.  $100\text{cm}^3 / \text{sec}$

C.  $300\text{cm}^3 / \text{sec}$

D. None of these

**Answer: A**



**Watch Video Solution**

**23.** A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic metre per hour. Then the depth of the wheat is increasing at the rate of:

A.  $1m^3 / h$

B.  $0.1m^3 / h$

C.  $1.1m^3 / h$

D.  $0.5m^3 / h$

**Answer: A**



**Watch Video Solution**

**24.** Find the slope of the tangent to the curve

$$y = 3x^4 - 4x \text{ at } x = 4 .$$

A. 674

B. 764

C. 476

D.  $-764$

**Answer: B**



**Watch Video Solution**

**25.** The slope of the normal to the curve

$$y = 3x^2 + 2 \sin x \text{ at } x = 0 \text{ is}$$

A. 3

B.  $1/3$

C.  $-3$

D.  $-\frac{1}{3}$

**Answer: D**



**Watch Video Solution**

**26.** The line  $y = x + 1$ , is a tangent to the curve  $y^2 = 4x$  at the point.

A.  $(1, 2)$

B.  $(2, 1)$

C.  $(1, -2)$



D.  $(-1, 2)$

**Answer: A**



**Watch Video Solution**

27. The slope of the tangent to the curve  $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$  at the point  $(2, -1)$  is:

A.  $\frac{22}{7}$

B.  $\frac{6}{7}$

C.  $\frac{7}{6}$

D.  $-\frac{6}{7}$

**Answer: B**



**Watch Video Solution**

**28.** The line  $y = mx + 1$ , is a tangent to the curve  $y^2 = 4x$  if the value of  $m$  is:

A. 1

B. 2

C. 3

D.  $\frac{1}{2}$

**Answer: A**



**Watch Video Solution**

**29.** The normal at the point (1,1) on the curve

$2y + x^2 = 3$  is:

A.  $x + y = 0$

B.  $x - y = 0$

C.  $x + y + 1 = 0$

D.  $x - y + 1 = 0$

**Answer: B**



**Watch Video Solution**

**30.** If  $f(x) = 3x^2 + 15x + 5$ , then the approximate value of  $f(3.02)$  is :

A. 47.66

B. 57.66

C. 67.66

D. 77.66

**Answer: D**



**Watch Video Solution**

**31.** The approximate change in the volume of a cube of side  $x$  metres caused by increasing the side by 3% is

A.  $0.06x^3 m^3$

B.  $0.6x^3m^3$

C.  $0.09x^3m^3$

D.  $0.9x^3m^3$

**Answer: C**



**Watch Video Solution**

**32.** The point on the curve  $x^2 = 2y$  which is nearest to the point  $(0, 5)$  is:

A.  $(2\sqrt{2}, 4)$

B.  $(2\sqrt{2}, 0)$

C.  $(0, 0)$

D.  $(2, 2)$

**Answer: A**



**Watch Video Solution**

**33.** For all real values of  $x$ , the minimum value of  $(1 + x + x^2)$  is

A. 0

B. 1

C. 3

D.  $\frac{1}{3}$

**Answer: D**



**Watch Video Solution**

**34.** The maximum value of

$[x(x - 1) + 1]^{\frac{1}{3}}, 0 \leq x \leq 1$  is:

A.  $\left(\frac{1}{3}\right)^{\frac{1}{3}}$



B.  $\frac{1}{2}$

C. 1

D. 0

**Answer: C**



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