MATHS

BOOKS - BETTER CHOICE PUBLICATION

APPLICATIONS OF DERIVATIVES

Solved Examples Section I Multiple Choice Questions

1. Find the rate of change of the area of a circle with respect to its radius r at r = 6 cm

A. 10π

B. 12π

C. 8π

D. 11π

Answer: B

Watch Video Solution

2. The slope of the tangent to the curve $y = 4 - x^2$ at x = 1 is

A. 1

B. -2

C. 2

D. 3

Answer: B

Watch Video Solution

3. The normal at the point (1,1) on the curve $2y + x^2 = 3$ is:

A. $x + y = 0$

B. $x + y + 1 = 0$

C. $x - y = 0$

D. $2x - y = 0$

Answer: C

Watch Video Solution

4. Which of the following function is strictly decreasing in $\left(0, \frac{\pi}{2}\right)$?

A. cos x

B. cos 2x

C. cos 3x

D. tan x

Answer: A

Watch Video Solution

5. The interval in which $y = x^2 e^{-x}$ is increasing is:

A. $(-\infty, \infty)$

B. $(-2, 0)$

C. $(2, \infty)$

D. $(0, 2)$

Answer: D

Watch Video Solution

Solved Examples Section Ii

1. Find the rate of change of the area of a circle with respect to its radius r when $r = 3cm$

Watch Video Solution

2. Find the rate of change of the area of a circle with respect to its radius r when $r = 4cm$

Watch Video Solution

3. The volume of a cube is increasing at the rate of $8c\frac{m^3}{s}$. How fast is the surface area increasing when the length of an edge is 12 cm?

Watch Video Solution

4. A stone is dropped into a quiet lake and waves move in circles at the speed of 5 cm/s. At the instant when the radius of the circular wave is 8 cm, how fast is the enclosed area increasing?

Watch Video Solution

5. The radius of a spherical spherical bubble is increasing at the rate of 0.3 cm per second. Find the rate of change of its volume, when the radius is 6 cm.

Watch Video Solution

6. The length x of a rectangle is decreasing at the rate of 2 cm/s and the width y is

increasing at the rate of 2 cm/s. When x = 12 cm and y = 5 cm find the rate of change of (a) the perimeter

Watch Video Solution

7. The length x of a rectangle is decreasing at the rate of 2 cm/s and the width y is increasing at the rate of 2 cm/s. When x = 12 cm and y = 5 cm find the rate of change of (b) the area of the rectangle .

Watch Video Solution

8. A balloon, which always remains spherical has a variable radius. Find the rate at which its volume is increasing with the radius when the later is 10 cm.

Watch Video Solution

9. Sand is pouring from a pipe at the rate of 12 cubic cm./sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of

the base. At which rate is the height of the sand-cone increasing when the height is 4 cm. ?

Watch Video Solution

10. A man of height $2m$ walks at a uniform speed of $5k\frac{m}{h}$ away from a lamp post which is $6m$ high. Find the rate at which the length of his shadow increases.

Watch Video Solution

11. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ?

Watch Video Solution

Solved Examples Section Iii

1. Show that the function given by $f(x) = 3x + 17$ is increasing on R.

Watch Video Solution

2. Show that the function : f(x) = $x^3 - 3x^2 + 6x - 100$ is increasing on R.

Watch Video Solution

3. f(x) = sin x is a strictly increasing in $\left(0, \frac{\pi}{2}\right)$

Watch Video Solution

4. f(x) = sin x is a strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$

Watch Video Solution

5. Prove that the function f given by f(x) = log sin x is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ and strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$.

Watch Video Solution

6. Find the intervals in which the function : $f(x) = 6 - 9x - 2x^2$ is

strictly increasing

Watch Video Solution

7. Find the intervals in which the following functions are strictly decreasing : $f(x) = 6 - 9x - x^2$

Watch Video Solution

8. Find the intervals in which the function f given by

$f(x) = 2x^3 - 3x^2 - 36x + 8$ is

(i) strictly increasing (ii) strictly decreasing

Watch Video Solution

9. Find the intervals in which the function $f(x) = -2x^3 - 9x^2 - 12x + 1$ is strictly increasing or strictly decreasing.

Watch Video Solution

10. Show that $y = \log(1+x) - 2\frac{x}{2+x}, x \succ 1,$ is an increasing function of x. throughout its domain.

Watch Video Solution

11. Find the values of x for which $y = [x(x-2)]^2$ is an increasing function.

Watch Video Solution

Solved Examples Section Iv

1. Find the slope of the tangent to the curve $y = \frac{x-1}{x-2}, x \neq 2$ at x=10

Watch Video Solution

2. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose x-coordinate is 3.

Watch Video Solution

3. Find the slope of the normal to the curve $x = a\cos^3\theta, y = a\sin^3\theta at\theta = \frac{\pi}{4}$

Watch Video Solution

4. Find the equations of the normal to the given curves at the indicated points:

$y = x^4 - 6x^3 \_ 13x^2 - 10x + 5$ at $(0, 5)$

Watch Video Solution

5. Find the equation of tangent and normal to the curve $x = \sin t, y = \cos t$ at $t = \frac{\pi}{4}$.

Watch Video Solution

6. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$, which is parallel to the line $2x - y + 9 = 0$.

Watch Video Solution

7. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is perpendicular to the line $5y - 15x = 13$

Watch Video Solution

8. Find points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to the x-axis.

Watch Video Solution

9. Find the points on the curve $y = 5x^2 - 2x^3$ at which the tangent is parallel to the line $y = 4x + 5$.

Watch Video Solution

10. Find the equation of all lines having slope 2 which are tangent to the curve $y = \frac{1}{x-3}, x \neq 3$.

Watch Video Solution

11. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles* if $8k^2 = 1$

Watch Video Solution

Solved Examples Section V

1. Using elementary transformations , find the inverse of the following matrice $\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

Watch Video Solution

2. Using differentials, find the approximate value of $\sqrt{25.3}$.

Watch Video Solution

3. Using differentials find the approximate value of $(0.009)^{1}/3$

Watch Video Solution

4. Using differential find approximate value of

$\sqrt[3]{26}$

Watch Video Solution

5. Using elementary transformations , find the inverse of the following matrice $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

Watch Video Solution

6. Find the approximate value of f(2.01) where $f(x) = 4x^2 + 5x + 2.$

Watch Video Solution

7. If the radius of a sphere is measured as 7 m with an error of 0.02 m, then find the approximate error in calculating its volume.

Watch Video Solution

Solved Examples Section Vi

1. Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals:

$f(x) = x^3, x \in [-2, 2]$

Watch Video Solution

2. Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals:

$f(x) = \sin x + \cos x, x \in [0, \pi]$

Watch Video Solution

3. Find the maximum profit that a company can make of the profit function is given by $p(x) = 41 - 24x - 18x^2$

Watch Video Solution

4. Find the points at which the function f given by $f(x) = (x-2)^4(x+1)^3$ has local maxima.

Watch Video Solution

5. Find the points at which the function f given by $f(x) = (x-2)^4(x+1)^3$ has local minima.

Watch Video Solution

6. Find the points at which the function f given by $f(x) = (x-2)^4(x+1)^3$ has point of inflexion.

Watch Video Solution

Solved Examples Section Vii Long Answer Type Questions

1. . Find two numbers whose sum is 24 and whose product is as large as possible.

Watch Video Solution

2. Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

Watch Video Solution

3. Show that of all rectangles with given perimeter square has maximum area

Watch Video Solution

4. Find the point on the curve $y^2 = 4x$, which is nearest to the point (2, 1).

Watch Video Solution

5. A window is in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

Watch Video Solution

6. A square piece of tin of side 18 cm is to be made into a box without top by cutting a square from each comer and folding up the

flaps to form a box. What should be the side of square to be cut off so that the volume of box is maximum and also find the volume of box ?

Watch Video Solution

7. Of all the closed cylindrical cans (right circular), of a given volume of $100cm^3$, find the dimensions of the can which has the minimum surface area?

Watch Video Solution

8. An open box with a square base is to be made out of a given Iron sheet of area 27 square meter. Show that the maximum volume of the box is 13.5 cubic meter.

Watch Video Solution

9. Show that of all rectangles inscribed in a given circle the square has maximum area.

Watch Video Solution

10. Prove that volume of largest cone, which can be inscribed in a sphere, is $\left(\frac{8}{27}\right)^{th}$ part of volume of sphere.

Watch Video Solution

11. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

Watch Video Solution

Assignment Most Important Questions For Practice Section I Multiple Choice Questions

1. The radius of a circle is increasing at the rate of 0.14 cm/sec. The rate of change of its area at r = 7 cm is

A. 1.96π

B. 0.98 cm

C. 14π

D. None of these

Answer: A

 Watch Video Solution

2. If the side of a square is increasing at the rate of 3cm/sec, then rate of change of its perimeter is

A. 3 cm/sec

B. 6 cm/sec

C. 9 cm/sec

D. 12 cm/sec

Answer: D

Watch Video Solution

3. Which of the following function is increasing for all values of x in its domain ?

A. sin x

B. log x

C. x^2

D. |x|

Answer: C

Watch Video Solution

4. The interval in which the function $f(x) = 3x^2 - 6x - 5$ is increasing, is :

A. $[-\infty, 1]$

B. $[1, \infty]$

C. $[3, 6]$

D. None of these

Answer: B

Watch Video Solution

5. The function , which is decreasing in the interval $\left[\pi, \frac{3\pi}{2}\right]$ is :

A. sin x

B. cos x

C. tan x

D. None of these

Answer: A

Watch Video Solution

6. The line $y = mx + 1$, is a tangent to the curve $y^2 = 4x$ if the value of m is:

A. 1

B. 2

C. 3

D. $\frac{1}{2}$

Answer: A

Watch Video Solution

7. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is:

A. 3

B. $\frac{1}{3}$

C. -3

D. $\frac{-1}{3}$

Answer: D

Watch Video Solution

8. The point where the tangent to the curve $y = x^2 - 4x + 5$ is parallel to x-axis, is :

A. (2, 1)

B. (1, 2)

C. (2, 4)

D. (-4, 5)

Answer: A

Watch Video Solution

9. The function $f(x) = 2x^2 - 4x + 7$ has a point of minima at x =

A. 0

B. -1

C. 1

D. 2

Answer: C

 Watch Video Solution

10. The function f(x) = sin x + cos x has maxima or minima at x =

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: A

Watch Video Solution

Assignment Short Answer Type Questions Section Ii

1. Find the rate of change of the area of a circle with respect to its radius 'r' when r = 2 cm.

Watch Video Solution

2. The radius of a circle is increasing uniformly at the rate of 7cm/sec. What is the rate of increase of its circumference ?

Watch Video Solution

3. The radius of a circle is increasing uniformly at the rate of 11 cm per second. Find the rate at which the area of the circle is increasing when the radius is 8 cm.

Watch Video Solution

4. The radius of a balloon is increasing at the rate of 10 cm/sec. At what rate is the surface area of the balloon increasing when its radius is 15 cm ?

Watch Video Solution

5. The volume of a balloon is changing at the rate of 25 $\mathrm{cm}^3/\mathrm{sec}$. Find the rate of change of its surface area when its radius is 5 cm.

Watch Video Solution

6. A balloon, which always remains spherical has a variable radius. Find the rate at which its volume is increasing with the radius when the later is 9 cm.

Watch Video Solution

7. The volume of a cube is increasing at the rate of 9 $\mathrm{cm}^3/\mathrm{sec}$. How fast is surface area

increasing when the length of an edge is 10 cm ?

Watch Video Solution

8. The radius of a spherical spherical bubble is increasing at the rate of 0.3 cm per second. Find the rate of change of its volume, when the radius is 6 cm.

Watch Video Solution

9. The radius of a spherical soap bubble is increasing at the rate of 0.3 cm/s. Find the rate of change of its (ii) surface area, when the radius is 8 cm.

Watch Video Solution

10. The radius of a spherical soap bubble is increasing at the rate of 0.2 cm/s. Find the rate of change of its (i) volume

Watch Video Solution

11. The radius of a spherical soap bubble is increasing at the rate of 0.2 cm/s. Find the rate of change of its (ii) surface area, when the radius is 4 cm.

Watch Video Solution

12. The radius of a spherical soap bubble is increasing at the rate of 0.4 cm/s. Find the rate of change of its (i) volume

Watch Video Solution

13. The radius of a spherical soap bubble is increasing at the rate of 0.4 cms. Find the rate of change of its (ii) surface area, when the radius is 10 cm.

 Watch Video Solution

14. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at

which the y-coordinate is changing 8 times as fast as the x-coordinate.

Watch Video Solution

15. A particle moves along the curve $y = \frac{4}{3}x^3 + 5$. Find the points on the curve at which the y-coordinate changes as fast as the x-coordinate.

Watch Video Solution

16. A particle moves along the curve $x^2 = 2y$. At what point the y-coordinate increases as x-coordinate decreases ?

Watch Video Solution

17. A man 160 cm tall, walks away from a source of light situated at the top of a pole 6 m high, at the rate of 1.1m/sec. How fast is the length of his shadow increasing when he is 1 m away from the pole ?

Watch Video Solution

18. Sand is pouring from a pipe at the rate of 9 cm^3 / s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 3 cm ?

Watch Video Solution

19. A man is walking at the rate of 6.5km/h towards the foot of a tower 120m high. At what rate is he approaching the top of tower when he is 50 m away from the tower ?

Watch Video Solution

20. A ladder 16 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 m/sec. How fast is its height on the wall

decreasing when the foot of the ladder is 4 m away from the wall ?

Watch Video Solution

Assignment Section Iii

1. Show that the function $f(x) = mx + c$ when $m > 0$ is strictly increasing on R.

Watch Video Solution

2. Show that the function $f(x) = \frac{1}{x}, x > 0$ is decreasing function.

Watch Video Solution

3. Show that $f(x) = a^x, a > 0$ is increasing for $\forall x \in R$.

Watch Video Solution

4. Prove that $f(x) = x^2 - 4x, x \geq 4$ is increasing function.

Watch Video Solution

5. f(x) = cos x is strictly decreasing in $(0, \pi)$

Watch Video Solution

6. Prove that $f(x) = \cos x$ is strictly decreasing $(0, \pi)$

Watch Video Solution

7. f(x) = cos x is neither increasing nor decreasing in $(0, 2\pi)$

Watch Video Solution

8. Prove that $f(x) = x^7 + 3x^5 + 15$ is increasing function.

Watch Video Solution

9. Prove that $f(x) = \frac{2}{x} + 5$ is strictly decreasing for all $x \in R, x \neq 0$.

Watch Video Solution

10. Prove that $f(x) = \tan x$ is strictly increasing in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Watch Video Solution

11. Prove that $f(x) = \sin x - x$ is decreasing $\forall x \in R.$

Watch Video Solution

12. Determine the intervals in which the following functions $f(x) = x^2 + 2x - 5$ are strictly increasing or strictly decreasing.

Watch Video Solution

13. Find the intervals in which the following functions are strictly increasing :

$f(x) = 6 - 9x - x^2$

Watch Video Solution

14. Find the intervals in which the following functions are strictly increasing or strictly decreasing

$2x^3 + 9x^2 + 12x + 20$

Watch Video Solution

15. Find the intervals in which the following functions are strictly increasing or strictly decreasing

$x^3 - 12x^2 + 36x + 17$

Watch Video Solution

16. Determine the intervals in which the following functions $f(x) = 2x^3 - 15x^2 + 36x + 1$ are strictly increasing or strictly decreasing.

Watch Video Solution

17. Find the intervals in which the following functions are strictly increasing or strictly decreasing

$2x^3 - 6x^2 - 48x + 17$

Watch Video Solution

18. Find the intervals in which the following functions are strictly increasing or strictly

decreasing

$2x^3 - 21x^2 + 36x - 40$

Watch Video Solution

19. Find the intervals in which the following functions are strictly increasing or strictly decreasing

$4x^3 - 6x^2 - 72x + 30$

Watch Video Solution

20. Find the intervals in which the following functions are strictly increasing or strictly decreasing

$x^3 - 6x^2 + 9x + 15$

Watch Video Solution

21. Find the intervals in which the function : $f(x) = 20 - 9x + 6x^2 - x^3$ is strictly increasing.

Watch Video Solution

22. Find the intervals in which the following functions are strictly increasing or strictly decreasing

$x^4 - 2x^2$

Watch Video Solution

23. Find the intervals in which the following functions are strictly increasing or strictly decreasing

$x^4 - 8x^3 + 22x^2 - 24x + 21$

Watch Video Solution

24. Find the intervals in which the following functions are strictly increasing or strictly decreasing

$(x-1)(x-2)^3$

Watch Video Solution

25. Find the intervals in which the following functions are strictly increasing or strictly

decreasing

$(x-1)^3(x-2)^2$

Watch Video Solution

Assignment Section Iv

1. Find the slope of the tangent to the curve $y = 8x^2 - 3$ at $x = \frac{1}{4}$

Watch Video Solution

2. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at x = 3.

Watch Video Solution

3. Find the slope of the tangent to the curve $x = \cos\theta$ and $y = \sin\theta$ at $\theta = \frac{\pi}{4}$.

Watch Video Solution

4. Find the slope of the normal to the curve $x = 1 - a\sin\theta, y = b\cos^2\theta$ at $\theta = \frac{\pi}{4}$.

Watch Video Solution

5. Find the equations of the tangent to the given curves at the indicated points: $y = x^3$ at $(1, 1)$

Watch Video Solution

6. Find the equations of the normal to the given curves at the indicated points:

$y = x^4 - 6x^3 \_ 13x^2 - 10x + 5$ at $(1, 3)$

Watch Video Solution

7. Find the equation of the tangent and normal to the given curve at the indicated points :

$y^2 = \frac{x^3}{4 - x}$ at (2, -2)

Watch Video Solution

8. Find the equation of the tangent and normal to the given curve at the indicated points :

$y = 2x^2 + 3\sin x$ at x = 0

Watch Video Solution

9. Find the equations of the tangent to the given curves at the indicated points: $y = x^2$ at $(0, 0)$

Watch Video Solution

10. Find the equation of the tangent and normal to the given curve at the indicated points :

$y = 2x^2 - 3x - 1$ at (1, -2)

Watch Video Solution

11. Find the equation of the tangent and normal to the given curve at the indicated

points :

$y = \sin^2 x$ at $x = \frac{\pi}{2}$

Watch Video Solution

12. Find the equation of the tangent and normal to the given curve at the indicated points :

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $\left(\sqrt{2}a, b\right)$

Watch Video Solution

13. Find the equation of the tangent to the curve $x = a\cos^3\theta, y = a\sin^3\theta at\theta = \frac{\pi}{4}$

Watch Video Solution

14. Find the equation of tangent to the curve : $x^2 + 3y = 3$ at which tangent line is parallel to the line $y - 4x + 5 = 0$.

Watch Video Solution

15. Find the equation of the tangent to the curve $2x^2 - y = -7$, which is parallel to the line $4x - y + 3 = 0$.

Watch Video Solution

16. Find the equation of the tangent to the curve $4x^3 - 3x + 10 = y$, which is perpendicular to the line $9y + x - 15 = 0$.

Watch Video Solution

17. Find the equation of the tangent to the curve $y = x^3 + 2x + 6$, which is perpendicular to the line $x + 14y + 4 = 0$.

Watch Video Solution

18. Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$

Watch Video Solution

19. Find the point on the curve $y = 2x^3 - 15x^2 + 36x - 21$ at which the tangent is parallel to x-axis. Also, find the equation of tangents.

Watch Video Solution

20. Find the equations of all lines having slope 0 which are tangent to the curve $y = \frac{1}{x2 - 2x + 3}$

Watch Video Solution

21. Find the equation of the normals to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$

Watch Video Solution

22. Find a point on the curve $y = x^3$, where the tangent to the curve is parallel to the chord joining the points (1, 1) and (3, 27).

Watch Video Solution

23. Show that the curve $xy = a^2$ and $x^2 + y^2 = 2a^2$ touch each other.

Watch Video Solution

24. Prove that the curves $y^2 = 4ax$ and $xy = c^2$ cut at right angles if $c^4 = 32a^4$

Watch Video Solution

Assignment Section V

1. Use differentials to find the approximate values of the following

$\sqrt{401}$

Watch Video Solution

2. Use differentials to find the approximate values of the following

$\sqrt{0.037}$

Watch Video Solution

3. Use differentials to find the approximate values of the following

$\sqrt{26}$

Watch Video Solution

4. Use differentials to find the approximate values of the following

$\sqrt{36.6}$

Watch Video Solution

5. Using differential find approximate value of

$\sqrt[3]{25}$

Watch Video Solution

6. Use differentials to find the approximate values of the following

$(26.57)^{1/3}$

Watch Video Solution

7. Use differentials to find the approximate values of the following

$(127)^{1/3}$

Watch Video Solution

8. Use differentials to find the approximate values of the following

$(81.5)^{1/4}$

Watch Video Solution

9. Use differentials to find the approximate values of the following

$(82)^{1/4}$

Watch Video Solution

10. Prove that f(x) = cosx is strictly increasing in $(\pi, 2\pi)$

Watch Video Solution

11. Find the approximate change in the volume V of a cube of side x meters caused by increasing the side by 2%

Watch Video Solution

12. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its volume.

Watch Video Solution

13. Find the approximate value of $f(3.02)$, where $f(x) = 3x^2 + 5x + 3$

Watch Video Solution

Assignment Section Vi

1. Find the absolute maximum value and the absolute minimum value of the following function in the given intervals :

$f(x) = x^3 - 3x, x \in [-2, 2]$

Watch Video Solution

2. Find the absolute maximum value and the absolute minimum value of the following function in the given intervals :

$f(x) = 1 - x - 2x^2, x \in [-1, 1]$

Watch Video Solution

3. Find the absolute maximum value and the absolute minimum value of the following

function in the given intervals :

$$f(x) = x^3 - \frac{5}{2} \times x^2 - 2x + 9, x \in [0, 3]$$

Watch Video Solution

4. Find the absolute maximum value and the absolute minimum value of the following function in the given intervals :

$$f(x) = \left(\frac{1}{2} - x\right)^2 + x^3, x \in [-2, 2.5]$$

Watch Video Solution

5. Find the absolute maximum value and the absolute minimum value of the following function in the given intervals :

$f(x) = 2\cos x + x \quad \text{in} \quad [0, \pi]$

Watch Video Solution

6. Find the absolute maximum value and the absolute minimum value of the following function in the given intervals :

$f(x) = 2\sin x + \sin 2x \quad \text{in} \quad [0, 2\pi]$

Watch Video Solution

7. Find the maximum profit that a company can make of the profit function is given by $p(x) = 65 - 20x - 8x^2$

Watch Video Solution

8. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values,

as the case may be:

$f(x) = x^3 - 6x^2 + 9x + 15$

Watch Video Solution

9. Find the points of local maxima and local minima if any. Also find the local maximum and local minimum value :

$f(x) = x^3 - 3x + 3$

Watch Video Solution

Assignment Section Vii Long Answer Type Questions

1. Find two positive numbers whose sum is 32 and product is large as possible.

Watch Video Solution

2. Find two numbers x and y such that $x + y = 15$ and x^2y^3 is maximum.

Watch Video Solution

3. Find two positive numbers whose sum is 24 and their sum of squares is minimum.

Watch Video Solution

4. Find two positive numbers x and y such that $x + y = 36$ and xy^3 is maximum.

Watch Video Solution

5. Find two positive numbers whose sum is 14 and sum of squares is minimum.

Watch Video Solution

6. Find the area of the largest rectangle having the perimeter 200 cm.

Watch Video Solution

7. Find the rate of change of the area of a circle with respect to its radius r when r = 3 cm.

Watch Video Solution

8. Find the point on the curve $x^2 = 8y$ which is nearest to the point (2, 4)

Watch Video Solution

9. A figure consists of a semi-circle with a rectangle on its diameter. Given perimeter of the figure, find the dimensions in order that the area may be maximum.

Watch Video Solution

10. A window consists if a semi-circle with a rectangel on its diameter. If the perimeter of the window is 30 meters, find the dimensions

of the window in order that its area may be maximum.

Watch Video Solution

11. A square piece of tin of side 18 cm is to be made into a box without top by cutting a square from each comer and folding up the flaps to form a box. What should be the side of square to be cut off so that the volume of box is maximum and also find the volume of box ?

Watch Video Solution

12. Show that the surface area of a closed cuboid with surface base and given volume is minimum when it is cube.

Watch Video Solution

13. Find two positive numbers whose sum is 14 and their sum of squares is minimum .

Watch Video Solution

14. Show that the height of the cylinder, open at the top of given surface area and greatest volume is equal to the radius of its base.

Watch Video Solution

15. Find the rate of change of the area of the circle with respect to its radius r when r = 6 cm.

Watch Video Solution

16. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $2\frac{R}{\sqrt{3}}$. Also find the maximum volume.

Watch Video Solution

17. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical

angle α is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 \tan^2 \alpha$

Watch Video Solution

18. The radius of the circle is increasing at the rate of 0.7 cm/s . What is the rate of increase of its circumference?

Watch Video Solution

19. A balloon which always remain spherical has a variable radius. Find the rate at which its volume is increasing with the radius when the later is 10 cm.

Watch Video Solution

Previous Year Board S Questions For Practice Multiple Choice Questions

1. The slope of tangent to the curve $y = 3 - x^2$ at x = 1 is

A. 1

B. -2

C. 2

D. 3

Answer: B

Watch Video Solution

2. The slope of tangent to the curve $y = 2 - x^2$ at x = 1 is

A. 1

B. 2

C. -2

D. 3

Answer: C

Watch Video Solution

3. The function $f(x) = 3x + 17$ is

A. strictly increasing on R

B. strictly decreasing on R

C. Neither increasing nor decreasing on R

D. None of these

Answer: A

 Watch Video Solution

Previous Year Board S Questions For Practice

1. Find the interval in which the function $f(x) = 2x^3 - 12x^2 + 18x - 5$ is strictly

increasing.

Watch Video Solution

2. Of all the rectangles, each of which perimeter 32 metres, find one having maximum area. Also find the area.

Watch Video Solution

3. Show that of all rectangles inscribed in a given circle the square has maximum area.

Watch Video Solution

4. Use differentials to approximate $\sqrt{49.5}$

Watch Video Solution

5. Find the point on the curve $y^2 = 4x$ which is nearest to the point (2,-8).

Watch Video Solution

6. Find the point on the curve $y^2 = 2x$, which is nearest to the point (1, -4).

Watch Video Solution

7. Of all the closed cylindrical cans (right circular), of a given volume of $100cm^3$, find the dimensions of the can which has the minimum surface area?

Watch Video Solution

8. The radius of a spherical soap bubble is increasing at the rate of 0.5 cm per second. Find the rate of change of its volume, when the radius is 10 cm.

Watch Video Solution

9. The radius of a spherical roap bubble is increasing at the rate of 0.2 cm per second. Find the rate of change of its volume, when the radius is 8 cm.

Watch Video Solution

10. The radius of a spherical roap bubble is increasing at the rate of 0.4 cm per second. Find the rate of change of its volume, when the radius is 7 cm.

Watch Video Solution

11. Find the equation of the tangent to the curve $y = 2x^3 + 7$, which is perpendicular to the line $x + 4y + 5 = 0$.

Watch Video Solution

12. Find the equation of the tangent to the curve $y = x^2 + 4x + 6$, which is perpendicular to the line $x - 2y + 1 = 0$.

Watch Video Solution

13. Find the equation of the tangent to the curve $x^3 + 4y = 4$, which is perpendicular to the line $x - 3y + 5 = 0$.

Watch Video Solution

14. Find the intervals in which the following functions are strictly increasing or strictly decreasing

$x^3 - 6x^2 + 9x + 15$

Watch Video Solution

15. Find the intervals in the following functions are strictly increasing

$f(x) = 2x^3 - 9x^2 + 12x + 5$

Watch Video Solution

16. Find the intervals in the following functions are strictly increasing

$f(x) = 2x^3 - 12x^2 + 18 - 5$

Watch Video Solution

17. Use differentials to approximate

$\sqrt{25.3}$

Watch Video Solution

18. Use differentials to approximate

$3\sqrt{26}$

Watch Video Solution

19. Use differentials to approximate

$\sqrt{0.60}$

Watch Video Solution

20. A wire of length 28 m is to be cut into two pieces, one of the pieces is to be made into a square and the other into a circle. What should be the length ·of two pieces so that the combined area of the square and the circle is minimum ?

Watch Video Solution

21. Using differentials to approximate $3\sqrt{0.009}$

Watch Video Solution

22. The radius of a circle is increasing uniformly at the rate of 5cm/sec. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

Watch Video Solution

23. Find the equations of the tangent and normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_0, y_0)

Watch Video Solution

24. Find the equation of tangent and normal to the hyprobola $x^2 = 4y$ at $\left(at^2, 2at\right)$

Watch Video Solution

25. Find both the maximum value and the minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval $[0, 3]$

Watch Video Solution

26. Find the maximum value of $2x^3 - 24x + 107$ in the interval $[1, 3]$. Find the maximum value of the same function in $[-3, -1]$.

Watch Video Solution

27. Find local maximum and local minimum value of $f(x) = x\sqrt{1 - x : x} > 0$

Watch Video Solution

28. Sand is pouring from a pipe at the rate of 12 cubic cm./sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. At which rate is the height of the sand-cone increasing when the height is 4 cm. ?

Watch Video Solution

29. Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at x = 4 .

Watch Video Solution

30. Find the slope of the tangent to the curve $y = \frac{x-1}{x-2}$ at x = 10.

Watch Video Solution

31. Find the equation of the tangents to the function $y = x^3 + 2x + 6$, which are perpendicular to the line $14y + x + 4 = 0$.

Watch Video Solution

32. Find the equation of the tangents to the function $y = 4x^3 - 3x + 5$ which are perpendicular to the line $x + 9y + 5 = 0$.

Watch Video Solution

33. Find the equation of the tangent line to the curve $y = x^2 - 2x + 9$, which is parallel to the line $2x - y + 7 = 0$.

Watch Video Solution

34. Find the equation of the tangent to the curve, $y = x^2 - 3x + 7$ which is parallel to the line $3x - y + 5 = 0$.

Watch Video Solution

35. Find the equation of the tangent to the curve $y = x^2 - 4x + 11$ which is parallel to the line $4x - y + 8 = 0$

Watch Video Solution

36. Find the intervals in which the following are strictly increasing and strictly decreasing

$f(x) = x^3 - 3x^2 - 105x + 25$

Watch Video Solution

37. Find the intervals in which the following are strictly increasing and strictly decreasing

$f(x) = 30 + 24x - 15x^2 + 2x^3$

Watch Video Solution

38. Find the intervals in which the following are strictly increasing and strictly decreasing

$f(x) = 20 - 12x + 9x^2 - 2x^3$

Watch Video Solution

39. Find the intervals in which the following are strictly increasing and strictly decreasing

$f(x) = 17 - 18x + 12x^2 - 2x^3$

Watch Video Solution

40. Find the intervals in which the following are strictly increasing and strictly decreasing

$f(x) = 2x^3 - 9x^2 + 12x + 30$

Watch Video Solution

41. A window is in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

Watch Video Solution

42. A window is in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 20 m. Find

the dimensions of the window to admit maximum light through the whole opening.

Watch Video Solution

43. Find the slope of the tangent to the curve $y = x^3 - x^2 + 1$ at the point whose x-coordinate is 2 .

Watch Video Solution

44. Using differentials, find the approximate value of $\sqrt{36.5}$.

Watch Video Solution

45. Use differentials to approximate $\sqrt{49.7}$

Watch Video Solution

46. Using differentials, find the approximate value of $\sqrt{25.5}$.

Watch Video Solution

47. Show that the function f given by, $f(x) = x^3 - 3x^2 + 4x, x \in R$ is increasing on R.

Watch Video Solution

48. Prove that $f(x) = \log\cos x$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$ and strictly increasing on $\left(\frac{\pi}{2}, \pi\right)$

Watch Video Solution

49. Find the equation of the normals to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$

Watch Video Solution

50. The length x of a rectangle is decreasing at the rate of 2 cm. per minute and the width is increasing at the rate of 3 cm. per minute. When x = 10 cm and y = 6 cm, find the rates of change of

the perimeter of the rectangle

Watch Video Solution

51. The length x of a rectangle is decreasing at the rate of 2 cm. per minute and the width is increasing at the rate of 3 cm. per minute.

When x = 10 cm and y = 6 cm, find the rates of change of the area of the rectangle

Watch Video Solution

52. The length 'x' of a rectangle is decreasing at the rate of 5 cm per minute and the width 'y' is increasing at the rate of 4 cm per minute, when x = 8 cm and y = 6 cm, find the rate of change of the perimeter of the rectangle.

Watch Video Solution

53. The length 'x' of a rectangle is decreasing at the rate of 5 cm per minute and the width 'y' is increasing at the rate of 4 cm per minute, when x = 8 cm and y = 6 cm, find the rate of change of the area of the rectangle.

Watch Video Solution

54. Find the intervals in which the function $f(x) = 6 + 12x + 3x^2 - 2x^3$ is strictly increasing and srictly decreasing.

Watch Video Solution

55. Find the points on the curve $y = x^3$ at which slope of tangent is equal to x-coordinate of the point.

Watch Video Solution

56. Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

Watch Video Solution

57. Find the equation of tangent to the curve $y = x^3 - 2x^2 - 2x$ at which tangent line is parallel to line $y = 2x - 3$.

Watch Video Solution

58. Find the point on the curve $y = x^3 - 2x^2 - 2x$ at which the tangent lines are parallel to the line $y = 2x - 2$.

Watch Video Solution

59. Find the equation of the tangent line to the curve $x^2 + 3y = 2$ which is parallel to the line $y - 4x + 5 = 0$.

Watch Video Solution

60. Find the point on the curve $x^2 = 4y$ which is nearest to the point (-1,2).

Watch Video Solution

61. Find the interval in which the function $f(x) = 10 - 6x - 2x^2$ is strictly increasing and strictly decreasing.

Watch Video Solution

62. Find a point on the curve $y = (x - 2)^2$ at which the tangent is parallel to the chord joining the points (2,0) and (4,4)

Watch Video Solution

63. Find points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to the x-axis.

Watch Video Solution

64. Use differentials to find the approximate values of the following

$(82)^{1/4}$

Watch Video Solution

65. Find the slope of the tangent of the curve $y = x^2 - 3x + 2$ at the point whose x-coordinate is 3.

Watch Video Solution

66. Evaluate $\int \frac{x}{\sqrt{x-1}} dx$

Watch Video Solution

67. Find two positive numbers whose sum is 16 and whose sum of cubes is minimum.

Watch Video Solution

68. Find the maximum profit that a company can make of the profit function is given by $p(x) = 41 - 24x - 18x^2$

Watch Video Solution

69. Find the equation of normal to the curve, $y = .x^3$ at the point (1,1).

Watch Video Solution

70. Using differentials, find the approximate value of $\sqrt{49.5}$.

Watch Video Solution

71. Find points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to the x-axis.

Watch Video Solution

72. A wire of length 28 m is to be cut into two pieces, one of the pieces is to be made into a square and the other into a circle. What should be the length ·of two pieces so that the combined area of the square and the circle is minimum ?

Watch Video Solution

73. A square sheet of tin of size 24 cm is to be made into a box without top by cutting off squares from each comer and folding up the

flaps. What should be the side of the 'square to be cut off so that the volume of the box is maximum ?

Watch Video Solution