MATHS

BOOKS - OMEGA PUBLICATION

RELATIONS AND FUNCTIONS

Questions

1. Determine whether each of the following relations are reflexive, symmetric and transitive :

(i) Relation R in the set A = {1, 2, 3,......, 13, 14} defined as $R = \{(x, y) : 3x - y = 0\}$

(ii) Relation R in the set N of natural numbers defined as $R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$

(iii) Relation R in the set A = {1, 2, 3, 4, 5, 6} defined as R = {(x, y): y is divisible by x}.

iv) Relation R in the set Z, of all integers defined as R = {(x, y) : x -y is an integer}.

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2. Check whether the relation R defined in the set {1, 2,3,4, 5, 6} as R = {(a, b) : b = a + 1} is reflexive, symmetric or transitive.

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3. Check whether the relation-R defined in the set of all real numbers as $R = \{(a, b) : a \leq b^3\}$ is reflexive, symmetric or transitive.

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4. Show that the relation R in the set A of all the books in a library of a college, given by R = {(x, y): x andy have same number of pages} is an equivalence, relation.

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5. Let L be the set of all lines in XY-pJane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1$ is parallel to $L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$

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6. Let T be the set of all triangles in a plane with R a relation in T given by : $R = \{(T_1, T_2) : T_1$ is congruent to T_2}. Show that R is an equivalence relation.

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7. Show that the relation R in, the set A = {1, 2, 3, 4, 5} given by $R = \{(a, b) : |a - b| \text{ is even}\}$, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5}. is related to any element of. {2, 4).

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8. Give any example of a relation, which is:

(i) symmetric: but neither reflexive nor transitive.

ii) transitive but neither reflexive nor symmetric.

iii) reflexive and symmetric but not transitive.

iv) reflexive and transitive but not symmetric.

v) symmetric and transitive but not reflexive.

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9. Prove that greatest integer function $f: R \rightarrow R$, given by $f(x) = [x]$, is neither one-one nor onto where [x] denotes the greatest integer less than or equal to x .

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10. Prove that Modulus Function $f: R \rightarrow R$ given by : $f(x) = |x|$ is neither one-one nor onto, where $|x|$ is x, if x is positive and $|x|$ is - x, if x is negative.

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11. Show that the signum function $f: R \rightarrow R$ given by:

$$f(x) = \begin{cases} 1 & \text{if } \; x > 0 \\ 0 & \text{if } \; x = 0 \\ -1 & \text{if } \; x < 0 \end{cases}$$ is neither one-one nor onto.

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12. Consider $f: R \to [-5, \infty]$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \left[\frac{\sqrt{y+6}-1}{3}\right]$

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13. Let $f: N \to N$ be defined by $f(n) = \begin{cases} \frac{n+1}{2} & \text{if n is odd} \\ \frac{n}{2} & \text{if n is even} \end{cases}$ for all $n \in N$.

State whether the function f is bijective.

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14. Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f: A \to B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

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15. Find gof and fog, if

$f(x) = 8x^3$ and $g(x) = x^{2/3}$.

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16. Let $f: \{1, 3, 4\} \rightarrow \{1, 2, 5\}$ and $g: \{1, 2, 5\} \rightarrow \{1, 3\}$ be given by f= {(1, 2), (3, 5), (4,1)} and g = {(1, 3), (2, 3), (5, 1)}. Find gof,

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17. If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$, show that (fof) (x) = x for all $\neq \frac{2}{3}$. What is inverse of 'f' ?

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18. Show that $f:[-1, 1] \rightarrow R,$ given by $f(x) = \frac{x}{x+2}$ is one-one. Find the inverse of the function f: [-1,1] $\rightarrow$ Range f.

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19. Consider f : R → R given by $f(x) = 4x + 3.$

Show that f is invertible. Find the inverse of f.

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20. Let $f : R \rightarrow R$, be defined as $f(x) = 10x + 7$.

Find the function $g : R \rightarrow R$ such that $gof = fog = 1_R$

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21. Find gof and fog if $f : R \rightarrow R$ and $g : R \rightarrow R$ are given by

f(x) = cos x and g (x) $= 3x^2$

Show that $gof \neq fog$

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22. Consider $f: R_+ \rightarrow [4, \infty)$ given by $f(x) = x^2 + 4$ Show that f is invertible with the inverse $f^{-}1, of f given by f^{-}1(y) = \sqrt{y-4}$, where R_+ is the set of all non-negative real numbers.

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23. Let $f: X \rightarrow Y$ be an invertible function. Show that the inverse of $f^{-}1$ is f, i.e. $(f^{-}1)^{-}1 = f$

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24. Consider the binary operation $\wedge$ on the set $\{1, 2, 3, 4, 5\}$ defined by $a \wedge b = \min\{a, b\}$ Write the operation table of the operation $\wedge$.

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25. Let '*' be the binary operation on the set {I, 2, 3, 4,5} defined by a*b = H.C.F. of a and b. Write the operation table of the operation '*'.

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26. Is $*$ defined on the set $\{1, 2, 3, 4, 5\}$ by $a \cdot b = L.\,C.\,M.\, of a$ and b a binary operation? Justify your answer.

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27. Let be the binary operation on N defined by a*b = H.C.F. of a and b: Is '*' commutative? Is * associative ? Does there exist identity for this binary operation on N ?

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28. Show that the binary operation on the set 'N' given by a * b =1 $\forall a, b \in N$ is commutative as well as associative.

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29. The binary operation $* : R \times R \to R$ is defined as $a * b = 2a + b$. Find $(2 * 3) * 4$

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30. Consider the binary operations * : $R \times R \to R$ and $o : R \times R \to R$ defined as a * b = |a-b| and a o b = a for all a,b in R`. Show that '*' is commutative but not associative, 'o' is associative but not commutative.

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31. Show that the binary operation on the set 'N' given by $a \cdot b = \frac{a+b}{2} \forall a, b \in N$ is commutative but not associative.

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32. Show that the binary operations on the set Q of rational numbers given by $a \cdot b = (a - b)^2$ is commutative but not associative.

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33. State whether the following statements are true or false. Justify.

(i) For an arbitrary binary operation on a set N, $a \times a = a \forall a \in N$.

ii) If * is a commutative binary operation on N, then $a \times (b \times c) = (c \times b) \times a$.

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34. Determine whether each of the following relations are reflexive, symmetric and transitive :

Relation R in the set A = {1, 2, 3,......., 13, 14} defined as

$R = \{(x, y) : 2x - y = 0\}$

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36. Check whether the relation-R defined in the set of all real numbers as $R = \{(a, b) : a \leq b^3\}$ is reflexive, symmetric or transitive.

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37. Show that the relation R in the set A of all the books in a library of a college, given by R = {(x, y): x andy have same number of pages} is an equivalence, relation.

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40. Show that the relation R in, the set A = {1, 2, 3, 4, 5} given by $R = \{(a, b) : |a - b| \text{ is even}\}$, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5}. is related to any element of. {2, 4).

41. Give any example of a relation, which is:

(i) symmetric: but neither reflexive nor transitive.

ii) transitive but neither reflexive nor symmetric.

iii) reflexive and symmetric but not transitive.

iv) reflexive and transitive but not symmetric.

v) symmetric and transitive but not reflexive.

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42. Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. Show that f is one-one.

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43. Prove that greatest integer function $f: R \to R$, given by $f(x) = [x]$, is neither one-one nor onto where [x] denotes the greatest integer less than or equal to x .

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44. Prove that Modulus Function $f: R \to R$ given by : $f(x) = |x|$ is neither one-one nor onto, where $|x|$ is x, if x is positive and $|x|$ is - x, if x is negative.

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45. Show that the signum function $f: R \to R$

given by:

$f(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$ is neither one-one nor

onto.

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46. Consider $f: R \to [-5, \infty]$ given by

$f(x) = 9x^2 + 6x - 5$. Show that f is invertible

with $f^{-1}(y) = \left[\dfrac{\sqrt{y+6} - 1}{3}\right]$

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47. Let $f: N \to N$ be defined by $f(n) = \begin{cases} \frac{n+1}{2} & \text{if n is odd} \\ \frac{n}{2} & \text{if n is even} \end{cases}$ for all $n \in N$.

State whether the function f is bijective.

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48. Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f: A \to B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

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49. Find gof and fog, if

(i) $f(x) = |x|$ and $g(x) = |5x - 2|$

(ii) $f(x) = 8x^3$ and $g(x) = x^{1/3}$.

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50. If f:(1,3,4} $\rightarrow$ {1, 2,5} and g:(1,2,5) {1,3} be given by f = {(1, 2), (3,5),(4, 1)} and g: {(1,3), (2, 3), (5, 1)}, write down gof.

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51. Let $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$ .show that fof (x) = x.

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52. Show that $f:[-1, 1] \to R,$ given by $f(x) = \frac{x}{x+2}$ is one-one. Find the inverse of the function f: [-1,1] $\to$ Range f.

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53. Consider f : R → R given by $f(x) = 4x + 3.$

Show that f is invertible. Find the inverse of f.

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54. Let $f: R \rightarrow R$, be defined as $f(x) = 10x + 7.$

Find the function $g: R \rightarrow R$ such that

$gof = fog = 1_R$

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55. Find gof and fog if $f: R \rightarrow R$ and $g: R \rightarrow R$

are given by

f(x) = cos x and g (x) $= 3x^2$

Show that $gof \neq fog$

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56. Consider $r \rightarrow [4, \infty]$ given by $f(x) = x^2 + 4$.

Show that f is invertible with the increase $f^{-1}off$

given by $f^{-1}(y) = \sqrt{y-4}$, where R is the set of

all non-negative real numbers.

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57. Let $f: X \to Y$ be an invertible function. Show that the inverse of $f^{-}1$ is f, i.e. $(f^{-}1)^{-}1 = f$

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58. Consider the binary operation '*' on the set {1,2,3,4,5} defined by a * b = min. {a,b}. Write the operation table of the operation '*'

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59. Let '*' be the binary operation on the set {I, 2, 3, 4,5} defined by a*b = H.C.F. of a and b. Write the operation table of the operation '*'.

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60. Is '*' defined on the set {1,2,3,4,5} by a*b = L.C.M. of a and b a binary operation ? Justif your answer.

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61. Show that the binary operation on the set 'N' given by a * b =1 $\forall a, b \in N$ is commutative as well as associative.

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62. The binary operation $* : R \times R \to R$ is defined as $a * b = 2a + b$. Find $(2 * 3) * 4$

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63. Consider the binary operations $* : R \times R \to R$ and $o : R \times R \to R$ defined as a * b = |a-b| and a o b = a for all a,b in R`. Show that '*' is commutative but not associative, 'o' is associative but not commutative.

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64. Show that the binary operation on the set 'N' given by $a \cdot b = \frac{a+b}{2} \forall a, b \in N$ is commutative but not associative.

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65. Show that the binary operations on the set Q of rational numbers given by $a \cdot b = (a - b)^2$ is commutative but not associative.

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66. State whether the following statements are true or false. Justify.

(i) For an arbitrary binary operation on a set N, $a \times a = a \forall a \in N$.

ii) If * is a commutative binary operation on N, then $a \times (b \times c) = (c \times b) \times a$.

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Important Questions From Miscellaneous Exercise

1. If $f: R \rightarrow R$ is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$.

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2. Let $f: W \rightarrow W$ be defined as $f(n) = n - 1$, if n is odd and $f(n) = n + 1$, if n is even. Show that f is invertible. Find the inverse of f. Here, W is the set of all whole numbers.

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3. Find the number of all onto functions formthe set {1,2,3,....,n} to itself.

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4. Let $S = \{a, b, c\}$ and $T = \{1, 2, 3\}$. Find $F^{-}1$ of the following functions F from S to T, if it exists: $F = \{(a, 3), (b, 2), (c, 1)\}$

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5. Let $f: R \rightarrow R$ be the signum function defined as $f(x)=\begin{cases}1 & x>0 \\ 0 & x=0 \\ -1 & x<0\end{cases}$ and $g: R \rightarrow R$ be the greatest integer function given by $g(x)=[x]$ where [x] is greatest integer less than or equal to x. Then, does fog and gof coincide in (0,1] ?

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6. Let $*$ be a binary operation on N given by $a * b$= HCF (a, b), $a, b, \in N$. Write the value of $22 * 4$.

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7. If $f: R \to R$ is defined by $f(x) = x^2 - 3x + 2$, find f(f(x)).

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8. Let $f: W \to W$ be defined as $f(n) = n - 1$, if n is odd and $f(n) = n + 1$, if n is even. Show that f is invertible. Find the inverse of f. Here, W is the set of all whole numbers.

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9. Find the number of all onto functions formthe set {1,2,3,.....,n} to itself.

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10. Let S = {a, b, c] and T = {1,2,3}. Find F^{-1} of the following functions F from S to T, if exists. (i) $F = \{(a, 3), (b, 2), (c, 1)\}$, (ii) $F = \{(a, 2), (b, 1), (c, 1)\}$.

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11. Let $f: R \to R$ be the signum function defined as $f(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$ and $g: R \to R$ be the greatest integer function given by $g(x) = [x]$ where [x] is greatest integer less than or equal to x. Then, does fog and gof coincide in (0,1] ?

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12. Let '*' be a binary operation on N given by a*b = H.C.F (a,b), a,b $\in$ N. Write the value of 22*4.

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Multiple Choice Questions Mcqs

1. Let the relation in the set {1, 2, 3, 4} given by R= {(1, 2), (2, 2), (1, 1), (4,4), (1, 3), (3, 3), (3, 2) then (a)R is reflexive and symmetric but not transitive (b)R is reflexive and transitive but not symmetric (c)R is symmetric and transitive but not reflexive (d)R is an equivalence relation

A. R is reflexive and symmetric but not transitive

B. R is reflexive and transitive but not symmetric

C. R is symmetric and transitive but not reflexive

D. R is an equivalence relation.

Answer: B

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2. Let R be the relation in the set N given by $R = \{(a, b) : a = b - 2, b > 6\}$. Choose the correct answer:

A. $(2, 4) \in R$

B. $(3, 8) \in R$

C. $(6, 8) \in R$

D. $(8, 7) \in R$

Answer: C

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3. Let $f: R - R$ be defined as $f(x) = x^4$, then (a) f is one-one (b) f is many-one onto (c) f is one-one but not onto (d) f is neither one-one nor onto

A. f is one-one onto

B. f is many-one onto

C. f is one-one but not onto

D. f is neither one-one nor onto

Answer: D

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4. Let $f : R \to R$ be defined as f(x) = 3x. Then

a. f is one-one onto

b. f is many-one onto

c. f is one-one but not onto

d. f is neither one-one nor onto

A. f is one-one onto

B. f is many-one onto

C. f is one-one but not onto

D. f is neither one-one nor onto

Answer: A

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5. If $f: R \rightarrow R$ be given by: $f(x) = \left(3 - x^3\right)^1 / 3,$

then f(f(x)) is:

A. $x^{1/3}$

B. x^3

C. x

D. $(3 - x^3)$

Answer: C

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6. Let $f: R-\left\{\frac{4}{3}\right\} \rightarrow R$ be a function defined as $f(x) = 4\frac{x}{3x+4}$ The inverse of f is the $(Map)g$: Range $f \rightarrow R-\left\{\frac{4}{3}\right\}$, given by :

A. $g(y) = \frac{3y}{3-4y}$

B. $g(y) = \dfrac{4y}{4 - 3y}$

C. $g(y) = \dfrac{4y}{3 - 4y}$

D. $g(y) = \dfrac{3y}{4 - 3y}$

Answer: B

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7. Consider a binary operation $*$ on N defined as $a \cdot b = a^3 + b^3$ Choose the correct answer: Is $*$ neither commutative nor associative?

A. operation '*' is both associative anc commutative

B. operation '*' is commutative but nol associative

C. operation '*' is associative but nol commutative

D. operation '*' is neither commutative not associative

Answer: B

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8. Let $A = \{1, 2, 3\}$ Then number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is :

A. 1

B. 2

C. 3

D. 4

Answer: A

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9. Let $A = \{1, 2, 3\}$ Then number of equivalence relations containing (1, 2) is:

A. 1

B. 2

C. 3

D. 4

Answer: B

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10. Number of binary operations on the set {a, b} is

:

A. 10

B. 16

C. 20

D. 8

Answer: B

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11. Let R be the relation in the set {1,2,3,4} given by:

$$R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$$

. Then:

A. R is reflexive and symmetric but not transitive

B. R is reflexive and transitive but not symmetric

C. R is symmetric and transitive but not reflexive

D. R is an equivalence relation.

Answer: B

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12. Let R be the relation in the set N given by $R = \{(a, b) : a = b - 2, g > 6\}$. Then

A. $(2, 4) \in R$

B. $(3, 8) \in R$

C. $(6, 8) \in R$

D. $(8, 7) \in R$

Answer: C

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13. Let $f: R \rightarrow R$ be defined as $f(x) = x^4$. Choose the correct answer.

A. f is one-one onto

B. f is many-one onto

C. f is one-one but not onto

D. f is neither one-one nor onto

Answer: D

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14. Let $f : R \rightarrow R$ be defined as f(x) = 3x. Then

a. f is one-one onto

b. f is many-one onto

c. f is one-one but not onto

d. f is neither one-one nor onto

A. f is one-one onto

B. f is many-one onto

C. f is one-one but not onto

D. f is neither one-one nor onto

Answer: A

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15. If $f: R \rightarrow R$ be given by: $f(x) = \left(3 - x^3\right)^1 / 3$,

then f(f(x)) is:

A. $x^{1/3}$

B. x^3

C. x

D. $\left(3 - x^3\right)$

Answer: C

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16. Let $f: R - \left\{-\frac{4}{3}\right\} \to R$ be a function defined as $f(x) = \frac{4x}{3x+4}$. The inverse of f is the map g : Range $f \to R = \left\{-\frac{4}{3}\right\}$ given by

A. $g(y) = \frac{3y}{3-4y}$

B. $g(y) = \frac{4y}{4-3y}$

C. $g(y) = \frac{4y}{3-4y}$

D. $g(y) = \frac{3y}{4-3y}$

Answer: B

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17. Consider a binary operation '*' on N defined as $a \times b = a^3 + b^3$. Then

A. operation '*' is both associative anc commutative

B. operation '*' is commutative but nol associative

C. operation '*' is associative but nol commutative

D. operation '*' is neither commutative not associative

Answer: B

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18. Let $A = \{1, 2, 3\}$ Then number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is :

A. 1

B. 2

C. 3

D. 4

Answer: A

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19. Let $A = \{1, 2, 3\}$ Then number of equivalence relations containing (1, 2) is:

A. 1

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C. 3

D. 4

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20. Number of binary operations on the set {a, b} is

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