MATHS

BOOKS - OMEGA PUBLICATION

CONTINUITY AND DIFFERENTIABILITY

Questions

1. Examine the continuity of the function $f(x) = 2x^2 - 1$ at $x = 3$

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2. Prove that the function $f(x) = x^n$, is continuous at $x = n$, where n is a positive integer.

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3. Find all the points of discontinuity, where f is defined by

$$f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x \geq 1 \\ -1, & \text{if } x \geq 0 \end{cases}$$

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4. Find all points of discontinuity of $f(x)$, where

$$f(x) = \begin{cases} \frac{x}{|x|} & \text{if} x \neq 0 \\ 0 & \text{if} x = 0 \end{cases}$$

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5. Discuss the continuity of $f(x) = \sin x - \cos x$

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6. Discuss the discontinuity of $f(x) = \sin x - \cos x$

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7. Find all the points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} 2x + 3, & \text{if } x \leq 2 \\ 2x - 3, & \text{if } x > 2 \end{cases}$$

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8. Find all the points of discontinuity, where f is defined by

$$f(x) = \begin{cases} x + 1, & \text{if } x \geq 1 \\ x^2 + 1, & \text{if } x < 1 \end{cases}$$

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9. Discuss the continuity of the function f, where f is defined by: $f(x) = \begin{cases} 2x & \text{if } x < 0 \\ 0 & \text{if } 0 \leq x \leq 1 \\ 4x & \text{if } x > 1 \end{cases}$

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10. Show that the function defined by $g(x) = x - [x]$ is discontinuous at all integral points. Here $[x]$ denotes the greatest integer less than or equal to x.

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11. Whether the function f defined by $f(x) = \begin{cases} \frac{\sin x}{x} & \text{if} x < 0 \\ x + 1 & \text{if } x \geq 0 \end{cases}$ at $x = 0$ is continuous or not ?

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12. Find the value of the constant k so that function $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2\pi} & \text{if} x \neq \frac{\pi}{2} \\ 3 & \text{if } x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$

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13. Find all point of discontinuity of f where f is defined by

$$f(x) = \begin{cases} x^3 - 3 & \text{if} x \leq 2 \\ x^2 & \text{if } x > 2 \end{cases}$$

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14. Discuss the continuity of the following function at $x = 0 : f(x) = \begin{cases} \frac{x^4 + 2x^3 + x^2}{\tan^{-1} x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

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15. Discuss the continuity of $f(x) = \sin x \cos x$.

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16. Find the value of the constant k so that the function

$$f(x) = \begin{cases} kx^3 & \text{if} x \leq 2 \\ 3 & \text{if} x > 2 \end{cases}$$ is continuous at $x = 2$

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17. Find the value of k so that the function f defined by

$$f(x) = \begin{cases} kx + 1 & \text{if} x \leq \pi \\ \cos x & \text{if} x > \pi \end{cases}$$ is continuous at point $x = \pi$.

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18. Find the values of k so that the function f, defined by $f(x) = \begin{cases} kx + 1 & \text{if} \quad x \leq 5 \\ 3x - 5 & \text{if} \quad x > 5 \end{cases}$ is continuous at x=5.

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19. Discuss the continuity of the cosine, cosecant, secant and cotangent functions.

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20. Find the relationship between a and b so that the function f defined by:

$$f(x) = \begin{cases} ax + 1 & \text{if } x \leq 3 \\ bx + 3 & \text{if } x > 3 \end{cases}$$ is continuous at $x = 3$

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21. Find the relationship between a and b so that the function f defined by:

$$f(x) = \begin{cases} ax + 1 & \text{if } x \leq 3 \\ bx + 3 & \text{if } x > 3 \end{cases}$$ is continuous at $x = 3$

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22. For what value of λ is the function defined by

$$f(x) = \{\lambda(x^2 - 2x), \quad \text{if} \quad x \leq 0, 4x + 1, \quad \text{if} \quad x > 0\}$$

continuous at x = 0? What about continuity at x = 1?

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23. Disuss the continuity of the function f where is defined by

$$f(x) = \begin{cases} 3 & \text{if} \quad 0 \leq x \leq 1 \\ 3 & \text{if} \quad 1 < x < 3 \\ 5 & \text{if} \quad 3 \leq x < 10 \end{cases}$$

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24. Show that the function defined by $f(x) = \cos\left(x^2\right)$ is a continuous function.

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25. Show that the function defined by $f(x) = |\cos x|$ is a continuous function.

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26. Examine if sin |x| is a continuous function.

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27. Find all the points of discontinuity of f defined by

$f(x) = |x| - |x + 1|$

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28. Differentiate the following functions with respect to x.

$\sin(ax + b)$

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29. Differentiate the following functions with respect to x.

$2\sqrt{\cot\left(x^2\right)}$

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30. Find the derivative of the function given by $f(x) = \sec\left(\tan\left(\sqrt{x}\right)\right)$.

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31. Find $\frac{dy}{dx}$, if $y = \cos x^3 . \sin^2\left(x^5\right)$

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32. Differentiate: $(\log x)^n + x^{\log x}$

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33. If $y^x = x^y$, find $\frac{dy}{dx}$.

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34. Prove that the functions given by $f(x) = |x - 1|, x \in R$ is not differentiable at $x = 1$.

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35. Prove that the greatest integer function defined by $f(x) = |X|, 0 < x < 3$ is not differential at $x = 1.$

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36. Find $\frac{dy}{dx}$ in the following:

$ax + by^2 = \cos y$

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37. Find $\frac{dy}{dx}$ in the following: $\sin^2 x + \cos^2 y = 1$

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38. Find $\frac{dy}{dx}$ in the following:

$\sin^2 y + \cos xy = \pi$

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39. Find $\frac{dy}{dx}$ in the following:

$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

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40. Find $\frac{dy}{dx}$ in the following:

$$y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$$

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41. Derivative of $\sin^{-1}\left(2x\sqrt{1-x^2}\right), -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$ w.r.t.x is .....

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42. If $y = \sec^{-1}\left(\frac{1}{2x^2 - 1}\right)$, then find $\frac{dy}{dx}$, given $0 < x < \frac{1}{\sqrt{2}}$

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43. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2} - 1}{x}\right)$ w.r.t.x .

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44. Differentiate the following w.r.t.x :

$\log(\log x), x < 1$

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45. Differentiate the following w.r.t x

$\sin\left(\tan^{-1}\left(e^{-x}\right)\right)$

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46. Differentiate w.r.t. x : $\log(\cos e^{x})$.

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47. Differentiate the following w.r.t. x

$e^{x}+e^{x^{3}}+\ldots.+e^{x^{5}}$

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48. Differentiate the following w.r.t x

cos(logx + e^x) , x > 0

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49. Differentiate the following w.r.t.x :

$\sqrt{e^{\sqrt{x}}}, x > 0$

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50. Differentiate the following w.r.t. x

$$\frac{\cos x}{\log x}, x > 0$$

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51. If $f(x) = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$, find $f'(x)$. Also find $f'\left(\frac{\pi}{2}\right)$.

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52. Differentiate the function w.r.t. x : $(\log x)^{\cos x}$

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53. Differentiate the function w.r.t. x :

$$\frac{\sqrt{(x-1)(x-2)}}{(x-3)(x-4)(x-5)}$$

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54. Find the derivative of the given functions

$$x^x - 2^{\sin x}$$

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55. Find the derivative of the given functions

$$(\sin x)^x + \sin^{-1}\sqrt{x}$$

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56. Differentiate the following w.r.t. x :

$x^{x^2-3}+(x-3)^{x^2}, x>3$

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57. Differentiate the function

$(x+3)^2.(x+4).(x+5)^4$ w.r.t.x.

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58. Differentiate $(\log x)^x + x^{\log x}$ w.r.t.x .

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59. Differentiate $(\sin x)^x + x^{\sin x}$, w.r.t. 'x'.

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60. Differentiate the following w.r.t. x:

$\sin\left(\tan^{-1} e^{-x}\right)$

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61. Differentiate $(\sin x)^{\sin x}$ w.r.t. x.

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62. Find $\frac{dy}{dx}$ of the function : $(\cos x)^y = (\cos y)^x$

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63. Find $\frac{dy}{dx}$ of the function : $xy = e^{x-y}$

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64. Differentiate the function w.r.t. x :

$$x^{x\cos x}+\frac{x^2+1}{x^2-1}$$

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65. Differentiate $x^{\sin x}+(\sin x)^{\cos x}$ w.r.t.x.

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66. Find $\frac{dy}{dx}$, if x and y are connected by the reaction,

$x=\cos\theta-\cos 2\theta$ and $y=\sin\theta-\sin 2\theta$

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67. If $x^y + y^x = 1$, find $\frac{dy}{dx}$

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68. For a positive constant a find $\frac{dy}{dx}$, where $y = a^{t+1}$ and $x = \left(t + \frac{1}{t}\right)^a$.

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69. If $y^x = x^y$, find $\frac{dy}{dx}$.

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70. If $x = \sqrt{a^{\sin^{-1}t}}$ and $y = \sqrt{a^{\cos^{-1}t}}$, then show that $\frac{dy}{dx} = -\frac{y}{x}$

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71. If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{\{\log(xe)\}^2}$

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72. $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$

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73. If $y = 3e^{2x} + 2e^{3x}$, prove that $d^2\frac{y}{dx^2} - 5\frac{dy}{dx} + 6y = 0.$

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74. $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$

Watch Video Solution

75. If $y = 3e^{2x} + 2e^{3x}$, prove that $d^2\frac{y}{dx^2} - 5\frac{dy}{dx} + 6y = 0.$

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76. Find the second order derivative of the following functions

$x^3 \log x$

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77. Find the second order derivative of $\log(\log x)$.

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78. If $y = \left(\tan^{-1} x\right)^2$, then show that $\left(x^2+1\right)^2 y_1 + 2x\left(x^2+1\right) y_1 = 2$

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79. If $y = 3\cos(\log x) + 4\sin(\log x)$ show that $x^2 y_2 + x y_1 + y = 0$

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80. If $e^y(x+1) = 1$ show that y'' $= (y')^2$

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81. If $y = Ae^{mx} + Be^{nx}$, Show that $\left(\frac{d^2}{dx^2}y\right) - (m+n)\frac{dy}{dx} + mny = 0$

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82. If $y = \sin^{-1} x$, prove that $\left(1 - x^2\right)y_2 - xy_1 = 0$

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83. If $y = A\sin x + B\cos x$then prove that $d^2\frac{y}{dx^2} + y = 0$

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84. If $y = 3e^{2x} + 2e^{3x}$, prove that $d^2\frac{y}{dx^2} - 5\frac{dy}{dx} + 6y = 0.$

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85. If $y = 500e^{7x} + 600e^{-7x}$ show that $\frac{d^2y}{dx^2} = 49y$

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86. If $x = \tan\left(\frac{1}{a}\log y\right)$, show that $\left(1 + x^2\right)\frac{d^2y}{dx^2} + (2x - a)\frac{dy}{dx} = 0$

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87. If $\sin y = x \sin(a + y)$, prove that $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$

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88. If $(\cos x)^y = (\sin y)^x$, then find $\frac{dy}{dx}$.

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89. Verify Rolle's theorem for the function $f(x) = x^2 + 2x - 8, x \in [-4, 2]$

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90. Examine if Rolle's theorem is applicable to any of the following functions. Can you say some thing about the converse of Rolle's theorem from these example? $f(x) = [x]$ for $x \in [-2, 2]$

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91. Examine if Rolle's theorem is applicable to any of the following functions. Can you say some thing about the converse of Rolle's theorem from these example? $f(x) = x^2 - 1$ for $x \in [1, 2]$

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92. Verify Mean Value Theorem, if $f(x) = x^2 - 4x - 3$, in the interval [a, b], where a = 1 and b = 4.

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93. Verify Mean Value Theorem, if $f(x) = x^3 - 5x^2 - 3x$, in the interval [a, b], where a = 1 and b = 3. Find all $c \in (1, 3)$ for which $f'(c) = 0$.

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94. Verify Lagrange's mean value theorem for the following functions

$f(x) = x^2 + 2x + 3$ in the interval $[4, 6]$

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Important Questions From Miscellaneous Exercise

1. Find $\frac{dy}{dx}$, if x and y are connected parametrically by the equations, given below without eliminating the parameter.

$y = 12(1 - \cos t), x = 10(t - \sin t), -\frac{\pi}{2} < t < \frac{\pi}{2}$

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2. If $\cos y = x\cos(a+y)$, then prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$ where cos $awhere$cos ane+-1` .

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3. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ then $\frac{dy}{dx}$ equals.

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4. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ then $\frac{dy}{dx}$ equals.

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5. Differentiate w.r.t. x the function : $\frac{\cos^{-1}\left(\frac{x}{2}\right)}{\sqrt{2x+7}}$, x lies between -2 and 2

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6. find $\frac{dy}{dx}$, if $y\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right], 0 > x > \frac{\pi}{2}$

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7. Differentiate the following w.r.t. x:

$(\log x)^{\log x}, x > 1$

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8. Find $\frac{dy}{dx}$, if $y = (\sin x - \cos x)^{\sin x - \cos x}, \frac{\pi}{4} > x > \frac{3\pi}{4}$

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9. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, find $\frac{d^2y}{dx^2}$

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10. If $y = e^{a\cos^{-1}x}$, $-1 \leq x \leq 1$, show that

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Multiple Choice Questions

1. Derivative of $\tan\left(\frac{\pi}{2} - x\right)$ is

A. $\sec^2\left(\frac{\pi}{2} - x\right)$

B. $-\cos ec^2 x$

C. $\cos ec^2 x$

D. None of these.

Answer: D

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2. The derivative of f(x) =|x | at x=0 is

A. 1

B. 0

C. -1

D. Does not exist.

Answer: A

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3. The derivative of e^{nx} is

A. $-\mathrm{ne}^{-nx}$

B. ne^{-nx}

C. $-\mathrm{ne}^{nx}$

D. ne^{nx}

Answer: C

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4. The derivative of $\log(ax+b)$ is

A. $\frac{1}{ax+b}$

B. $\frac{b}{ax+b}$

C. $\frac{a}{ax+b}$

D. $\frac{a+b}{ax+b}$

Answer: C

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5. Find the derivative of $(ax+b)^n$

A. na^{n-1}

B. nx^{n-1}

C. $na(ax+b)^{n-1}n$

D. nx^{n}

Answer: D

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6. If $y = a^{\log x}, a > 0$ then $\frac{dy}{dx}$ equals

A. $\frac{a^{\log x}}{x}$

B. $\frac{a^{\log x}}{\log a}$

C. $\frac{a^{\log x}}{\log x}$

D. None of these.

Answer: A

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7. The derivative of a^x is

A. $a^x \log a$

B. a^x

C. $\frac{a^x}{\log a}$

D. None of these.

Answer: A

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8. The derivative of $\log(\log x)$ w.r.t.x is

A. $(x \log x)^{-1}$

B. $x \log x$

C. $\frac{\log x}{x}$

D. None of these.

Answer: A

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9. The derivative of $e^{\sin 1x}$ is

A. $\dfrac{e^{\sin 1x}}{\sqrt{1}-x^2}$

B. $\dfrac{e^{\sin -1x}}{\sqrt{1}+x^2}$

C. $e^{\sin x}\left(\sqrt{1}-x^2\right)$

D. None of these.

Answer: B

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10. The derivative of $\cos 6(-1)x$ is

A. $\frac{1}{\sqrt{1-x^2}}$

B. $-\frac{1}{\sqrt{1-x^2}}$

C. $\frac{1}{1+x^2}$

D. $\frac{1}{\sqrt{x^2-1}}$

Answer: C

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11. The second order derivative of $\log x$ is

A. $\frac{1}{x}$

B. $\frac{1}{x^2}$

C. $\frac{-1}{x^2}$

D. None of these.

Answer:

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