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# MATHS

## BOOKS - OMEGA PUBLICATION

## VECTOR ALGEBRA

**Question**

**1.** Classify the following as scalar and vector quantities.

(i) Time period, (ii) Force, (iii) Work done

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**2.** Find the values of x and y so that the vectors $2\vec{i} + 3\vec{j}$ and $x\vec{i} + y\vec{j}$ are equal.

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**3.** Find the unit vector in the direction of vector $\overrightarrow{PQ}$, where P and Q are the points (1,2,3) and (4,5,6) respectively.

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**4.** Show that the points with position vectors $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$ respectively form the vertices of a right angled triangle.

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**5.** Show that the vector $\vec{i} + \vec{j} + \vec{k}$ is equally inclined to the axes OX, OY and OZ.

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**6.** Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i}+2\hat{j}-\hat{k}$ and $-\hat{i}+\hat{j}+\hat{k}$ respectively, in the ratio $2:1$ internally

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**7.** Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2,respectively having $\vec{a}\cdot\vec{b}=\sqrt{6}$

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**8.** Find the projection of the vector $\hat{i}-\hat{j}$ on the vector $\hat{i}+\hat{j}$.

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**9.** Find the projection of the vector $\hat{i}+3\hat{j}+7\hat{k}$ on the vector $7\hat{i}-\hat{j}+8\hat{k}$.

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**10.** Find $\left|\vec{a}-\vec{b}\right|$, if two vectors $\vec{a}$ and $\vec{b}$ are such that $\left|\vec{a}\right|=2, \left|\vec{b}\right|=3$ and $\vec{a}\cdot\vec{b}=4$.

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**11.** Find $\left|\vec{a}\right|$ and $\left|\vec{b}\right|$, if $\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}-\vec{b}\right)=8$ and $\left|\vec{a}\right|=8\left|\vec{b}\right|$.

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**12.** If $\vec{p}$ is a unit vcetor and $\left(\vec{x}-\vec{p}\right)\cdot\left(\vec{x}+\vec{p}\right)=80$, then find $\left|\vec{x}\right|$.

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**13.** Find the magnitude of two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude and such that the angle between them is $60^o$ and their scalar

product is $\frac{1}{2}$.

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**14.** If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of $\lambda$.

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**15.** If $\vec{a} \cdot \vec{a} = 0$ and $\vec{a} \cdot \vec{b} = 0$, then what can be concluded about the vector $\vec{b}$ ?

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**16.** Three vectors $\vec{a}, \vec{b}, \vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ Evaluate the quantity $\mu = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ if $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 2$

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**17.** If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ is

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**18.** If either vector $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, then $\vec{a} \cdot \vec{b} = \vec{0}$. But the converse need not be true. Justify your answer with an example.

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**19.** If the vertices A, B, C of a triangle ABC are (1,2,3), (-1,0,0), (0,1,2) respectively, then find $\angle ABC$ [ $\angle ABC$ is the angle between the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$ ]

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**20.** Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 4, \left|\vec{c}\right| = 5$ and each one of them being perpendicular to the sum of the other two, find $\left|\vec{a} + \vec{b} + \vec{c}\right|$

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**21.** Show that the points A(1,2,7), B(2,6,3) and C(3,10,-1) are collinear.

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**22.** If $\left|\vec{a}\right| = \sqrt{3}, \left|\vec{b}\right| = 2$ and angle between $\vec{a}$ and $\vec{b}$ is $60^\circ$. find $\vec{a} . \vec{b}$

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**23.** Find the area of the triangle with vertices $A(1, 12), B(2, 3, 5), C(1, 5, 5)$

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**24.** Find a unit vector perpendicular to each of the vector $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$

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**25.** Prove that $\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}+\vec{b}\right)=\left|\vec{a}\right|^2+\left|\vec{b}\right|^2$, if and only if $\vec{a}$, $\vec{b}$ are perpendicular, given $\vec{a}\neq\vec{0}$, $\vec{b}\neq\vec{0}$.

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**26.** If a unit vector $\vec{a}$, makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ wih $\hat{j}$ and an acute angle $\theta$ with $\hat{k}$, then find $\theta$ and hence, the components of $\vec{a}$.

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**27.** Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \vec{i} - \vec{j} + 3\vec{k}$ and $\vec{b} = 2\vec{i} - 7\vec{j} + \vec{k}$

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**28.** The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of $\lambda$.

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**29.** Find $\left|\vec{x}\right|$, if for a unit vector $\vec{a}$, $\left(\vec{x} - \vec{a}\right) \cdot \left(\vec{x} + \vec{a}\right) = 12$

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**30.** Find $\left|\vec{a}\ \vec{b}\ \vec{c}\right|$ if $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{c} = 3\hat{i} + \hat{j} - 2\hat{k}$

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**31.** Find $\lambda$, if the vector $\vec{a} = \hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{c} = \hat{i} + \lambda\hat{j} - 3\hat{k}$ are coplanar.

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**32.** Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. Then : if $c_1 = 1$ and $c_2 = 2$ find $c_3$ which makes $\vec{a}$ , $\vec{b}$ , $\vec{c}$ coplanar.

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**33.** Show that the four points with position vectors $4\hat{i} + 8\hat{j} + 12\hat{k}$, $2\hat{i} + 4\hat{j} + 6\hat{k}$, $3\hat{i} + 5\hat{j} + 4\hat{k}$ and $5\hat{i} + 8\hat{j} + 5\hat{k}$ are coplanar.

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**34.** Find $'x'$ such that the four points : A(3,2,1),B(4,x,5),C(4,2,-2) and D(6,5,-1) are coplanar.

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**35.** If $\vec{a}, \vec{b}, \vec{c}$ are coplanar then show that $\vec{a}+\vec{b}, \vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are also coplanar.

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## Important Question From Miscellaneous Exercise

**1.** Write down a unit vector in XY-plane, making an angle of $30^{o}$ with the positive direction of x-axis.

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**2.** A girl walks 4 km towards west, then she walks 3 km in a direction $30^o$ east of north and stops. Determine the girl's displacement from her initial point of departure.

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**3.** If $\hat{i}+\hat{j}+\hat{k}, 2\hat{i}+5\hat{j}, 3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ are the position vectors of points A, B, C and D respectively, then find the angle between AB and CD. Deduce that AB and CD are collinear.

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**4.** Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$

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**5.** The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. Find the unit vector Parallel to its diagonal. Also, find its area.

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**6.** Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{p}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p} . \vec{c} = 18$.

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**7.** Find the vector in the direction of the vector $\hat{i} - 2\hat{j}$ that has magnitude 7 units.

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**8.** Show that the direction cosines of a vector equally inclined to the axes OX, OY and OZ are $\left(\frac{1}{\sqrt{3}}\right), \left(\frac{1}{\sqrt{3}}\right), \left(\frac{1}{\sqrt{3}}\right)$

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**9.** Find the value of '$p$' is : $\left(2\hat{i}+6\hat{j}+27\hat{k}\right)\times\left(\hat{i}+3\hat{j}+p\hat{k}\right)=\vec{0}$.

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**10.** Show that the vectors $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}=-2\hat{i}+3\hat{j}-4\hat{k}$ and $\vec{c}=\hat{i}-3\hat{j}+5\hat{k}$ are coplanar.

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Multiple Choice Questions Mcqs

**1.** Prove that in any triangle ABC , c = a cos(360-B) + b cos(360-A)

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**2.** If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following is incorrect ?

A. $\vec{b} = \lambda \vec{a}$, for some scalar $\lambda$

B. $\vec{a} = \pm \vec{b}$

C. the respective com ponerits of $\vec{a}$ and $\vec{b}$ proportional

D. both the vectors $\vec{a}$ and $\vec{b}$ have same direction but different magnitudes.

**Answer: D**

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**3.** If a is a non-zero vector of magnitude V a is a non-zero scalar, then X am is unit vector.

A. $\lambda = 1$

B. $\lambda = -1$

C. $a = |\lambda|$

D. $a = \frac{1}{|\lambda|}$

**Answer: D**

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**4.** Let the vectors $\vec{a}$ and $\vec{b}$ be such that $\left|\vec{a}\right| = 3$ and $\left|\vec{b}\right| = \frac{\sqrt{2}}{3}$, then $\vec{a} \times \vec{b}$ is a unit vector if the angle between $\vec{a}$ and $\vec{b}$ is :

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

**Answer: B**

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**5.** Area of a rectangle having vertices : $A\left(-\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}\right), B\left(\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}\right), C\left(\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}\right), D\left(-\hat{i}-\frac{1}{2}\right.$ is :

A. $\frac{1}{2}$

B. 1

C. 2

D. 4

**Answer: C**

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**6.** If $\theta$ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a} \cdot \vec{b} \geq 0$ only when:

A. $0 < \theta < \frac{\pi}{2}$

B. $0 \leq \theta \leq \frac{\pi}{2}$

C. $\theta = \frac{\pi}{2}$

D. $\theta = \frac{2\pi}{3}$

**Answer: B**

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**7.** Let $\vec{a}$ and $\vec{b}$ be two unit vectors and $\theta$ is the angle between them. Then $\vec{a} + \vec{b}$ is a unit vector if :

A. $\theta = \frac{\pi}{4}$

B. $\theta = \frac{\pi}{3}$

C. $\theta = \frac{\pi}{2}$

D. $\theta = \frac{2\pi}{3}$

**Answer: D**

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**8.** If $\left(\hat{i}, \hat{j}, \hat{k}\right)$ are the usual three perpendicular unit vectors, then the value of : $\hat{i} \cdot \left(\hat{j} \times \hat{k}\right) + \hat{j} \cdot \left(\hat{i} \times \hat{k}\right) + \hat{k} \cdot \left(\hat{i} \times \hat{j}\right)$ is :

A. 0

B. $-1$

C. 1

D. 3

**Answer: C**

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**9.** Using vectors, find the area of the triangle having vertices A (1, 1, 1), B (1, 2, 3) and C (2, 3, 1).

A. 21 square unit

B. $\frac{1}{2}\sqrt{21}$ square unit

C. $\sqrt{21}$ sqaure unit

D. $\frac{21}{2}$ square unit

**Answer: B**

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**10.** the angle between vectors $\vec{a} \times \vec{b}$ and $\vec{b} \times \vec{a}$ is

A. $180^\circ$

B. $90^\circ$

C. $45^\circ$

D. None of these

**Answer: A**

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**11.** Show that the area of the parallelogram with diagonals $\vec{a}$ and $\vec{b}$, is $\frac{1}{2}\left|\vec{a} \times \vec{b}\right|$.

A. $\left|\vec{a} \times \vec{b}\right|$

B. $\frac{1}{2}\left|\vec{a} \times \vec{b}\right|$

C. $\frac{1}{3}\left|\vec{a} \times \vec{b}\right|$

D. None of these

**Answer: B**

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**12.** The area of triangle having adjacent sides $\vec{a}$ and $\vec{b}$ is :

A. $\frac{1}{2}\left|\vec{a} \times \vec{b}\right|$

B. $\left|\vec{a} \times \vec{b}\right|$

C. $\frac{1}{2}\left|\vec{a} . \vec{b}\right|$

D. None of these

**Answer: A**

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**13.** The value of : $\hat{i} \cdot \left(\hat{j} \times \hat{k}\right) + \hat{j} \cdot \left(\hat{i} \times \hat{k}\right) + \hat{k} \cdot \left(\hat{i} \times \hat{j}\right)$ is :

A. 0

B. $-1$

C. 1

D. 3

**Answer: A**

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**14.** If $\theta$ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\left|\vec{a} \cdot \vec{b}\right| = \left|\vec{a} \times \vec{b}\right|$ when $\theta$ is equal to :

A. 0

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. $\pi$

**Answer: B**

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**15.** If $\vec{a} \cdot \vec{b} = -\left|\vec{a}\right|\left|\vec{b}\right|$, then $\theta =$

A. 0

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

D. $\pi$

**Answer: D**

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**16.** The projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ is :

A. $\frac{10}{\sqrt{6}}$

B. $-\frac{10}{\sqrt{6}}$

C. $\frac{5}{\sqrt{6}}$

D. $-\frac{5}{\sqrt{6}}$

**Answer: A**

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**17.** The product of $\vec{a} . \vec{b}$ equals to:

A. $\vec{b} . \vec{a}$

B. $\vec{b} . \vec{a}$

C. 0

D. None of these

**Answer: B**

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**18.** The product of $\vec{a} . \vec{b}$ equals to:

A. $\vec{a}^2$

B. $\left|\vec{a}\right|$

C. $\left|\vec{a}^2\right|$

D. $\left|\vec{a}\right|^2$

**Answer: D**

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**19.** The vectors a and b are perpendicular, if

A. $\vec{a} . \vec{b} = 0$

B. $\vec{a} \times \vec{b} = 0$

C. $\vec{a} . \vec{b} = 0$

D. None of these

**Answer: C**

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**20.** If $\vec{a} \times \vec{b} = \left|\vec{a}\right|\left|\vec{b}\right| \sin\theta \hat{n}$ which one is correct

A. $\hat{n}$ is unit vector $\perp$ to both $\vec{a}$ and $\vec{b}$

B. $\hat{n}$ is a unit vector || to both $\vec{a}$ and $\vec{b}$

C. $\hat{n}$ is unit vector $\perp$ nor || to $\vec{a}$ and $\vec{b}$

D. None of these

**Answer: A**

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**21.** Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively and when $\vec{a} \cdot \vec{b} = 1$.

A. $\frac{\pi}{3}$

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{2}$

D. None

**Answer: A**

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**22.** Find the angle between the vectors : $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

A. $\cos^{-1}\left(\frac{1}{3}\right)$

B. $\sin^{-1}\left(\frac{1}{3}\right)$

C. $\sin^{-1}\left(-\frac{1}{3}\right)$

D. $\cos^{-1}\left(-\frac{1}{3}\right)$

**Answer: A**

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**23.** Find the area of parallelogram whose adjacent sides are given by the vectors : $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

A. $\frac{1}{2}\sqrt{42}$ sq. unit

B. 42 sq. unit

C. $\sqrt{42}$ sq. unit

D. $\sqrt{21}$ sq. unit

**Answer: A**

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