MATHS

BOOKS - PSEB

THREE DIMENSIONAL GEOMETRY

Example

1. If a line makes angle $90^o, 60^o, 30^o$ with the positive direction of x, y and z-axis respectively, find its direction cosines.

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2. If a line has direction ratios 2, – 1, – 2, determine its direction cosines.

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3. The direction cosines of the line joining the points (-2, 4, -5) and (1, 2, 3) is :

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4. Find the direction cosines of x, y and z-axis.

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5. Show that the points $A(2, 3, -4), B(1, -2, 3), C(3, 8, -11)$ are collinear.

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6. Find the vector and the Cartesian equations of the line through the point (5, 2, – 4) and which is

parallel to the vector $3\hat{i} + 2\hat{j} - 8\hat{k}$

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7. Find the vector equation for the line passing through the points (–1, 0, 2) and (3, 4, 6).

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8. The Cartesian equation of a line is $\frac{x+3}{2} = \frac{y-5}{4} = \frac{z+6}{2}$. Find the vector equation for the line.

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9. Find the angle between the pair of lines given by $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$ and $\vec{r} = 5\hat{i} - 2\hat{j} + \mu\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$

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10. Find the angle between the pair of lines $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$

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11. 1 Find the shortest distance between the lines l_1 and l_2 whose vector equations are:

$\vec{r} = \hat{i} + \hat{j} + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$, and

$\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu\left(3\hat{i} - 5\hat{j} + 2\hat{k}\right)$

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12. Find the distance between the lines l_1 and l_2 given by :

$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$ and

$\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right) C\tilde{O}$

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13. Find the vector equation of the plane which is at a distance of $\frac{6}{\sqrt{29}}$ from the origin and its normal vector from the origin is $2\hat{i} - 3\hat{j} + 4\hat{k}$. Also find its cartesian form.

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14. Find the direction cosines of the unit vector perpendicular to the plane

$\vec{r} \cdot \left(6\hat{i} - 3\hat{j} - 2\hat{k}\right) + 1 = 0$ passing through the origin.

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15. Find the distance of the plane $2x - 3y + 4z - 6 = 0$ from the origin.

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16. Find the coordinates of the foot of the perpendicular drawn from the origin to the

plane $2x - 3y + 4z - 6 = 0$

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17. Find the vector and cartesian equations of the plane which passes through the point (5, 2, – 4) and perpendicular to the line with direction ratios 2, 3, – 1.

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18. Find the vector equations of the plane passing through the points $R(2, 5, -3), S(-2, -3, 5), T(5, 3, -3)$

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19. Find the equation of the plane with intercepts 2, 3 and 4 on the x, y and z-axis respectively.

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20. Find the vector equation of the plane passing through the intersection of the planes $\vec{r} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = 6$ and $\vec{r} \cdot \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) = -5$, and the point (1, 1, 1).

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21. Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar.

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22. Find the angle between the two planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$ using vector method.

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23. Find the angle between the two planes $3x - 6y + 2z = 7$ and $2x + 2y - 2z = 5$

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24. Find the distance of a point (2, 5, – 3) from the plane $\vec{r} \cdot \left(6\hat{i} - 3\hat{j} + 2\hat{k}\right) = 4$

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25. Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$

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26. If a line marks angles α, β and γ with the coordinates axes, prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

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27. Find the equation of the plane passing through the point (1, -1 , 2) and perpendicular to the planes 2x + 3y - 2z = 5 and x + 2y - 3z = 8.

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28. Find the distance between the point $P(6, 5, 9)$ and the plane determined by the points

$A(3, -1, 2), B(5, 2, 4), C(-1, -1, 6)$

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29. Show that the lines $\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha+\delta}$ and $\frac{x-b+c}{\beta-\gamma} = \frac{y-b}{\beta} = \frac{z-b-c}{\beta+\gamma}$ are coplanar.

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30. Find the coordinates of the point where the line through the points $A(3, 4, 1)$ and $B(5, 1, 6)$ crosses the XY-plane.

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Exercise

1. A line makes 90°, 135°, 45° with x, y and z axes respectively than its direction cosines are

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2. Find the direction cosines of a line which makes equal angles with the coordinate axes.

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3. If a line has the direction ratios –18, 12, – 4, then what are its direction cosines ?

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4. Show that the points (2, 3, 4), (– 1, – 2, 1), (5, 8, 7) are collinear.

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5. Find the direction cosines of the sides of the triangle whose vertices are (3, 5, – 4), (– 1, 1, 2) and (– 5, – 5, – 2).

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6. Show that the three lines with direction cosines

$$\frac{12}{13}, -\frac{3}{13}, -\frac{4}{13}, \frac{4}{13}, \frac{12}{13}, \frac{3}{13}, \frac{3}{13}, -\frac{4}{13}, \frac{12}{13}$$

are mutually perpendicular.

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7. Show that the line through the points (1, – 1, 2), (3, 4, – 2) is perpendicular to the line through the points (0, 3, 2) and (3, 5, 6).

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8. Show that the line through the points (4, 7, 8), (2, 3, 4) is parallel to the line through the points (– 1, – 2, 1), (1, 2, 5).

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9. Find the equation of the line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$

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10. Find the equation of the line in vector and in cartesian form that passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction $\hat{i} + 2\hat{j} - \hat{k}$

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11. Find the cartesian equation of the line which passes through the point (– 2, 4, – 5) and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$

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12. The cartesian equation of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$. Write its vector form.

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13. Find the vector and the cartesian equations of the lines that passes through the origin and (5, – 2, 3).


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14. Find the vector and the cartesian equations of the line that passes through the points (3, – 2, – 5), (3, – 2, 6).


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15. Find the angle between the following pair of lines: $\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ and $\vec{r} = 7\hat{i} - 6\hat{j} + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$

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16. Find the angle between the following pair of lines: $\vec{r} = 3\hat{i} + \hat{j} - 2\hat{k} + \lambda\left(\hat{i} - \hat{j} - 2\hat{k}\right)$ and $\vec{r} = 2\hat{i} - \hat{j} - 56\hat{k} + \mu\left(3\hat{i} - 5\hat{j} - 4\hat{k}\right)$

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17. Find the angle between the lines $\frac{x-2}{2}=\frac{y-1}{5}=\frac{z+3}{-3}$ and $\frac{x+2}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$

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18. Find the angle between the lines $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{x-5}{4}=\frac{y-2}{1}=\frac{z-3}{8}$

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19. Find the values of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2}p = \frac{z-3}{2}$ and $\frac{7-7x}{3}p = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles.

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20. Show that the lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other

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21. Find the shortest distance between the lines

$$\vec{r} = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$$ and

$$\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$$

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22. Find the shortest distance between the lines

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$ and

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

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23. Find the shortest distance between the lines whose vector equations are $\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - 3\hat{j} + 2\hat{k}\right)$ and $\vec{r} = 4\hat{i} + 5\hat{j} + 6\hat{k} + \mu\left(2\hat{i} + 3\hat{j} + \hat{k}\right)$

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24. Find the shortest distance between the lines whose vector equations are $\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$

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25. In the following case, determine the direction cosines of the normal to the plane and the distance from the origin: $z = 2$

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26. In the following case, determine the direction cosines of the normal to the plane and the distance from the origin: $x + y + z = 1$

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27. In the following case, determine the direction cosines of the normal to the plane and the distance from the origin: $2x + 3y - z = 5$

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28. In the following case, determine the direction cosines of the normal to the plane and the distance from the origin: $5y + 8 = 0$

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29. Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector $3\hat{i} + 5\hat{j} - 6\hat{k}$.

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30. Find the Cartesian equation of the following plane: $\vec{r} \cdot \left(\hat{i} + \hat{j} - \hat{k}\right) = 2$

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31. Find the Cartesian equation of the following plane: $\vec{r} \cdot \left(2\hat{i} + 3\hat{j} - 4\hat{k}\right) = 1$

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32. Find the Cartesian equation of the following plane:

$\vec{r} \cdot \left[(s - 2t)\hat{i} + (3 - t)\hat{j} + (2s + t)\hat{k}\right] = 15$

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33. In the following case, find the coordinates of the foot of the perpendicular drawn from the origin: $2x + 3y + 4z - 12 = 0$

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34. In the following case, find the coordinates of the foot of the perpendicular drawn from the origin: $3y + 4z - 6 = 0$

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35. In the following case, find the coordinates of the foot of the perpendicular drawn from the origin: $x + y + z = 1$

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36. In the following case, find the coordinates of the foot of the perpendicular drawn from the origin: $5y + 8 = 0$

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37. Find the vector and cartesian equations of the plane that passes through the point (1, 0, –2) and the normal to the plane is $\hat{i} + \hat{j} - \hat{k}$

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38. Find the vector and cartesian equations of the plane that passes through the point (1,4, 6) and the normal vector to the plane is $\hat{i} - 2\hat{j} + \hat{k}$

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39. Find the equations of the plane that passes through three points : (1,1,-1), (6,4,-5), (-4,-2,3)

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40. Find the equations of the plane that passes through three points : (1,1,0), (1,2,1), (-2,2,-1)

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41. Find the intercepts cut off by the plane $2x + y - z = 5$

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42. Find the equation of the plane with intercept 3 on the y-axis and parallel to ZOX plane.

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43. Find the equation of the plane through the intersection of the planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and the point (2, 2, 1).I

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44. Find the vector equation of the plane passing through the intersection of the planes $\vec{r} \cdot \left(2\hat{i} + 2\hat{j} - 3\hat{k}\right) = 7, \vec{r} \cdot \left(2\hat{i} + 5\hat{j} + 3\hat{k}\right) = 9$ and through the point (2, 1, 3)

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45. Find the equation of the plane through the line of intersection of the planes given by the equations x + y + z = 1 and 2x + 3y + 4z = 5 which is perpendicular to the plane given by the equation x - y + z = 0.

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46. Find the angle between the planes whose vector equations are $\vec{r} \cdot \left(2\hat{i} + 2\hat{j} - 3\hat{k}\right) = 5$ and $\vec{r} \cdot \left(3\hat{i} - 3\hat{j} + 5\hat{k}\right) = 3$

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47. In the following case, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between

them:$7x + 5y + 6z + 30 = 0$and

$3x - y - 10z + 4 = 0$

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48. In the following case, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them: $2x + y + 3z - 2 = 0$ and $x - 2y + 5 = 0$

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49. In the following case, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them: $2x - 2y + 4z + 5 = 0$ and $3x - 3y + 6z - 1 = 0$

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50. પIn the following case, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them: $2x - y + 3z - 1 = 0$ and $2x - y + 3z + 3 = 0$

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51. In the following case, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them: $4x + 8y + z - 8 = 0$ and $y + z - 4 = 0$

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52. In the following case, find the distance of each of the given point from the corresponding

given plane: Point (0,0,0) Plane $3x - 4y + 12z = 3$

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53. In the following case, find the distance of each of the given point from the corresponding given plane: Point (3,-2,1) Plane $2x - y + 2z + 3 = 0$

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54. In the following case, find the distance of each of the given point from the corresponding given plane:Point (2,3,-5) Plane $x + 2y - 2z = 9$

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55. In the following case, find the distance of each of the given point from the corresponding given plane: Point (-6,0,0) Plane $2x - 3y + 6z - 2 = 0$

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56. Show that the line joining the origin to the point (2, 1, 1) is perpendicular to the line determined by the points (3, 5, – 1), (4, 3, – 1).

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57. If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both of these are $m_1n_2 - m_2n_1, n_1l_2 - n_2l_1, l_1m_2 - l_2 - m_1$

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58. Find the angle between the lines whose direction ratios are a, b, c and b-c, c-a, a-b

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59. Find the equation of a line parallel to x-axis and passing through the origin.

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60. If the coordinates of the points A, B, C, D be (1, 2, 3), (4, 5, 7), (– 4, 3, – 6) and (2, 9, 2) respectively, then find the angle between the lines AB and CD.

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61. If the lines $\frac{x-1}{-3} = \frac{y-2}{2}k = \frac{z-3}{2}$ and $\frac{x-1}{3}k = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular, find the value of k

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62. Find the vector equation of the line passing through (1, 2, 3) and perpendicular to the plane $\vec{r} \cdot \left(\hat{i} + 2\hat{j} - 5\hat{k}\right) + 9 = 0$

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63. Find the equation of the plane passing through (a, b, c) and parallel to the plane $\vec{r} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = 2$

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64. Find the shortest distance between the lines given by the line : $\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$ and $\vec{r} = 4\hat{i} + \hat{k} + \mu\left(3\hat{i} - 2\hat{j} - 2\hat{k}\right)$.

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65. Find the coordinates of that point when the line passing through two points (5, 1, 6) and (3, 4, 1) crosses YZ plane.

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66. Find the coordinates of the point where the line through (3, – 4, – 5) and (2, – 3, 1) crosses the plane $2x + y + z = 7$

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67. Find the equation of the plane passing through the point (-1, 3, 2) and perpendicular to each of the planes : $x + 2y + 3z = 5$ and $3x + 3y + z = 0.$

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68. If the points (1, 1, p) and (– 3, 0, 1) be equidistant from the plane $\vec{r} \cdot \left(3\hat{i} + 4\hat{j} - 12\hat{k}\right) + 13 = 0$ then find the value of p.

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69. Find the equation of the plane passing through the line of intersection of the planes $\vec{r} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = 1$ and $\vec{r} \cdot \left(2\hat{i} + 3\hat{j} - \hat{k}\right) + 4 = 0$ and parallel to x-axis.

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70. If O be the origin and the coordinates of P be (1, 2, – 3), then find the equation of the plane passing through P and perpendicular to OP.

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71. Find the equation of the plane which contains the line of intersection of the planes $\vec{r} \cdot \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - 4 = 0$ and $\vec{r} \cdot \left(2\hat{i} + \hat{j} - \hat{k}\right) + 5 = 0$ and which is

perpendicular to the plane $\vec{r} \cdot \left(5\hat{i} + 3\hat{j} - 6\hat{k}\right) + 8 = 0$

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72. Find the distance of the point (-1 , -5 , -10) from the point of intersection of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$ and the plane $\vec{r} . \left(\hat{i} - \hat{j} + \hat{k}\right)$=5.

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73. Find the vector equation of the line passing through (1, 2, 3) and parallel to the planes $\vec{r}.\left(\hat{i}-\hat{j}+2\hat{k}\right)=5$ and $\vec{r}.\left(3\hat{i}+\hat{j}+\hat{k}\right)=6.$

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74. Find the equations of the straight line passing through the point (1, 2, - 4) and is perpendicular to the lines : $\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$.

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75. Prove that if a plane has the intercepts a, b, c and is at a distance of p units from the origin, then $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$

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76. Distance between the two planes: $2x+3y+4z=4$ and $4x+6y+8z=12$ is:

A. 2 units

B. 4 units

C. 8 units

D. $\frac{2}{\sqrt{29}} units$

Answer:

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77. The planes $2x - y + 4z = 5$ and $5x - 2.5y + 10z = 6$ are :

A. Perpendicular

B. Parallel

C. ntersect y-axis

D. passes through $\left(0, 0, \frac{5}{4}\right)$

Answer:

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