MATHS

BOOKS - PSEB

RELATIONS AND FUNCTIONS

Example

1. Let A be the set of all students of a boys school. Show that the relation R in A given by R= {(a,b) : a is sister of b}is the empty relation and R' = {(a, b) : the difference between heights of a and b is less than 3 meters} is the universal relation.

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2. Let T be the set of all triangles in a plane with R a relation in T given by : $R = \{(T_1, T_2) : T_1$ is congruent to T_2}. Show that R is an equivalence relation.

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3. Let a relation $R = \{(L_1, L_2) : L_1$ is perpendicular to $L_2\}$, be defined on the set of all lines L in a plane. Show that R is symmetric metric but neither reflexive nor transitive.

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4. Show that the relation R in the set {1, 2, 3} given by R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)} is reflexive but neither symmetric nor transitive.

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5. Show that the relation R in the set Z of integers given by R = {(a, b) : 2 divides a – b} is an equivalence relation.

6. Let R be the relation defined in the set A = {1, 2, 3, 4, 5, 6, 7} by R = {(a, b) : both a and b are either odd or even}. Show that R is an equivalence relation. Further, show that all the elements of the subset {1, 3, 5, 7} are related to each other and all the elements of the subset {2, 4, 6} are related to each other, but no element of the subset {1, 3, 5, 7} is related to any element of the subset {2, 4, 6}.

7. Let A be the set of all 50 students of class XII in a school. Let $f: A \rightarrow N$ be the function defined by : `f (x) = Roll number of the student x. Prove that 'f' is one-one but not onto.

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8. Show that the function $f: N \rightarrow N$ given by $f(x) = 2x$ is one-one but not onto.

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9. Prove that the function $f: R \rightarrow R$ given by $f(x) = 2x$ is one-one and onto.

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10. Show that the function: $f: N \rightarrow N$ given by $f(1) = f(2) = 1$ and $f(x) = x - 1$, for every $x > 2$ is onto but not one-one.

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11. Show that function $f: R \rightarrow R$ given by $f(x) = x^2$ is neither one-one nor onto.

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12. Show that $f: N \rightarrow N$, given by : $f(x) = \begin{cases} x+1 & \text{if} \quad xisodd \\ x-1 & \text{if} \quad xiseven \end{cases}$ is both one-one and onto.

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13. Show that an onto function $f:\{1, 2, 3\} \rightarrow \{1, 2, 3\}$ is always one-one.

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14. Show that a one-one function $f:\{1, 2, 3\} \rightarrow \{1, 2, 3\}$ must be onto.

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15. Let $f:(2, 3, 4, 5\} \rightarrow (3, 4, 5, 9\}$ and $g = (3, 4, 5, 9\} \rightarrow (7, 11, 15\}$ be functions defined as: $f(2) = 3,\ f(3) = 4,\ f(4) = f(5) = 5$ and $g(3) = g(4) = 7,$ and $g(5) = g(11) = 11.$ Find gof

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16. Show that if $f: A \to B$ and $g: B \to C$ are one-one, then $gof: A \to C$ is also one-one.

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17. Consider functions f and g such that composite gof is defined and is one-one. Are $f$and g both necessarily one-one.

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18. Are f and g both necessarily onto, if gof is onto?

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19. Let $f: \{1, 2, 3\} \rightarrow \{a, b, c\}$ be one-one and onto function given by $f(1) = a$, $f(2) = b$ and $f(3) = c$. Show that there exists a function $g: \{a, b, c\} \rightarrow \{1, 2, 3\}$ such that $gof = I_x$ and $fog = l_y$. where $X = \{1, 2, 3\}$ and $Y = \{a, b, c\}$.

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20. Let $f: N \rightarrow Y, beafunctiondef \in edasf(x) = 4x + 3,$ where, $Y = \{y \in N: y = 4x + 3$ for some $x \in N\}$. Show that f is invertible. Find the inverse.

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21. Let $Y = \{n^2 : n \in N\}$ sub N$Consider$f : N rarr Yasf (n) = n^2` Show that f is invertible. Find the inverse of f.

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22. Let $f: N \rightarrow R$, be a function defined as $f(x) = 4x^2 + 12x + 15, \forall x \in N$, show that $f: N \rightarrow S$ where S, is range of f is invertible.

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23. Consider $f: N \rightarrow N, g: N \rightarrow N$ and $h: N \rightarrow R$ defined as $f(x) = 2x, \quad g(y) = 3y + 4$ and $h(z) = \sin z, \forall x, y$ and z Show that $ho(gof) = (hog)of$.

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24. Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be two invertible functions. Then gof is also invertible with $(gof)^{-}1 = f^{-}1ofg^{-}1$

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25. Let $S = \{1, 2, 3\}$ Determine whether the functions $f: S \rightarrow S$ defined as below have inverses. Find $f^{-}1$, , if it exists:

$f = \{(1, 2), (2, 1), (3, 1)\}$

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26. Let $S = \{1, 2, 3\}$ Determine whether the functions $f: S \rightarrow S$ defined as below have inverses. Find $f^{-}1$, , if it exists:

$f = \{(1, 3), (3, 2), (2, 1)\}$

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27. Show that addition, subtraction and multiplication are binary operations on R, but division is not a binary operation on R. Further, show that division is a binary operation on the set R of nonzero real numbers.

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28. Show that subtraction and division are not binary operations on N.

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29. Show that $\cdot : R \times R \to R, givenby(a, b) \to a + 4b^2$ is a binary operation.

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30. Let P be the set of all subsets of a given set X. Show that $\cup : P \times P \rightarrow P, givenby(A, B) \rightarrow A \cup B$ and $\cap : P \times P \rightarrow P, givenby(A, B) \rightarrow A \cap B$ are binary operations on the set P.

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31. Show that $\vee : R \times R \rightarrow R$ given by $(a, b) \rightarrow \max . [a, b)$ and $\wedge : R \times R \rightarrow R$ given by $(a, b) \rightarrow \min (a, b)$ are binary operations.

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32. Show that $+ : R \times R \to R$ and $\times : R \times R \to R$ are commutative binary operations, but $\div R \times R \to R$ and $\div : R \times R \to R$ are not commutative.

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33. Show that $\cdot : R \times R \to R$ defined by $a \cdot b = a + 2b$ is not commutative.

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34. Show that addition and multiplication are associative binary operation on R. But subtraction is not associative on R. Division is not associative on R_*.

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35. Show that $\cdot : R \times R \to R$ defined by $a \cdot b = a + 2b$ is not commutative.

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36. Show that zero is the identity for addition on R and 1 is the identity for multiplication on R. But there is no identity element for the operations $- : R \times R \to R$ and $\div R_. \times R_. \to R_.$

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37. Show that $-a$ is the inverse of a for the addition operation $' + '$ on R and $\frac{1}{a}$ is the inverse of $a \neq 0$ for the multiplication operation $` \times '$ on R.

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38. Show that -a is not the inverse of $a \in N$ for the addition operation $' + '$ on N and $\frac{1}{a}$ is not the inverse of $a \in N$ for multiplication operation ‘ $\times$ ’on N , for $a \neq 1$.

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39. If R_1 and R_2 are equivalence relations in a set A, show that $R_1 \cap R_2$ is also an equivalence relation

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40. Let R be a relation on the set A of ordered pairs of positive integers defined by $R, (x, y)R(u, v)$, if and only if $xv = yu$.

Show that R is an equivalence relation.

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41. Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Let R_1 be a relation in X given by $R_1 = \{(x, y) : x - y$ is divisible by 3} and R_2 $bea \neg herrelationonXgivenby$R_2 = {(x, y): {x, y} sub {1, 4, 7} or {x, y} sub {2, 5, 8} or {(x, y} sub {3, 6, 9}. $Showt$^R_1 = R_2`.

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42. Let $f : X \rightarrow Y$ be a function. Define a relation R in X given by $R = \{(a, b) : f(a) = f(b)\}$ Examine whether R is an equivalence relation or not.

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43. Determine whether the following binary operation on the set N is associative and commutative $a * b = 1 \forall a, b \in N$.

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44. Determine the following binary operations on the set R are associative or commutative: $a \cdot b = \frac{a+b}{2}, \forall a, b \in R$

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45. Find the number of all one-one functions from set $A = \{1, 2, 3\}$ to itself.

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46. Let $A = \{1, 2, 3\}$. Then show that the number of relations containing (1, 2) and (2, 3) which are reflexive and transitive but not symmetric is three.

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47. Show that the number of equivalence relation in the set {1, 2, 3} containing (1, 2) and (2, 1) is two.

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48. Show that the number of binary operations on {1, 2} having 1 as identity and having 2 as the inverse of 2 is exactly one.

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49. Consider the identity function $I_N : N \to N$ defined as $I_N(x) = x, \forall x \in N$ Show that although I_N is onto but $I_N + I_N : N \to N$ defined as : $(I_N + I_N)(x) = I_N(x) + I_N(x) = x + x = 2x$ is not onto.

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50. Consider a function $f : \left[0, \frac{\pi}{2}\right] \to R$ given by $f(x) = \sin x$ and $g : \left[0, \frac{\pi}{2}\right] \to R$ given by $g(x) = \cos x$. Show that f and g are one-one, but $f + g$ is not one-one.

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Exercise

1. Determine whether the following relations are reflexive, symmetric and transitive: Relation R in the set $A = \{1, 2, 3, \ldots, 13, 14\}$ defined as $R = \{(x, y) : 3x - y = 0\}$

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2. Determine whether the following relations are reflexive, symmetric and transitive:Relation R in the set N of natural numbers defined as $R = \{(x, y) : y = x + 5$ ਅਤੇ $x < 4\}$

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3. Determine whether the following relations are reflexive, symmetric and transitive:Relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(x, y) : yis \div isib \leq byx\}$

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4. Prove that the following relation R in Z of integers is an equivalence relation : $R = [(x, y) : x - y$ is an integer}

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5. Determine whether the following relations are reflexive, symmetric and transitive: Relation R in the set A of human beings in a town at a particular time given by, R = {(x, y) : x and y work at the same place}

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6. Determine whether the following relations are reflexive, symmetric and transitive: Relation R in the set A of human beings in a town at a particular time given by, R = {(x, y) : x and y live in the same locality}

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7. Determine whether the following relations are reflexive, symmetric and transitive:Relation R in the set A of human beings in a town at a particular time given by, R = {(x, y) : x is exactly 7 cm taller than y}

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8. Determine whether the following relations are reflexive, symmetric and transitive: Relation R in the set A of human beings in a town at a particular time given by, $R = \{(x, y) : xisw \quad \text{if} \quad eofy\}$

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9. Determine whether the following relations are reflexive, symmetric and transitive:Relation R in the set A of human beings in a town at a particular time given by, $R = \{(x, y) : xisfatherofy\}$

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10. Show that the relation R in the set R of real numbers, defined as $R = \{(a, b) : a \leq b^2\}$, is neither reflexive nor symmetric nor transitive.

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11. Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive.

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12. Show that the relation R in the set R of real numbers defined as $R = ((a, b) : a \leq b\}$, is reflexive and transitive but not symmetric.

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13. Check whether the relation R in R defined by R = {(a, b) : a le b^3} is reflexive, symmetric or transitive.

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14. Show that the relation R in the set {1, 2, 3} given by $R = \{(1, 2), (2, 1)\}$ is symmetric but neither reflexive nor transitive.

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15. Show that the relation R in the set A of all the books in a library of a college, given by

$R = \{(x, y) : x \text{ and } yhavesamevmberofpa \geq s\}$ is an equivalence relation.

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16. Show that the relation in the set $A = \{1, 2, 3, 4, 5\}$, given by : $R = \{(a, b) : Ia - bIiseven\}$ is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of { 2, 4} are related to each other. But no element of { 1, 3, 5} is related to any element of {2, 4}.

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17. Show that the relation R in the set : $R = \{x : x \in Z, 0 \leq x \leq 12\}, givenby$: R = {(a, b) : |a - b|` is a multiple of 4} is an equivalence relation.

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18. Show that each of the relation R in the set $A = \{x \in Z : 0 \le x \le 12\}$, given by: $R = \{(a, b) : a = b\}$, is an equivalence relation. Find the set of all elements related to 1 in each case.

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19. Give an example of a relation. Which is: Symmetric but neither reflexive nor transitive.

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20. Give an example of a relation. Which is: Transitive but neither reflexive nor symmetric.

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21. Give an example of a relation. Which is: Reflexive and symmetric but not transitive.

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22. Give an example of a relation which is reflexive and transitive but not symmetric.

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23. Give an example of a relation which is symmetric and transitive but not reflexive.

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24. Show that the relation R in the set A of points in a plane given by R = {(P, Q) : distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further, show that the set of all points related to a point $P \neq (0, 0)$ is the circle passing through P with origin as centre.

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25. Show that the relation R, defined by the set A of all triangles as : $R = \{(T_1, T_2) = T_1$ is similar to T_2}` is an equivalence relation. Consider three right-angled triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 1 3 and T_3 with sides 6, 8, 10.

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26. Show that the relation R defined in the set A of all polygons as $R = \{(P1, P2) : P1 \text{ and } P2$ have same number of sides}`, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?

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27. Let T be the set of all triangles in a plane with R a relation in T given by : $R = \{(T_1, T_2) : T_1$ is congruent to T_2}. Show that R is an equivalence relation.

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28. Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4,4), (1, 3), (3, 3), (3, 2)}. Choose the correct answer.

A. R is reflexive and symmetric but not transitive.

B. R is reflexive and transitive but not symmetric.

C. R is symmetric and transitive but not reflexive.

D. R is an equivalence relation.

Answer:

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29. Let R be the relation in the set N given by $R = \{(a, b) : a = b - 2, b > 6\}$. Choose the correct answer:

A. $(2, 4) \in R$

B. $(3, 8) \in R$

C. $(6, 8) \in R$

D. $(8, 7) \in R$

Answer:

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30. Show that the function $f : R_{\cdot} \to R_{\cdot}$ defined by $f(x) = \frac{1}{x}$ is one-one and onto, where $R_{\cdot}$ is the set of all non-zero real

numbers. Is the result true, if the domain $R.$ is replaced by N with co-domain being same as $R.$?

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31. Check the injectivity and surjectivity of the following function: $f: N \to N$ given by $f(x) = x^2$

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32. Check the injectivity and surjectivity of the following function: $f: Z \to Z$ given by $f(x) = x^2$

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33. Check the injectivity and surjectivity of the following function: $f : R \rightarrow R$ given by $f(x) = x^2$

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34. Check the injectivity and surjectivity of the following function: $f : N \rightarrow N$ given by $f(x) = x^3$

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35. Check the injectivity and surjectivity of the following function: $f : Z \rightarrow Z$ given by $f(x) = x^3$

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36. Check the injectivity and surjectivity of the following function: $f: R \to R$ given by $f(x) = x^3$

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37. Prove that greatest integer function $f: R \to R$, given by $f(x) = [x]$, is neither one-one nor onto where [x] denotes the greatest integer less than or equal to x .

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38. Prove that Modulus Function $f: R \to R$ given by : $f(x) = |x|$ is neither one-one nor onto, where $|x|$ is x, if x is positive and $|x|$ is - x, if x is negative.

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39. Show that the Signum Function $f: R \rightarrow Rgivenbyf(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$, is neither one-one nor onto.

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40. Let $A = \{1, 2, 3\}, B = \{4, 5, 6, 7\}$ and $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. Show that f is one-one.

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41. In the following case, state whether the function is one-one, onto or bijective. Justify your answer: $f: R \rightarrow R$ defined by

$f(x) = 3 - 4x$

A.

B.

C.

D.

Answer:

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42. In the following case, state whether the function is one-one, onto or bijective. Justify your answer: $f: R \rightarrow R$ defined by $f(x) = 1 + x^2$

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43. Let $f: N \to N$ be defined by, $f(n) = \begin{cases} \frac{n+1}{2} & \text{if } n is odd \\ \frac{n}{2} & \text{if } n is even \end{cases}$ for all $n \in N$. State whether the function f is bijective. Justify your answer.

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44. Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f: A -> B$. defined by $f(x) = \frac{x-2}{x-3}$ Is f one one or Onto? Justify your answer

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45. Let $f: R \to R$ be defined as $f(x) = 3x$ Choose the correct answer.

A. f is one-one onto

B. f is many-one onto

C. f is one-one but not onto

D. f is neither one-one nor onto.

Answer:

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46. Let $f:(2, 3, 4, 5\} \rightarrow (3, 4, 5, 9\}$ and $g = (3, 4, 5, 9\} \rightarrow (7, 11, 15\}$ be functions defined as: $f(2) = 3,\ f(3) = 4,\ f(4) = f(5) = 5$ and $g(3) = g(4) = 7,$ and $g(5) = g(11) = 11$. Find gof

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47. Let f, g and h be functions from R to R. Show that

$(f+g)oh = foh + goh\ (f.\ g)oh = (foh).\ (goh)$

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48. Find gof and fog, if $f(x) = |x|$ and $g(x) = |5x - 2|$

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49. Find gof and fog, if $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$

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50. If $f(x) = \frac{4x-3}{6x-4}$,$x \neq \frac{2}{3}$show that $f \circ f(x) = x$for all $x \neq \frac{2}{3}$.what is inverse of f?

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51. State with reason whether following functions have inverse:

$f: \{1, 2, 3, 4\} \rightarrow \{10\}$ with

$f = \{(1, 10), (2, 10), (3, 10), (4, 10)\}$

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52. State with reason whether following functions have inverse:

$g: \{5, 6, 7, 8\} \rightarrow \{1, 2, 3, 4\}$ with

$g = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$

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53. State with reason whether following functions have inverse:

$h: \{2, 3, 4, 5\} \rightarrow \{7, 9, 11, 13\}$ with

$f = \{(2, 7), (3, 9), (4, 11), (5, 13)\}$

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54. Consider f : R → R given by $f(x) = 4x + 3$. Show that f is invertible. Find the inverse of f.

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55. Consider $f: R_+ \rightarrow [4, \infty)$ given by$f(x) = x^2 + 4$ Show that f is invertible with the inverse $f^-1, offgivenbyf^-1(y) = \sqrt{y - 4}$, where R_+ is the set of all non-negative real numbers.

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56. Consider

$f: \{1, 2, 3\} \rightarrow \{a, b, c\}, givenby f(1) = a, f(2) = b$ and $f(3) = c$

. Find $f^{-}1$ and show that $(f^{-}1)^{-}1 = f$

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57. Let $f: X \rightarrow Y$ be an invertible function. Show that the inverse of $f^{-}1$ is f, i.e. $(f^{-}1)^{-}1 = f$

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58. If $f: R \rightarrow R, begivenby f(x) = (3 - x^3)^{\frac{1}{3}}$, then $fof(x)$ is :

A. $\frac{x^1}{3}$

B. x^3

C. x

D. $(3 - x^3)$

Answer:

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59. Let $f: R - \left\{\frac{4}{3}\right\} \rightarrow R$ be a function defined as $f(x) = 4\frac{x}{3x+4}$ The inverse of f is the $(Map)g$: Range $f \rightarrow R - \left\{\frac{4}{3}\right\}$, given by :

A. $g(y) = 3\frac{y}{3+4y}$

B. $g(y) = 4\frac{y}{4-3y}$

C. $g(y) = 4\frac{y}{3-4y}$

D. $g(y) = 3\frac{y}{4 - 3y}$

Answer:

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60. Determine whether or not each of the definition of '*' given below gives a binary operation. In the event that '*' is not a binary operation, give justification for this: $OnZ^+, def \in e \cdot bya \cdot b = a - b$

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61. Determine whether or not each of the definition of '*' given below gives a binary operation. In the event that '*' is not a binary operation, give justification for this: $On Z^{+}, def \in e' \cdot ' by a * b = ab$

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62. Determine whether or not each of the definition of '*' given below gives a binary operation. In the event that '*' is not a binary operation, give justification for this: $On Z^{+}, def \in e' \cdot ' by a * b = ab^2$

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63. Determine whether or not each of the definition of '*' given below gives a binary operation. In the event that '*' is not a binary operation, give justification for this: $On Z^{+}, def \in e' \cdot ' by a * b = |a - b|$

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64. Determine whether or not each of the definition of '*' given below gives a binary operation. In the event that '*' is not a binary operation, give justification for this: $OnZ^{+}, def \in e' \cdot 'bya * b = a$

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65. For operation $*$ defined below, determine whether $*$ is binary, commutative or associative: On Z, define $a \cdot b = a - b$

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66. For operation $*$ defined below, determine whether $*$ is binary, commutative or associative: On Q, define $a \cdot b = ab + 1$

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67. For operation $*$ defined below, determine whether $*$ is binary, commutative or associative: On Q, define$a \cdot b = a\frac{b}{2}$

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68. For operation $*$ defined below, determine whether $*$ is binary, commutative or associative: On Z^+, define$a \cdot b = 2^{ab}$

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69. For operation $*$ defined below, determine whether $*$ is binary, commutative or associative: On Z^+, define$a \cdot b = a^b$

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70. For operation $*$ defined below, determine whether $*$ is binary, commutative or associative: On $R - \{-1\}$, define

$$a \cdot b = \frac{a}{b+1}$$

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71. Consider the binary operation $\wedge$ on the set $\{1, 2, 3, 4, 5\}$ defined by $a \wedge b = \min\{a, b\}$ Write the operation table of the operation $\wedge$.

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72. Consider a binary operation $*$ on the set $\{1, 2, 3, 4, 5\}$ given by the following multiplication table

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

: Is $*$ commutative?

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73. Consider a binary operation $*$ on the set $\{1, 2, 3, 4, 5\}$ given by the following multiplication table

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

:

Compute (2*3)*(4*5)

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74. Let $*$ be the binary operation on N given by $\cdot a \cdot b = L.C.M. of a$ and b. Find : $5 \cdot 7, 20 \cdot 16$

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75. Let $*$ be the binary operation on N given by $\cdot a \cdot b = L.C.M.\, of a$ and b. Find : Is $*$ commutative?

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76. Let $*$ be the binary operation on N given by $\cdot a \cdot b = L.C.M.\, of a$ and b. Find : Is $*$ associative?

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77. Let $*$ be the binary operation on N given by $\cdot a \cdot b = L.C.M.\, of a$ and b. Find : Find the identity of $*$ in N

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78. Let $*$ be the binary operation on N given by $\cdot a \cdot b = L.C.M.\,of a$ and b. Find : Which elements of N are invertible for the operation $*$?

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79. Is $*$ defined on the set $\{1, 2, 3, 4, 5\}$ by $a \cdot b = L.C.M.\,of a$ and b a binary operation? Justify your answer.

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80. Let $*$ be the binary operation on N defined by $a \cdot b = H.C.F.\,of a$ and b. Is $*$ commutative?

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81. Let $*$ be the binary operation on N defined by $a \cdot b = H.C.F. of a$ and b. Is $*$ associative?

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82. Let $*$ be the binary operation on N defined by $a \cdot b = H.C.F. of a$ and b. Does there exist identity for this binary operation on N?

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83. Let $*$ be a binary operation on the set Q of rational numbers as follows: $a \cdot b = a - b$ find is it commutative?

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84. Let $*$ be a binary operation on the set Q of rational numbers as follows: $a \cdot b = a - b$ find is it associative?

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85. Let $*$ be a binary operation on the set Q of rational numbers as follows: $a \cdot b = a2 - b2$ find is it commutative?

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86. Let $*$ be a binary operation on the set Q of rational numbers as follows: $a \cdot b = a2 - b2$ find is it associative?

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87. Let $*$ be a binary operation on the set Q of rational numbers as follows: $a \cdot b = a + ab$ find is it commutative?

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88. Let $*$ be a binary operation on the set Q of rational numbers as follows: $a \cdot b = a + ab$ find is it associative?

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89. Let $*$ be a binary operation on the set Q of rational numbers as follows: $a \cdot b = (a - b)^2$ find is it commutative?

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90. Let $*$ be a binary operation on the set Q of rational numbers as follows: $a \cdot b = (a - b)^2$ find is it associative?

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91. Let $*$ be a binary operation on the set Q of rational numbers as follows: $a \cdot b = \frac{a^b}{4}$ find is it commutative?

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92. Let $*$ be a binary operation on the set Q of rational numbers as follows: $a \cdot b = \frac{a^b}{4}$ find is it associative?

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93. Let $*$ be a binary operation on the set Q of rational numbers as follows: $a \cdot b = (ab)^2$ find is it commutative?

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94. Let $*$ be a binary operation on the set Q of rational numbers as follows: $a \cdot b = (ab)^2$ find is it associative?

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95. Let $A = N \times N$ and $*$ be the binary operation on A defined by $(a, b) \cdot (c, d) = (a + c, b + d)$ Show that $*$ is commutative and associative. Find the identity element for $*$ on A, if any.

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96. State whether the following statement is true or false. Justify:For an arbitrary binary operation $\cdot$ on a set N, $a \cdot a = a, f$ or $alla \in N$

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97. State whether the following statement is true or false. Justify: If $*$ is a commutative binary operation on N, then $a \cdot (b \cdot c) = (c \cdot b) \cdot a$

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98. Consider a binary operation $*$ on N defined as $a \cdot b = a^3 + b^3$ Choose the correct answer: Is $*$ neither commutative nor associative?

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99. Consider a binary operation $*$ on N defined as $a \cdot b = a^3 + b^3$ Choose the correct answer:Is $*$ commutative but not associative?

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100. Consider a binary operation $*$ on N defined as $a \cdot b = a^3 + b^3$ Choose the correct answer: Is $*$ neither commutative nor associative?

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101. Let $f: R \rightarrow R$, be defined as $f(x) = 10x + 7$. Find the function $g: R \rightarrow R$ such that $gof = fog = 1_R$

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102. Let $f: W \rightarrow W$, be defined as $f(n) = n - 1$, if n is odd and $f(n) = n + 1$, if n is even. Show that f is invertible. Find the inverse of f. Here, W is the set of all whole numbers.

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103. If $f: R \rightarrow R$ is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$

.

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104. Show that the function $f: R \rightarrow \{x \in R: -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}, x \in R$ is one one and onto function.

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105. Show that the function $f: R \rightarrow R$ given by $f(x) = x^3$ is injective.

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106. Give examples of two functions $f: N \rightarrow Z$ and $g: Z \rightarrow Z$ such that gof is injective but g is not injective.

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107. Give examples of two functions $f: N \rightarrow N$ and $g: N \rightarrow N$ such that gof is onto but f is not onto.

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108. Given a non empty set X, consider $P(X)$ which is the set of all subsets of X. Define the relation R in $P(X)$ as follows: For subsets A, B in P(X), ARB if and only if $A \subset B$. Is R an equivalence relation on P(X)? Justify your answer.

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109. Find the number of all one-one functions from set $A = \{1, 2, 3\}$ to itself.

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110. Let $S = \{a, b, c\}$ and $T = \{1, 2, 3\}$. Find $F^{-}1$ of the following functions F from S to T, if it exists: $F = \{(a, 3), (b, 2), (c, 1)\}$

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111. Let $S = \{a, b, c\}$ and $T = \{1, 2, 3\}$. Find $F^{-}1$ of the following functions F from S to T, if it exists: $F = \{(a, 2), (b, 1), (c, 1)\}$

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112. Consider the binary operations $\cdot : R \times R \to R$ and $o : R \times R \to R$ defined as $a \cdot b = |a - b|$ and

$aob = a, \forall a, b \in R$ Show that $*$ is commutative but not associative, o is associative but not commutative. Further, show that ∀ $a, b, c \in R, a \cdot (boc) = (a \cdot b)o(a \cdot c)$. [If it is so, we say that the operation $*$ distributes over the operation o]. Does o distribute over $*$? Justify your answer.

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113. Given a non-empty set X, let $\cdot : P(X) \times P(X) \to P(X)$, be defined as $A \cdot B = (A - B) \cup (B - A), \forall A, B \in P(X)$ .Show that the empty set ϕ is the identity for the operation * and all the elements A of P(X) are invertible with $A^{-}1 = A$.

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114. Define a binary operation * on the set $\{0, 1, 2, 3, 4, 5\}$ as

$a \cdot b = \{a + b, \quad \text{if} \quad a + b < 6 a + b - 6, \quad \text{if} \quad a + b \geq 6.$

Show that zero is the identity for this operation and each element 'a' of the set is invertible with (6-a) being the inverse of a.

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115. Let $A = \{-1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2\}$ and $f, g: A \rightarrow B$, be functions defined by $f(x) = x^2 - x$, $x \in A$ and g(x)= 2|x-(1/2)|-1, x in A`. Are f and g equal? Justify your answer.

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116. Let $A = \{1, 2, 3\}$ Then number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is :

A. 1

B. 2

C. 3

D. 4

Answer:

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117. Let $A = \{1, 2, 3\}$ Then number of equivalence relations containing (1, 2) is:

A. 1

B. 2

C. 3

D. 4

Answer:

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118. Number of binary operations on the set {a, b} is :

A. 10

B. 16

C. 20

D. 8

Answer:

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