MATHS

BOOKS - BETTER CHOICE PUBLICATION

CONTINUITY AND DIFFERENTIABILITY

Solved Examples Multiple Choice Questions

1. The function $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x & \text{if } x \neq 0 \\ k & \text{if } x = 0 \end{cases}$ is

continuous at $x = 0,$ then then value of $'k'$ is

A. 3

B. 2

C. 1

D. 1.5

2. The derivative of $f(x) = x$ at $x = 1$ equals

A. 1

B. -1

C. $= 0$

D. 2

3. If $y = a^5$ then $\dfrac{dy}{dx}$ is equal to :

A. $5a^4$

B. $a^5 \log a$

C. $\dfrac{a^5}{\log a}$

D. 0

Answer: D

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4. If $x = 2at, y = at^2$, then $\dfrac{dy}{dx}$ equals

A. 2

B. $2a$

C. $2at$

D. t

Answer: D

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5. Derivative of $\sin^{-1}(\cos x)$ w.r.t. x equals

A. -1

B. 1

C. $\cos x$

D. $\sin x$

Answer: A

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6. Derivative of $\left(\tan^{-1}x + \cot^{-1}x\right)$ w.r.t 'x' is equal to :

A. 1

B. 0

C. 1

D. 2

Answer: B

7. If $x^2 + y^2 = 100$, then $\dfrac{dy}{dx}$ equals

A. 0

B. $\dfrac{2x}{y}$

C. $\dfrac{-x}{y}$

D. None of these

Answer: C

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1. Prove that the function

$f(x) = 5x - 3, is cont Nuous at x = 0, at x = -3$ and $at x = 5$

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2. Prove that the function $f(x) = x + |x|$ is continuous at $x = 0$.

3. Examine the continuity of the function $f(x) = 2x^2 - 1$ at $x = 3$

4. Discuss the continuity of function f(x) at 0 , if f(x) = |x| .

5. Discuss the continuity of $f(x) = 0$ at $x = 0$ if

$$f(x) = \begin{cases} \dfrac{\sqrt{1+4x} - \sqrt{1-4x}}{\sin x}, & x \neq 0 \\ 4 & x = 0 \end{cases}$$

6. Show that the function $f(x) = \begin{cases} x^2 + 3 & \text{if} x \neq 0 \\ 1 & \text{if} x = 0 \end{cases}$ is not continous at $x = 0$.

7. Determine the constant k, so that function f (x) is continuous at the indicated points :

$$f(x) = \begin{cases} \dfrac{x^2 - 3x + 2}{x - 1} & \text{if} \quad x \neq 1 \\ k & \text{if} \quad x = 1 \end{cases}$$ at x=1.

8. Determine, k if the following function is continuous at

$x = 0$

$$f(x) = \begin{cases} \frac{\sin 3x}{4x} & x \neq 0 \\ k & x = 0 \end{cases}$$

9. Find the values of k so that the function f is continuous at the indicated point : $f(x) = \begin{cases} kx^2 & \text{if } x \leq 2 \\ 3 & \text{if } x > 2 \end{cases}$ at

$x = 2$

10. Find the value of k so that the function f is continuous at the indicated point in following:

11. Find the relationship between a and b so that the function f defined by: $f(x) = \begin{cases} ax + 1 & \text{if } x \leq 3 \\ bx + 3 & \text{if } x > 3 \end{cases}$ is continuous at $x = 3$

12. For what value of λ is the function defined by $f(x) = \begin{cases} \lambda(x^2 - 2x) & \text{if } x \leq 0 \\ 4x + 1 & \text{if } x > 0 \end{cases}$ continuous at $x = 0$? What about continuity at $x = 1$?

13. Find all points of discontinuity of f, where f is defined by :

$$f(x) = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

14. Is the function defined by

$$f(x) = \begin{cases} x + 5 & \text{if } x \leq 1 \\ x - 5 & \text{if } x > 1 \end{cases}$$ a continuous function?

15. Determine if f defined by :

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$ is a continuous

function?

16. Find the values of a and b such that the function defined

by : `f(x)={(5,,,, if x le2),(ax+b,,,, if 2

Solved Examples Short Answer Type Questions Section Iii

1. Prove that the function f given by

$f(x) = |x - 1|, x \in R, x = 1$ is not differentiable at x = 1.

2. Show that the function $f(x) = \begin{cases} -x^2 & x \le 0 \\ x^2 & x > 0 \end{cases}$ is continuous at $x = 0$. Also show that f is differentiable at $x = 0$.

3. if $f(x) = \begin{cases} x^2 & x \le 1 \\ ax + b & x > 1 \end{cases}$ is differentiable at $x = 1$. Find the values of a and b.

Solved Examples Short Answer Type Questions Section Iv

1. Differentiate $\left(\dfrac{2x - 1}{2x + 1} \right)^2$ w.r.t. x.

2. Differentiate $\sqrt{3x + 2} + \dfrac{1}{\sqrt{2x^2 + 4}}$ w.r.t.x.

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3. Differentiate $\log\left(x + \sqrt{a^2 + x^2}\right)$ w.r.t.x .

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4. Find $\dfrac{dy}{dx}$ if $y = \log\left(x + 3 + \sqrt{x^2 + 6x + 3}\right)$

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5. Differentiate $\sqrt{a + \sqrt{a + \sqrt{a + x^2}}}$ w.r.t.x

6. Differentiate $\sin\left(x^2 + 5\right)$ w.r.t.x.

7. Differentiate $\cos x^3 . \sin^2\left(x^5\right)$ w.r.t.x .

8. Differentiate the w.r.t x $\dfrac{\sin(ax + b)}{\cos(cx + d)}$.

9. Differentiate $e^{3x} \log(\sin 2x)$ w.r.t.x.

10. If $y = \log \sqrt{\dfrac{1 + \cos^2 x}{1 - e^{2x}}}$, find $\dfrac{dy}{dx}$.

Solved Examples Short Answer Type Questions Section V

1. Find $\dfrac{dy}{dx}$ if $2x + 3y = \sin y$

2. Find $\dfrac{dy}{dx}$ if $xy^2 + y^2 = \tan x + y$

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3. If $x^2 + xy + y^2 = 100$. Find $\dfrac{dy}{dx}$.

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4. Find $\dfrac{dy}{dx}$ if $x^3 + x^2y + xy^2 + y^3 = 81$

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5. Find $\dfrac{dy}{dx}$ when $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 2008$.

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6. Find $\dfrac{dy}{dx}$, if $\sin^2 y + \cos xy = \pi$

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7. Find $\dfrac{dy}{dx}$, if $\sin^2 x + \cos^2 y = 1$

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8. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ show that $\dfrac{dy}{dx} = -\dfrac{1}{(1+x)^2}$

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9. If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \ldots . \infty}}}$, prove that

$(2y - 1)\dfrac{dy}{dx} = \dfrac{1}{x}$

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10. If $\cos y = x \cos(a + y)$, with $\cos a \neq \pm 1$, prove that

$\dfrac{dy}{dx} = \dfrac{\cos^2(a + y)}{\sin a}$

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11. Differentiate $\sin^{-1}(x\sqrt{x})$ w.r.t.x.

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12. Differentiate $\tan^{-1}\left(\dfrac{\sin x}{1+\cos x}\right)$ w.r.t.x

13. Find $\dfrac{dy}{dx}$ when: $y = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$

14. Find $\dfrac{dy}{dx}$ in the following:$y = \sec^{-1}\left(\dfrac{1}{2x^2-1}\right)$, $0 < x < \dfrac{1}{\sqrt{2}}$

15. Find $\dfrac{dy}{dx}$, if $y = \tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right)$, $\dfrac{-1}{\sqrt{3}} < x < \dfrac{1}{\sqrt{3}}$

16. Find $\dfrac{dy}{dx}$, if $y = \tan^{-1}\left(\dfrac{x^{\frac{1}{3}} + a^{\frac{1}{3}}}{1 - x^{\frac{1}{3}}a^{\frac{1}{3}}}\right)$

17. Differentiate $\tan^{-1}\left(\dfrac{\sqrt{1 + x^2} - 1}{x}\right)$ w.r.t.x .

18. Differentiate $\tan^{-1}\left(\sqrt{1 + x^2} - x\right)$ w.r.t. x.

19. Differentiate w.r.t x :

$$\cot^{-1}\left\{\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\}, 0 < \theta < \frac{\pi}{2}$$

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20. Differentiate $\tan^{-1}\left\{\dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right\}$ w.r.t.x

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21. If $y = \sin^{-1} x^2\sqrt{1-x^2} + x\sqrt{1-x^4}$, show that

$$\frac{dy}{dx} - \frac{2x}{\sqrt{1-x^4}} = \frac{1}{\sqrt{1-x^2}}$$

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22. If $y = \tan^{-1}\left(\dfrac{5ax}{a^2 - 6x^2}\right)$. Prove that $\dfrac{dy}{dx} = \dfrac{3a}{a^2 + 9x^2} + \dfrac{2a}{a^2 + 4x^2}$

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23. If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$ using inverse trignometric function by a suitable substitution , prove that $\dfrac{dy}{dx} = \dfrac{\sqrt{1 - y^2}}{\sqrt{1 - x^2}}$

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Solved Examples Short Answer Type Questions Section Vii

1. Differentiate $x^2 e^{3x}$ w.r.t. $(\log x)^2$.

2. Differentiate $\sin^2 x$ w.r.t. $e^{\cos x}$

3. Differentiate $\sin^{-1}\left(\dfrac{x}{\sqrt{1+x^2}}\right)$ w.r.t. $\tan^{-1} x$.

4. Differentiate $\tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t. $\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$.

5. Differentiate $\sqrt{1 + x^2}$ w.r.t. $\tan^{-1} x$.

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1. Differentiate $x^{\sin x}$, $x > 0$ w.r.t. x.

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2. Differentiate $(\sin x)^{\sin x}$ w.r.t. x.

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3. Differentiate $\left(x^x\right)^x$ w.r.t.x .

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4. Differentiate x^{x^x} w.r.t. x.

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5. Differentiate $\left(\sin x - \cos x\right)^{\sin x - \cos x}$, $\dfrac{-\pi}{4} < x < \dfrac{3\pi}{4}$

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6. If $x^Y = e^{X-Y}$, prove that $\dfrac{dy}{dx} = \dfrac{\log x}{\left(1 + \log x\right)^2}$.

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7. If $y = x^y$, prove that $\dfrac{dy}{dx} = \dfrac{y^2}{x(1 - y\log x)}$

8. Differentiate the function w.r.t. x : $\dfrac{\sqrt{(x-1)(x-2)}}{(x-3)(x-4)(x-5)}$

9. Find the derivative of the function given by $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$ and hence find $f'(1)$

1. Find $\dfrac{dy}{dx}$ if $x = at^2$, $y = 2at$

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2. For a positive constant a find $\dfrac{dy}{dx}$, where $y = a^{t + \left(\frac{1}{t} \right)}$, and $x = \left(t + \dfrac{1}{t} \right)^a$

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3. Find $\dfrac{dy}{dx}$ if $x = \sin t$ and $y = \cos 2t$.

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4. Find $\dfrac{dy}{dx}$ when y = log(x) + 4

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5. Find $\dfrac{dy}{dx}$ when $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$.

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6. $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$ find dy/dx

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Solved Examples Short Answer Type Questions Section X

1. Find the second order derivative of $x \cos x$.

2. Find the second order derivatives of the function :

$e^{6x} \cos 3x$

3. Find $\dfrac{d^2 y}{dx^2}$ if $y = \tan^{-1} x$.

4. If $e^y (x+1) = 1$ show that $\left(d^2 \dfrac{y}{dx^2} \right) = \left(\dfrac{dy}{dx} \right)^2$ ਹੈ।

5. If $y = Ae^{mx} + Be^{nx}$, Show that

$$\left(\frac{d^2}{dx^2}y\right) - (m + n)\frac{dy}{dx} + mny = 0$$

6. If $y = 3\cos(\log x) + 4\sin(\log x)$ show that

$$x^2 y_2 + xy_1 + y = 0$$

7. If $y = \left[\tan^{-1} x\right]^2$, then prove that :

$$\left(x^2 + 1\right)^2 y_2 + 2x\left(x^2 + 1\right)y_1 = 2.$$

8. If $y = \left(\sin^{-1} x\right)^2$, then prove that

$$(1 - x)^2 \frac{d^2 y}{dx^2} - x\frac{dy}{dz} = 2$$

9. If $y = \left(\cos^{-1} x\right)^2$, prove that:

$$\left(1 - x^2\right)\left(\frac{d^2 y}{dx^2}\right) - x\left(\frac{dy}{dx}\right) - 2 = 0.$$

10. If $y = \log\left(x + \sqrt{x^2 + 1}\right)$, $provet\widehat{:}$ $\left(x^2 + 1\right)d^2\dfrac{y}{dx^2} + x\dfrac{dy}{dx} = 0$

.

11. If $y = \sin(2\sin^{-1}x)$, prove that:

$$(1 - x^2)\left(d^2\frac{y}{dx^2}\right) - x\left(\frac{dy}{dx}\right) + 4y = 0.$$

12. If $y = e^{a\cos^{-1}x}$, $-1 \le x \le 1$, show that

$$(1 - x^2)\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right) - a^2y = 0$$

13. If $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

Solved Examples Short Answer Type Questions Section Xi

1. Verify Rolle's theorem for the function $f(x) = x^2 + 2x - 8, x \in [-4, 2]$

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2. Verify Rolle's theorem for the function $f(x) = (x-3)(x-5)^2$ in the interval $(3, 5)$.

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3. Verify Rolle's theorem for function $f(x) = \sin x + \cos x$ in the interval $[0, 2\pi]$.

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4. Discuss the appllicability of Rolle's theorem for the function $f(x) = x^2 - 1$ for $x \in (1, 2)$.

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5. Verify Lagrange's Mean Value Theorem for the function : $f(x) = x(x - 1)(x - 2)(x - 3)$ in the interval $[0, 4]$

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6. Verify Mean Value Theorem, if $f(x) = x^3 - 5x^2 - 3x$, in the interval [a, b], where a = 1 and b = 3. Find all $c \in (1, 3)$ for which $f'(c) = 0$.

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Solved Examples Long Answer Type Questions Section Xii

1. If $y = \dfrac{2}{\sqrt{a^2 - b^2}} \tan^{-1}\left(\sqrt{\dfrac{a+b}{a-b}} \ \tan \dfrac{x}{2}\right)$, prove that $\dfrac{dy}{dx} = \dfrac{1}{a - b\cos x}$

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2. Differentiate $(\log x)^x + x^{\log x}$ w.r.t.x .

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3. Differentiate : $x^{\sin x} + (\sin x)^x$ w. r. tx:

4. Find $\dfrac{dy}{dx}$ when $x^y + y^x = a^b$

Assignment Most Important Questions For Practice Section I

Multiple Choice Questions

1. The function $f(x) = \begin{cases} \dfrac{\sin x}{x} + \cos x & \text{if } x \neq 0 \\ k & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$, then then value of $'k'$ is

A. 1

B. 2

C. $\dfrac{1}{2}$

D. 0

Answer: D

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2. The value of 'k' for which $f(x) = \dfrac{\sin 2x}{kx}, x \neq 0$ and $f(0) = 2$ is continous is

A. 1

B. 2

C. $\dfrac{1}{2}$

D. 0

Answer: A

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3. The value of $\dfrac{d}{dx}\left[\cos^{-1}(\cos x)\right]$ is

A. $\dfrac{1}{\sqrt{1-x^2}}$

B. 1

C. 0

D. None of these

Answer: B

4. The derivative of $\log(\log x)$ w.r.t. x is :

A. $\dfrac{1}{x \log x}$

B. $\dfrac{1}{\log x}$

C. $\dfrac{1}{x}$

D. None of these

Answer: A

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5. If $x = \log t^2,\ y = \log t^3,$ then $\dfrac{dy}{dx}$ is

A. t

B. $\dfrac{1}{t}$

C. $\dfrac{3}{2}$

D. $\dfrac{2}{3}$

Answer: C

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6. if `y=sec(tan^(-1)x), then (dy)/dx is equal to

A. $\dfrac{x}{\sqrt{1+x^2}}$

B. $\dfrac{-x}{\sqrt{1+x^2}}$

C. $\pm \dfrac{x}{\sqrt{1+x^2}}$

D. None of these

Answer: A

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7. $\dfrac{d}{dx}\left(\dfrac{\sin^{-1}x}{a}\right)$

A. $\dfrac{1}{\sqrt{a^2 - x^2}}$

B. $\dfrac{a}{\sqrt{a^2 - x^2}}$

C. $\dfrac{1}{\sqrt{x^2 - a^2}}$

D. None of these

Answer: A

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8. If $y = \sqrt{\sin x + y}$, then $\dfrac{dy}{dx}$ is equal to

A. $\dfrac{\cos x}{2y - 1}$

B. $\dfrac{\cos x}{1 - 2y}$

C. $\dfrac{\sin x}{1 - 2y}$

D. $\dfrac{\sin x}{2y - 1}$

Answer: A

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1. Prove that the function $f(x) = 2x + 3$ is continouns at $x = 1$.

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2. Prove that the function $f(x) = x^n$, is continuous at $x = n$, where n is a positive integer.

3. Prove that $f(x) = |\sin x|$ is continous at all point of its domnin.

4. Discuss the continuity of $f(x)$ at $x = 0$ of

$$f(x) = \begin{cases} \frac{\sqrt{1+x}}{\sin x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

5. Discuss the continuity of the function $f(x)$ at $x = 5$, if

$$f(x) = \begin{cases} \frac{x^2 - 25}{x - 5}, & \text{if } x \neq 5 \\ 10, & \text{if } x = 5 \end{cases}$$

6. A function f is defined as

$$f(x) = \begin{cases} \frac{x^2 - x - 6}{x^2 - 2x - 3}, & x \neq 3 \\ \frac{5}{3}, & x = 3 \end{cases}$$

Prove that f is discontinous at $x = 3$.

7. Test the continuity of the function at $x = 3$, where

$$f(x) = \begin{cases} \frac{|x - 3|}{x(x - 3)}, & x \neq 3 \\ 0, & x = 3 \end{cases}$$

8. Discuss the continuity of the function

$$f(x) = \begin{cases} \frac{|x-2|}{2-x}, & x \neq 2 \\ -1, & x = 2 \end{cases} \text{ at } x = 2.$$

9. Examine the continuity of $f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ at $x = 0$

10. If $f(x) = \begin{cases} \frac{x}{|x|+2x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$

Prove that $f(x)$ remain discontinous at $x = 0$ for any real

value of k.

11. Discuss the continuity of the function defined by

$$f(x) = \begin{cases} \frac{\sin 3x}{x}, & x \neq 0 \\ t, & x = 0 \end{cases} \quad \text{at } x = 0$$

12. Discuss the continuity of the function defined by

$$f(x) = \begin{cases} \frac{\tan 2x}{3x}, & x \neq 0 \\ \frac{3}{2}, & x = 0 \end{cases}$$

13. Test the continuity of the following functions at indicated

points :

$$f(x) = \begin{cases} x^2\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases} \text{ at } x = 0$$

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14. Test the continuity of the following functions at indicated

points :

$$f(x) = \begin{cases} (x)\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases} \text{ at } x = 0$$

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15. Test the continuity of the following functions at indicated

points :

$$f(x) = \begin{cases} (x-a)\cos\frac{1}{x-a}, & x \neq a \\ 0, & x = a \end{cases} \quad \text{at } x = a$$

16. Determine k, so that $f(x) = \begin{cases} \frac{x^2-36}{x-6}, & \text{if } x \neq 6 \\ k, & \text{if } x = 6 \end{cases}$ is

continous at x = 6.

17. Find k, so that $f(x) = \begin{cases} \frac{\sin kx}{x}, & x \neq 0 \\ 4+x, & x = 0 \end{cases}$ is continous at

$x = 0$

18. Determine k, if the following function is continuous at

$x = 0$

$$f(x) = \begin{cases} \frac{1 - \cos 2x}{2x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

19. For what value of k is the function defined by

$$f(x) = \begin{cases} k(x^2 + 2), & \text{if} x < 0 \\ 3x + 1, & \text{if } x > 0 \end{cases} \quad \text{continous at } x = 0 ?$$

20. Find the value of k so that $f(x)$ is continous at $x = 0$

$$f(x) = \begin{cases} \frac{1 - \cos 8x}{4x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

21. Find the value of k so that $f(x)$ is continouns at $x = 0$

$$f(x) = \begin{cases} \frac{1 - \cos 2x}{4x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

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22. Determine the constant k so that the function

$$f(x) = \begin{cases} 3k - 2x, & \text{if } x < 1 \\ 2k + 1, & \text{if } x \geq 1 \end{cases} \text{ is continous at } x = 1.$$

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23. Determine the constant λ , so that the function is

continuous at x=1

$$f(x) = \begin{cases} \frac{x-1}{x+1}, & \text{if } x \neq 1 \\ \lambda - 1, & \text{if } x = 1 \end{cases}$$

24. For what value of k is the following function continous at

$x = 2$?

$$f(x) = \begin{cases} 2x + 1, & x < 2 \\ k, & x = 2 \\ 3x - 1 & x > 2 \end{cases}$$

25. Find all the points of discontinuity, where f is defined by

$$f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x \geq 1 \\ -1, & \text{if } x \geq 0 \end{cases}$$

26. Find all the points of discontinuity, where f is defined by

$$f(x) = \begin{cases} x + 1, & \text{if } x \geq 1 \\ x^2 + 1, & \text{if } x < 1 \end{cases}$$

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27. Find all the points of discontinuity, where f is defined by

$$f(x) = \begin{cases} x^3 - 3, & \text{if } x \leq 2 \\ x^2 + 1, & \text{if } x > 2 \end{cases}$$

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28. Discuss the continuity of the function $f(x)$ at $x = \dfrac{1}{2}$

where

$$f(x) = \begin{cases} \frac{1}{2} + x, & x \leq x < \frac{1}{2} \\ 1, & x = \frac{1}{2} \\ \frac{3}{2} + x & \frac{1}{2} < x < 1 \end{cases}$$

Assignment Most Important Questions For Practice Section Iii Short Answer Type Questions

1. Prove that $f(x) = |x|$ is continous but not differentiable at $x = 0$

2. Prove that $f(x) = [x], 0 < x < 3$ is not differentiable at $x = 1$ but $x = 2$.

3. Show that the function

$$f(x) = \begin{cases} 2 + x, & \text{if } x \geq 0 \\ 1, & \text{if } x < 0 \end{cases}$$ is not continuous at x = 0

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1. Differentiate the following w.r.t.x.

$$\left(\frac{3x - 1}{2x + 1} \right)^2$$

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2. Differentiate the following w.r.t.x.

$$\sqrt{15x^2 - x + 1}$$

3. Differentiate the following w.r.t.x.

$$\log\left(\frac{x + \sqrt{x^2 - a^2}}{x - \sqrt{x^2 - a^2}}\right)$$

4. Differentiate the following w.r.t.x.

$$\log\left[x - 3 + \sqrt{x^2 - 6x + 1}\right]$$

5. Differentiate the following w.r.t.x.

$$\frac{e^x . \log x}{x^2}$$

6. Differentiate the following w.r.t.x.

$$\sqrt{\frac{1-x^2}{1+x^2}}$$

7. Differentiate the following w.r.t.x.

$$\tan(2x+3)$$

8. Differentiate the following w.r.t.x.

$$\left(\sin\left(\cos x^2\right)\right)$$

9. Differentiate the following w.r.t.x.

$$\sin^4 \left(ax + b \right)^2$$

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10. Differentiate the following w.r.t.x.

$$2\sqrt{\cot(x)^2}$$

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11. Differentiate the following w.r.t.x.

$$\log \tan \left(\frac{\pi}{4} + \frac{x}{2} \right)$$

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12. Differentiate the following w.r.t.x.

$$\log \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

13. Differentiate the following w.r.t.x.

$$\log \tan \left(\frac{\pi}{4} + \frac{x}{2} \right)$$

14. Differentiate the following w.r.t.x.

$$\log \sqrt{\frac{1 + \sin x}{1 - \sin x}}$$

Short Answer Type Questions

1. Find $\dfrac{dy}{dx}$ in the following :

$y + \sin y = \cos x$

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2. Find $\dfrac{dy}{dx}$ in the following: $2x + 3y = \sin x$

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3. Find $\dfrac{dy}{dx}$ in the following :

$x^3 + y^3 + 3a \times y = 0$

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4. Find $\dfrac{dy}{dx}$ in the following :

$$x^2 + y^4 + 4xy = 100$$

5. Find $\dfrac{dy}{dx}$ in the following :

$$x^6 + y^6 + 6x^2y^2 = 16$$

6. Find $\dfrac{dy}{dx}$ in the following :

$$x^2 + y^2 + 2yx + 2 + y + c = 0$$

7. Find $\dfrac{dy}{dx}$ in the following :

$$ax^2 + 2hxy + by^2 + 2gx + 2 + y + c = 0$$

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8. Find $\dfrac{dy}{dx}$ in the following :

$$x^3 + y^3 = \sin(x + y)$$

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9. Find $\dfrac{dy}{dx}$ in the following :

$$x^3 y + y^3 x = a^4$$

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10. Find $\dfrac{dy}{dx}$ in the following :

$$\left(x^2 + y^2\right)^2 = xy$$

▶ Watch Video Solution

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11. Find $\dfrac{dy}{dx}$ in the following :

$$x^{2/3} + y^{2/3} = a^{2/3}$$

▶ Watch Video Solution

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12. Find $\dfrac{dy}{dx}$ in the following :

$$y \sec x + \tan x + x^2 y = 0$$

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13. If $e^x + e^y = e^{x+y}$, prove that $\dfrac{dy}{dx} = \dfrac{e^x(e^y - 1)}{e^y(e^x - 1)}$

14. If $\log\left(\sqrt{1 + x^2} - x\right) = y\sqrt{1 + x^2}$, show that $(1 - x^2)\dfrac{dy}{dx} + xy + 1 = 0$

15. If $x^2 + 2xy + y^2 = 18$, verify that $\dfrac{dy}{dx} \times \dfrac{dx}{dy} = 1$

16. If $\sin y = x\sin(a + y)$, prove that $\dfrac{dy}{dx} = \dfrac{\sin^2(a + y)}{\sin a}$

17. If $\sin x = y \sin(x + b)$, show that $\dfrac{dy}{dx} = \dfrac{\sin b}{\sin^2(x + b)}$

18. If $x^{16} y^9 = \left(x^2 + y\right)^{17}$, $Prove$(dy)/(dx) = (2y)/(x)`

19. If $y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots\ldots\infty}}}$, show that $(2y - 1)\dfrac{dy}{dx} = 1$

1. Differentiate the following w.r.t.x.

$$\tan^{-1} x^4$$

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2. Differentiate the following w.r.t.x.

$$\cos^{-1}(\sin x)$$

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3. Differentiate the following w.r.t.x.

$$\cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$$

4. Differentiate the following w.r.t.x.

$$\tan^{-1}\left(\frac{\sqrt{1 - \cos 2x}}{\sqrt{1 + \cos 2x}}\right)$$

5. Differentiate the following w.r.t.x.

$$\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$$

6. Differentiate the following w.r.t.x.

$$\tan^{-1}(\sec x + \tan x)$$

7. Differentiate $\dfrac{x \cos^{-1} x}{\sqrt{1 - x^2}}$ w.r.t.x.

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8. Differentiate the following w.r.t.x.

$\sin^{-1}\left(2x\sqrt{1 - x^2}\right)$

▶ Watch Video Solution

9. Differentiate the following w.r.t.x.

$\sin^{-1}\left(3x - 4x^3\right), \quad -\dfrac{1}{2} < x < \dfrac{1}{2}$

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10. Differentiate the following w.r.t.x.

$$\cos^{-1}\left(4x^3 - 3x\right)$$

11. Differentiate the following w.r.t.x.

$$\cos ec^{-1}\frac{1 + x^2}{2x}$$

12. Differentiate the following w.r.t.x.

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right)$$

13. Differentiate the following w.r.t.x.

$$\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$$

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14. Differentiate the following w.r.t.x.

$$\tan^{-1}\left(\sqrt{1+x^2}+x\right)$$

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15. Differentiate the following w.r.t.x.

$$\cot^{-1}\left(\sqrt{1+x^2}-x\right)$$

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16. Differentiate $\sin^{-1} x + \sin^{-1} \sqrt{1 - x^2}$ w.r.t.x.

17. If $y = \sin^{-1}\left(\dfrac{2x}{1 + x^2}\right) + \sec^{-1}\left(\dfrac{1 + x^2}{1 - x^2}\right)$, prove that $\dfrac{dy}{dx} = \dfrac{4}{1 + x^2}$, 0 < x < 1

18. If $y = \tan^{-1}\left(\dfrac{\sqrt{1 + x^2} - \sqrt{1 - x^2}}{\sqrt{1 + x^2} + \sqrt{1 - x^2}}\right)$, then show that $\dfrac{dy}{dx} = \dfrac{x}{\sqrt{1 - x^4}}$

19. If $y = \sin^{-1} x^2 \sqrt{1 - x^2} + x\sqrt{1 - x^4}$, show that

$$\frac{dy}{dx} - \frac{2x}{\sqrt{1 - x^4}} = \frac{1}{\sqrt{1 - x^2}}$$

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20. Differentiate $\tan^{-1}\left(\dfrac{\sqrt{1 + x} - \sqrt{1 - x}}{\sqrt{1 + x} + \sqrt{1 - x}}\right)$ w.r.t. x .

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21. If $\sqrt{1 - x^6} + \sqrt{1 - y^6} = a^3 (x^3 - y^3)$ prove that

$$\frac{dy}{dx} = \frac{x^2 \sqrt{1 - y^6}}{y^2 \sqrt{1 - x^6}}$$

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1. Differentiate $\dfrac{x^2}{1+x^2}$ w.r.t.x.

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2. Differentiate $\log(xe^x)$ w.r.t.x log x.

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3. Differentiate $\tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right)$ w.r.t. $\tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$

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4. Differentiate $\cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$ w.r.t. $\tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right)$

5. Differentiate $\tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$ w.r.t. $\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$

6. Prove that derivative of $\tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t. $\tan^{-1}x$ is independent of x.

7. Prove that derivative of $\sec^{-1}\left(\dfrac{1}{2x^2 - 1}\right)$, $x > 0$, w.r.t. $\sqrt{1 - x^2}$ is equal to derivative of $\log_e x^2$ w.r.t.x.

8. Differentiate $\tan^{-1}\left(\dfrac{\sqrt{1 - x^2}}{x}\right)$ w.r.t. $\cos^{-1}\left(2x\sqrt{1 - x^2}\right)$

9. Differentiate $\tan^{-1}\left(\dfrac{\sqrt{1 - x^2}}{x}\right)$ w.r.t. $\cos^{-1}\left(2x\sqrt{1 - x^2}\right)$ w.r.t.x $\sin^{-1}\left(\dfrac{2x}{1 + x^2}\right)$

1. Differentiate the following w.r.t.x.

$$x^x$$

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2. Differentiate the following w.r.t.x.

$$x^{x^2}$$

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3. Differentiate the following w.r.t.x.

$$(5x)^{3\cos 2x}$$

4. Differentiate the following w.r.t.x.

$$(\sin x)^x$$

5. Differentiate the following w.r.t.x.

$$(\sin x)^{\tan x}$$

6. Differentiate the following w.r.t.x.

$$(\log x)^{\log x}$$

7. Differentiate the following w.r.t.x.

$(\log x)^{\cos x}$

8. Differentiate the following w.r.t.x.

$\left(\sin^{-1} x\right)^x$

9. Differentiate the following w.r.t.x.

If $y^x = e^{y-x}$, prove that $\dfrac{dy}{dx} = \dfrac{(1 + \log y)^2}{\log y}$

10. Differentiate the following w.r.t.x.

If $x^y = y^x$, prove that $\dfrac{dy}{dx} = \dfrac{\frac{y}{x} - \log y}{\frac{x}{y} - \log x}$

11. Differentiate the following w.r.t.x.

$y = x^{x^{x^{\infty}}}$, prove that $x\dfrac{dy}{dx} = \dfrac{y^2}{1 - y \log x}$

12. Differentiate the following w.r.t.x.

$y = e^{x + ex + ex - \text{to} \infty}$, prove that $\dfrac{dy}{dx} = \dfrac{y}{1 - y}$

13. Differentiate the following w.r.t.x .

$$(x + 3)^2 (x + 4)^3 (x + 5)^4$$

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14. Differentiate the following w.r.t.x .

$$e^x \cos^3 x \sin^2 x$$

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15. Differentiate the following w.r.t.x .

$$\frac{\sqrt{(x - 3)(x^2 + 4)}}{3x^2 + 4x + 5}$$

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1. Find $\dfrac{dy}{dx}$ in the following

$$x = 2at^2, y = at^4$$

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2. Find $\dfrac{dy}{dx}$ in the following

$$x = \dfrac{1 - t^2}{1 + t^2}, y = \dfrac{2t}{1 + t^2}$$

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3. Find $\dfrac{dy}{dx}$ in the following

$$x = a\left(\dfrac{1 + t^2}{1 - t^2}\right), y = \dfrac{2t}{1 - t^2}$$

4. Find $\dfrac{dy}{dx}$ in the following

$x = a\cos\theta, y = a\sin\theta.$

5. Find $\dfrac{dy}{dx}$ in the following

$x = a\sec\theta, y = b\tan\theta\,.$

6. Find $\dfrac{dy}{dx}$ in the following

$x = a(1 - \cos\theta), y = a(\theta + \sin\theta)$

7. Find $\dfrac{dy}{dx}$ in the following

$x = a(\theta - \sin\theta),\ y = a(1 + \cos\theta)$

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8. Find $\dfrac{dy}{dx}$ in the following

$x = a\sin\theta,\ y = a\left(\cos\theta + \dfrac{\log\tan\theta}{2}\right)$

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9. Find $\dfrac{dy}{dx}$ in the following

$x = \cos 2\theta + 2\cos\theta,\ y = \sin 2\theta - 2\sin\theta$

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10. Find $\dfrac{dy}{dx}$ in the following

$$x = e^\theta(\sin\theta + \cos\theta), \; y = e^\theta(\sin\theta - \cos\theta)$$

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11. If $(x = a\sin(2t)(1 + \cos(2t))$ and $(y = b\cos(2t)(1 - \cos(2t)),$ then show that $\left(\left(\dfrac{dy}{dx}\right)_{t=\frac{\pi}{4}} = \dfrac{b}{a}\right).$

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Assignment Most Important Questions For Practice Section X Short Answer Type Questions

1. Find the second order derivatives of the function : $\tan^{-1} x$

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2. Find the second order derivative of the following functions

$x^3 \log x$

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3. Find the second order derivative of the following functions

$e^x \sin 3x$

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4. Find the second order derivative of the following functions

If $y = x + \tan x$, prove that $\cos^2 x . \dfrac{d^2y}{dx^2} - 2y + 2x = 0$

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5. Find the second order derivative of the following functions

If $y = \cos^{-1} x$, find $\dfrac{d^2y}{dx^2}$ in terms of y alone.

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6. Find the second order derivative of the following functions

If $y = Pe^{ax} + Qe^{bx}$ show that $\dfrac{d^2y}{dx^2} - (a + b)\dfrac{dy}{dx} + aby = 0$

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7. Find the second order derivative of the following functions

If $y = ae^{mx} + be^{-mx}$, prove that $\dfrac{d^2y}{dx^2} - m^2y = 0$

8. If $y = 500e^7x + 600e^{-7}x$ show that $\left(d^2\dfrac{y}{dx^2}\right) = 49y$

9. If $y = A\sin x + B\cos x$ then prove that $d^2\dfrac{y}{dx^2} + y = 0$

10. Find the second order derivative of the following functions

$y = A \cos nx + B \sin nx$, Prove that $\dfrac{d^2 y}{dx^2} + n^2 y = 0$

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11. Find the second order derivative of the following functions

$y = (\log x)^2$, prove $x^2 y_2 + x y_1, \; - 2 = 0$

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12. Find the second order derivative of the following functions

If $\quad y = a \cos(\log x) + b \sin(\log x), \quad$ prove $\quad$ that $x^2 y_2 + x y_1 + y = 0.$

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13. Find the second order derivative of the following functions

If $y = \left(\cos^{-1} x\right)$, $then P.\ T \left(1 - x^2\right) \dfrac{d^2 y}{dx^2} - x \dfrac{dy}{dx} = 0.$

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14. Find the second order derivative of the following functions

If $y = \cos^{-1} x$ show that $\left(1 - x^2\right) \dfrac{d^2 y}{dx^2} - x \dfrac{dy}{dx} = 0$

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15. If $y = \tan^{-1} x$, show that : $\left(1 + x^2\right) d^2 \dfrac{y}{dx^2} + 2x \dfrac{dy}{dx} = 0$

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16. Find the second order derivative of the following functions

If $y = \dfrac{\sin^{-1} x}{\sqrt{1 - x^2}}$, prove that $(1 - x^2) y_2 - 3x y_1 - y = 0$

17. Find the second order derivative of the following functions

If $y = \log\left[x + \sqrt{x^2 + a^2}\right]$, show that $(x^2 + a^2)\dfrac{d^2 y}{dx^2} + x\dfrac{dy}{dx} = 0$

18. If $y = \left[\log\left(x + \sqrt{x^2 + 1}\right)\right]^2$ then show that $(x^2 + 1) y_2 + x y_1 = 0$

19. Find the second order derivative of the following functions

If $\quad y = \sin(m\tan^{-1}x),\quad$ prove $\quad$ that

$(1+x)^2 y_2 + 2x(1+x^2) + m^2 y = 0$

20. Find the second order derivative of the following functions

If $y = e^{\tan x}$, prove that $\cos^2 x \dfrac{d^2 y}{dx^2} - (1+\sin 2x)\dfrac{dy}{dx} = 0$

21. If $\quad y = e^{2\tan^{-1}x},\quad$ then $\quad$ show $\quad$ that

$(1+x^2)^2 \dfrac{d^2 y}{dx^2} + 2x(1+x^2)\dfrac{dy}{dx} = 4y$

22. Find $\dfrac{d^2y}{dx^2}$ in the following

$x = a\cos^3\theta,\ y = a\sin^3\theta$

23. If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, find $\dfrac{d^2y}{dx^2}$

24. Find $\dfrac{d^2y}{dx^2}$ in the following

If $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$, find $\dfrac{d^2y}{dx^2}$ at $\theta = \dfrac{\pi}{2}$

Assignment Most Important Questions For Practice Section Xi Short Answer Type Questions

1. Verify the truth of Rolle's Theorem for the function $f(x) = x^2 - 5x + 4$ in the interval [1,4]

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2. Verify Rolle's theore for the following functions $f(x) = 4x^2 - 12x + 9$ in the inveral $[0, 3]$.

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3. Verify Rolle's theore for the following functions

$f(x) = 8x - x^2$ in the interval $[0, 8]$.

4. Verify Rolle's theore for the following functions

$f(x) = x(x^2 - 4)$ in the interval $[-2, 2]$.

5. Verify Rolle's theore for the following functions

$f(x) = x(x - 3)^2$ in the interval $[0, 3]$.

6. Verify Rolle's Theorem for the following functions :

$$f(x) = (x^2 - 1)(x - 2) \in [-1, 2]$$

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7. Verify Rolle's theore for the following functions

$$f(x) = \sin x + \cos x \text{ in the interval } \left[0, \frac{\pi}{2}\right]$$

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8. Verify Rolle's theore for the following functions

$$f(x) = \cos 2\left(x - \frac{\pi}{4}\right) \text{ in the interval } \left[0, \frac{\pi}{2}\right].$$

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9. Verify Rolle's theore for the following functions

$f(x) = \sin x + \cos x - 1$ in the inverval $\left[0, \dfrac{\pi}{2}\right]$

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10. Verify Rolle's theore for the following functions

$f(x) = \sin x - \sin 2x$ in the interval $[0, 2\pi]$

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11. Verify Rolle's theore for the following functions

$f(x) = \cos 2x$ in the interval $[0, \pi]$.

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12. Discuss the applicability of Rolle's theorem for the following functions

$f(x) = [x], x \in [5, 9]$

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13. Discuss the applicability of Rolle's theorem for the following functions

$f(x) = |x - 1|, x \in [0, 2]$

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14. Verify Lagrange's mean value theorem for the following functions

$f(x) = x^2 + x - 1$ in the interval $[0, 4]$

15. Verify Lagrange's mean value theorem for the following

functions

$f(x) = x^2 + 2x + 3$ in the interval $[4, 6]$

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16. Verify Lagrange's mean value theorem for the following

functions

$f(x) = x(x - 2)$ in the interval $[1, 3]$

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17. Verify Lagrange's mean value theorem for the following functions

$f(x) = x(x - 1)^2$ in the interval $[0, 1]$

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18. Verify Lagrange's mean value theorem for the following functions

$f(x) = x^3 - 5x^2 - 3x$ in the interval $[1, 3]$

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19. Verify Lagrange's mean value theorem for the following functions

$f(x) = \sin x$ in the interval $\left[\dfrac{\pi}{2}, \dfrac{5\pi}{2}\right]$

20. Verify Lagrange's mean value theorem for the following functions

$f(x) = (x - 1)^{2/3}$ in the interval $[1, 2]$

21. Verify Lagrange's mean value theorem for the following functions

$f(x) = x^3 - 2x^2 - x + 3$ in the interval $[0, 1]$

1. If $y = \dfrac{2}{\sqrt{a^2 - b^2}} \tan^{-1}\left(\sqrt{\dfrac{a+b}{a-b}}\ \tan\ \dfrac{x}{2}\right)$, prove that $\dfrac{dy}{dx} = \dfrac{1}{a - b\cos x}$

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1. If $y = \dfrac{x \sin^{-1} x}{\sqrt{1 - x^2}} + \log\sqrt{1 - x^2}$, prove that $\dfrac{dy}{dx} = \dfrac{\sin^{-1} x}{(1 - x^2)^{3/2}}$

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2. If $y = \cos^{-1}\left(\dfrac{3 + 5\cos x}{5 + 3\cos x}\right)$, prove that

$\cos x = 2\tan^{-1}\left(\dfrac{1}{2}\tan\left(\dfrac{x}{2}\right)\right)$

3. Differentiate the following w.r.t.x.

$x^{\tan x} + (\tan x)^x$.

4. Differentiate the following w.r.t.x.

$x^{\cot x} + (\sin x)^x$.

5. Differentiate the following w.r.t.x.

$$(\log x)^x + x^{\cos x}$$

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6. Differentiate the following w.r.t.x.

$$x^2 + (\cos x)^{2x}$$

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7. Differentiate the following w.r.t.x.

$$(\tan x)^{\cot x} + (\cot x)^{\tan x}$$

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8. Differentiate the following w.r.t.x.

$$(\sin x)^x + \sin^{-1}\sqrt{x}$$

9. If $x^y + y^x = 2$, find $\dfrac{dy}{dx}$.

10. Find $\dfrac{dy}{dx}$, if $y^x + x^y + x^x = a^b$

11. If $x^p \cdot y^q = (x+y)^{p+q}$, show that $\dfrac{dy}{dx} = \dfrac{y}{x}$

12. If $x^{16} \cdot y^9 = \left(x^2 + y\right)^{17}$, prove that $\dfrac{dy}{dx} = \dfrac{2y}{x}$.

Previous Years Boards Questions For Practice Multiple Choice Questions

1. Derivative of $\cos^{-1}(\sin x)$ w.r.t. x equals

   A. -1

   B. 1

   C. $\cos x$

   D. $\sin x$

Answer: A

2. Derivative of $\tan^{-1}(\cot x)$ w.r.t. x equals

A. 1

B. -1

C. tan x

D. cotx

Answer: B

3. The derivative of $f(x) = |x|$ at $x = 2$ equals :

A. 1

B. -1

C. 0

D. 2

Answer: A

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4. The derivative of $f(x) = |x|$ at $x = 3$ equals

A. 1

B. -1

C. 0

D. 2

Answer: A

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5. If $y = n^x$, $n > 0$ then $\dfrac{dy}{dx}$ is equal to

A. xn^{x-1}

B. $\dfrac{x}{\log x}$

C. $n^x \log n$

D. None of these

Answer: C

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6. If $y = x^a$, $a \in R$ then $\dfrac{dy}{dx}$ is equal to :

A. $x^a \log x$

B. $x^a \log a$

C. $a^x \log a$

D. ax^{a-1}

Answer: D

7. If $x = 3at$, $y = at^3$, then $\dfrac{dy}{dx}$ is equal to :

A. 3

B. $3a$

C. $3at$

D. t^2

Answer: D

8. If $x = 4at, y = at^4$, then $\dfrac{dy}{dx}$ is equal to :

A. t^3

B. $4at$

C. $4a$

D. 4

Answer: A

9. Derivative of $\left(\sin^{-1} x + \cos^{-1} x\right)$ w.r.t 'x' is equal to :

A. -1

B. 0

C. 1

D. 2

Answer: B

10. Derivative of $\left(\sec^{-1}x + \cos ec^{-1}x\right)$ w.r.t 'x' is equal to :

A. 0

B. -1

C. 1

D. 2

Answer: A

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11. The derivative of $\sqrt{\log(\sin x)}$ w.r.t. x is

A. $1\dfrac{1}{2\sqrt{\tan x}}$

B. $\dfrac{1}{2}\sec^2 x$

C. $\dfrac{\cot x}{2\sqrt{\log \sin x}}$

D. None of these

Answer: C

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12. If function defined by $f(x) = \begin{cases} \frac{\sin 3x}{2x}, & x \neq 0 \\ k + 1, & x = 0 \end{cases}$ is continuous at $x = 0$, then value of k is

A. 0

B. $\dfrac{3}{2}$

C. $\dfrac{1}{2}$

D. 1

Answer: C

13. The derivative of a^x is

A. $a^x \log a$

B. a^x

C. $\dfrac{a^x}{\log a}$

D. None of these

Answer: A

14. Discuss the continuity of the function (x) at $x = 1$ if

$$f(x) = \begin{cases} \dfrac{x-1}{\sqrt{x^2-1}}, & x \neq 1 \\ 0, & x = 1 \end{cases}$$

15. Discuss the continuity of the function $f(x)$ at $x = 3$ if

$$f(x) = \begin{cases} \dfrac{x-3}{\sqrt{x^2-9}}, & x \neq 3 \\ 0, & x = 3 \end{cases}$$

16. Find all the points of discontinuity of f, where f is defined

by

$$f(x) = \begin{cases} 2x + 3, & \text{if } x \leq 2 \\ 2x - 3, & \text{if } x > 2 \end{cases}$$

17. Find $\dfrac{dy}{dx}$ if $y = \log\left(\sqrt{2x + \sqrt{4x^2 + a^2}}\right)$

18. If $x^y = e^{x+y}$, show that $\dfrac{dy}{dx} = \dfrac{\log x - 2}{(1 - \log x)^2}$

19. If $\sin y = x \sin(a + y)$, prove that $\dfrac{dy}{dx} = \dfrac{\sin^2(a + y)}{\sin a}$

20. Differentiate $\cos^{-1}\left(\dfrac{1 - x^2}{1 + x^2}\right), 0 < x < 1.$ w.r.t.x

21. Diferentiate $\tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$ w.r.t.x.

22. If $y = a\sin(\log x) + b\cos(\log x)$, then prove that :

$$x^2 d^2 \frac{y}{dx^2} + x\frac{dy}{dx} + y = 0.$$

23. If $y = \sin(m\sin^{-1}x)$, prove that

$$\left(1 - x^2\right)y_2 - xy_1 + m^2 y = 0$$

24. Discuss the continuity of $f(x) = 0$ at $x = 0$ if

$$f(x) = \begin{cases} \dfrac{\sqrt{1+4x} - \sqrt{1-4x}}{\sin x}, & x \neq 0 \\ 4 & x = 0 \end{cases}$$

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25. Discuss the continuity of following at $x = 0$ if

$$f(x) = \begin{cases} \dfrac{\sqrt{1+3x} - \sqrt{1-3x}}{\sin x}, & x \neq 0 \\ 3, & x = 0 \end{cases}$$

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26. Determine the value of constant k so that the function

$$f(x) = \begin{cases} \dfrac{x^2 - 2x - 3}{x + 1}, & \text{if } x \neq 1 \\ k, & \text{if } x = 1 \end{cases}$$

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27. Determine the constant k, so that the function

$$f(x) = \begin{cases} \frac{x^2 - 9}{x - 3}, & x \neq 3 \\ k, & x = 3 \end{cases}$$ is continuous at x=3.

28. Examine the continuity of the following function at $x = 2$:

$$f(x) = \begin{cases} 2x + 1, & x \leq 2 \\ 3x - 1, & x > 2 \end{cases}$$

29. Examine the continuity of the following function at $x = 2$:

$$f(x) = \begin{cases} 3x + 1, & x \leq 2 \\ 4x - 1, & x > 2 \end{cases}$$

30. Differentiate the following w.r.t.x

$$y = \log\left(x + 4 + \sqrt{x^2 + 8x + 4}\right)$$

31. Differentiate the following w.r.t.x

$$y = \log\left(x + 2 + \sqrt{x^2 + 4x + 1}\right)$$

32. $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \dots \text{ to } \infty}}}$ prove that

$$(2y - 1)\frac{dy}{dx} = -\sin x$$

33. $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + ...1\infty}}}$ prove that

$(2y - 1)\dfrac{dy}{dx} = \cos x$

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34. $y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + ... to\infty}}}$ prove that

$(2y - 1)\dfrac{dy}{dx} = \sec^2 x.`$

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35. If $\sin y = x \cos(a + y)$, prove that $\dfrac{dy}{dx} = \dfrac{\cos^2(a + y)}{\cos a}$

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36. $x = a(\theta - \sin\theta), y = a(1 + \cos\theta)$ find dy/dx

37. If $y = (\sin x)^{\cos x}$ find $\dfrac{dy}{dx}$.

38. If $y = (\cos x)^{\sin x}$ find $\dfrac{dy}{dx}$

39. Differentiate $\cos^{-1}\left(\dfrac{1 - x^2}{1 + x^2}\right)$ w.r.t. $\tan^{-1} x$.

40. Differentiate $\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$ w.r.t. $\tan^{-1}x$.

41. Differentiate $\dfrac{\tan^{-1}(2x)}{1-x^2}$ w.r.t. $\tan^{-1}x$.

42. Differentiate $\cot^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t.x.

43. $y = \left[x + \sqrt{x^2 + 1} \right]^p$, prove that

$\left(x^2 + 1 \right) y_2 + x y_1 - p^2 y = 0$

44. Verify Rolle's theorem for the following functions

$f(x) = (x + 2)(x - 6)$ in the interval $\left[-2, 6 \right]$

45. Verify Rolle's theorem for the following functions

$f(x) = (x - 2)(x - 6)$ in the interval $[2, 6]$

46. Verify Rolle's theorem for the following functions

$f(x) = (x - 2)(x - 4)^2$ in the interval $[2, 4]$

47. Verify Rolle's theorem for the following functions

$f(x) = (x - 1)(x - 3)^2$ in the interval $[1, 3]$

48. Discuss the continuity of the following function:

$f(x) = \sin x - \cos x$

49. Discuss the continuity of the following function:

$$f(x) = \sin x \cdot \cos x$$

50. Discuss the continuity of the following functions

$$f(x) = \begin{cases} x + 2, & \text{if } x < 0 \\ -x + 2, & \text{if } x \geq 0 \end{cases}$$

51. Determine the value of k so that following functions are

continuous at $x = 0$

$$f(x) = \begin{cases} \frac{\sin 2x}{x}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$$

52. Determine, k if the following function is continuous at

$x = 0$

$$f(x) = \begin{cases} \frac{\sin 3x}{4x} & x \neq 0 \\ k & x = 0 \end{cases}$$

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53. Differentiate the following w.r.t. x

$$\left(\frac{3x - 1}{3x + 1} \right)^2$$

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54. Differentiate the following w.r.t. x

$$\log\left(x + \sqrt{1 + x^2} \right)$$

55. If $y = \sqrt{2^x + \sqrt{2^x + \sqrt{2^x + \ldots\ldots\ldots \infty}}}$, then prove that

: $(2y - 1)\dfrac{dy}{dx} = 2^x \log 2.$

56. If $y = \sqrt{5^x + \sqrt{5^x + \sqrt{5^x + \ldots\ldots\ldots \infty}}}$, then prove that

: $(2y - 1)\dfrac{dy}{dx} = 5^x \log 5.$

57. Differentiate the function w.r.t. x : $\dfrac{\sqrt{(x-1)(x-2)}}{(x-3)(x-4)(x-5)}$

58. Differentiate the following w.r.t. x.

$$(\sin x)^{\cos x} + (\cos x)^{\sin x}$$

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59. Differentiate the following w.r.t. x.

$$(\sin x)^{\tan x} + (\cos x)^{\sec x}$$

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60. Differentiate $\cos^{-1}\left(\dfrac{1}{\sqrt{1+x^2}}\right)$ w.r.t.x.

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61. Diferentiate $\tan^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right)$ w.r.t.x.

62. If $y = \cos\left(2\cos^{-1}x\right)$, then prove that

$$\left(1-x^2\right)\dfrac{d^2y}{dx} - x\dfrac{dy}{dx} + 4y = 0$$

63. If $y = \sin\left(2\cos^{-1}x\right)$, then prove that

$$\left(1-x^2\right)\dfrac{d^2y}{dx} - x\dfrac{dy}{dx} + 4y = 0$$

64. Verify Rolle's theorem for the function :

$f(x) = (x + 1)(x - 4)$ in the interval [-1, 4]`

65. Verify Rolle's theorem for the following functions

$f(x) = (x + 2)(x - 3) \text{in} \ [-2, 5]$

66. Verify Rolle's theorem for the following functions

$f(x) = (x + 3)(x - 6) \text{in} (-3, 6)$

67. Find the values of k so that the function f is continuous at

the indicated point : $f(x) = \begin{cases} kx + 1 & \text{if } x \leq \pi \\ \cos x & \text{if } x > \pi \end{cases}$ at

$x = \pi$

68. Find the values of k so that the function f is continuous at

the indicated point : $f(x) = \begin{cases} kx + 1 & \text{if } x \leq 5 \\ 3x - 5 & \text{if } x > 5 \end{cases}$ at

$x = 5$

69. If $(\cos x)^y = (\cos y)^x$, find $\dfrac{dy}{dx}$.

70. Differentiate the following w.r.t. x :

$$\left(x + \frac{1}{x}\right)^x + x^{x + \frac{1}{x}}$$

71. Differentiate the following w.r.t. x :

$$x^{x^2 - 3} + (x - 3)^{x^2}, \, x > 3$$

72. For what value of k, function $f(x)$ is continuous at x = 0

where $f(x) = \begin{cases} \frac{1 - \cos 4x}{8x^2} & , & \left(\begin{array}{l} x \neq 0 \\ x = 0 \end{array}\right) \\ k \end{cases}$

73. Find $\dfrac{dy}{dx}$ when $x^y + y^x = a^b$

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74. Differentiate $\tan^{-1}\left(\dfrac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right)$ w.r.t.x .

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75. $y = e^{m\sin^{-1}x}$, prove that $(1-x^2)y_2 - xy_1 = m^2 y$

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76. If $y = e^{m\tan^{-1}x}$, then show that $(1+x^2)y_2 + (2x-m)y_1 = 0.$

77. Examine the continuity of $f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ at $x = 0$

78. If $xy = e^{x-y}$, prove that $\dfrac{dy}{dx} = \dfrac{y(x-1)}{x(y+1)}$

79. Differentiate $e^{\sec^2 x} + 3\cos^{-1} x$ w.r.t.x.

80. Find k so that $f(x) = \begin{cases} \frac{x^2 - 25}{x - 5}, & \text{if } x \neq 5 \\ k, & \text{if } x = 5 \end{cases}$ is continuous

at $x = 5$.

81. If $f(x)$ is continuous at $x = 0$, find a, where

$$f(x) = \begin{cases} \frac{1 - \cos ax}{x \sin x}, & x \neq 0 \\ \frac{1}{2}, & x = 0 \end{cases}$$

82. Differentiate $\tan^{-1}\left(\frac{\sqrt{1 + x^2} - 1}{x}\right)$ w.r.t.x .

83. Differentiate $\tan^{-1}\left(\dfrac{\sqrt{1+a^2x^2}-1}{ax}\right)$ w.r.t.x.

84. Verify LMV theorem for the following functions

$$f(x) = x(x-1)(x-2) \text{ in } \left[0, \frac{1}{2}\right]$$

85. Verify LMV theorem for the following functions

$$f(x) = x^2 + x - 1 \quad \text{in} \quad [0, 4]$$

86. Discuss the continuly of the following functions

$$f(x) = \begin{cases} 2x, & \text{if } x \neq 0 \\ 0, & \text{if } 0 \leq x \leq 1 \\ 4x, & \text{if } x > 1 \end{cases}$$

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87. Discuss the continuly of the following functions

$$f(x) = \begin{cases} 2, & \text{if } x < -1 \\ 2x, & \text{if } -1 \leq x \leq 1 \\ 2, & \text{if } x > 1 \end{cases}$$

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88. Find $\dfrac{dy}{dx}$ if $ax + by^2 = \cos y$

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89. Find $\dfrac{dy}{dx}$ if $2x + 3y = \sin x$

90. Find $\dfrac{dy}{dx}$ if $x^y = y^x$.

91. $y = 5\cos x - 3\sin x$, prove that $\dfrac{d^2 y}{dx^2} + y = 0$

92. Prove that the function $f(x) = \begin{cases} x^{\frac{\cos 1}{x}}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$

is continuous at $x = 0$

93. Discuss the continuity of the function

$$f(x) = \begin{cases} 3, & \text{if } 0 \le x \le 1 \\ 4, & \text{if } 1 < x < 3 \\ 5, & \text{if } 3 \le x \le 10 \end{cases}$$

94. Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ w.r.t.x.