MATHS

NCERT - FULL MARKS MATHS(TAMIL)

APPLICATIONS OF DIFFERENTIAL CALCULUS

Example

1. For the function $f(x) = x^2, x \in [0, 2]$ compute the average rate of changes in the

subintervals $[0, 0.5]$, $[0.5, 1]$, $[1, 1.5]$, $[1.5, 2]$ and the instantaneous rate of changes at the points $x = 0.5, 1, 1.5, 2$

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2. The temperature in celsius in a long rod of length 10m, insulated at both ends, is a function of length x given by $T = x(10 - x)$. Prove that the rate of change of temperature at the midpoint of the rod is zero.

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3. A person learnt 100 words for an English test. The number of words the persons remembers in t days after learning is given by $W(t) = 100 \times (1 - 0.1)^2, \ 0 \le t \le 10.$ What is the rate at which the person forgets the words 2 days after learning ?

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4. A particle moves so that the distance moved in according to the law $s(t) = \frac{t^3}{3} - t^2 + 3.$ At what time the velocity and acceleration are zero respectively ?

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5. A particle is fired straight up from the ground to reach a height of s feet in t seconds, where $s(t) = 128t - 16t^2$.

(1) Compute the maximum height of the particle reached .

(2) What is the velocity when the particle hits the ground ?

6. A particle moves along a horizontal line such that its position at any time $t \geq 0$ is given by $s(t) = t^3 - 6t^2 + 9t + 1$, where s is measusred in metres and t in seconds ?

(a) At what time the particle is at rest ? (3) Find the total distance travelled by the particle in the first 2 seconds.

7. If we blow air into a balloon of spherical shape at a rate of $1000cm^3$ per second . At what rate the radius of the baloon changes when the radius is 7cm ? Also compute the rate at which the surface changes.

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8. The price of a product is related to the number of units available (supply) by th equation $Px + 3P - 16x = 234$, where P is the price of the product per unit in Rupees (Rs.) and x is the number of units. Find the rate at which the price is changing with respect to time when 90 units are available and the supply is increasing at a rate of 15 units week .

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9. Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high ?

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10. A road running north to south crosses a road going east to west at the point P. Car A is driving north along the first road, and car B is driving east along the second road. At a particular time car A is 10 kilometres to the north of P and traveling at 80 km/hr, while car

B is 15 kilometers to the east of P and travelling at 100 km/hr. How fast is the distance between the two cars changing ?

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11. Find the equations of tangent and normal to the curve $y = x^2 + 3x - 2$ at the point $(1, 2)$.

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12. For what value of x the tangent of the curve $y = x^3 - 3x^2 + x - 2$ is parallel to the line $y = x$.

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13. Find the equation of the tangent and normal to the Lissajous curve given by $x = 2\cos 3t$ and $y = 3\sin 2t, t \in \mathbb{R}$.

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14. Find the acute angle between $y = x^2$ and $y = (x - 3)^2$.

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15. Find the acute angle between the curves $y = x^2$ and $x = y^2$ at their points of intersection $(0, 0), (1, 1)$.

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16. Find the angle of intersection of the curve $y = \sin x$ with the positive x-axis.

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17. Compute the value of 'c' satisfied by the Rolle's theorem for the function $f(x) = x^2(1-x)^2, x \in [0, 1]$.

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18. Find the values in the interval $\left(\frac{1}{2}, 2\right)$ satisfied by the Rolle's theorem for the function $f(x) = x + \frac{1}{x}, x \in \left[\frac{1}{2}, 2\right]$

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19. Expand : $\lim_{x \to 1} \left(\frac{x^2 - 3x + 2}{x^2 - 4x + 3}\right)$

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20. Compute the limit $\lim_{x \to a} \left(\frac{x^n - a^n}{x - a}\right)$.

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21. Evaluate the limit $\lim_{x \to 0}\left(\frac{\sin mx}{x}\right)$

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22. Evaluate the limit $\lim_{x \to 0}\left(\frac{\sin x}{x^2}\right)$

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23. Evaluate : $\lim_{x \to 0^+}\left(\frac{1}{x} - \frac{1}{e^x - 1}\right)$.

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24. Evaluate : $\lim_{x \to \infty} \left(\frac{e^x}{x^m}\right), m \in N$

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25. Evaluate : $\lim_{x \to \infty} (1 + 2x)^{\frac{1}{2 \log x}}$

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26. Evaluate : $\lim_{x \to 1} x^{\frac{1}{1 - x}}$

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27. Find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 + 3x^2 - 12x$ on $[-3, 2]$

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28. Find the intervals of monotonicity and local extrema of the function $f(x) = x \log x + 3x$

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29. Find the intervals of monotonicity and local extrema of the function $f(x)=\frac{1}{1+x^2}$.

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30. Find the intervals of monotoncitiy and local extrema of the function $f(x)=\frac{x}{1+x^2}$

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31. Find the local extremum of the function $f(x) = x^4 + 32x.$

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32. Find the local extremum of the function $f(x) = 4x^6 - 6x^4.$

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33. We have 12 square unit piece of thin material and want to make an open box by cutting small squares from the corners of our material and folding the sides up. The question is, which cut produces the box of maximum volume ?

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34. Find the points on the unit circle $x^2 + y^2 = 1$ nearest and farthest from $(1, 1)$

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35. A steel plant is capable of producing x tonnes per day of a low- grade steel and y tonnes per day of a high-grade steel , where $y = \frac{40 - 5x}{10 - x}$. If the fixed market price of low gradesteel is half that of high-grade steel then what should be optimal productions in low-grade steel and high grade steel in order to have maximum receipts

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36. Find the asymptotes of the curve

$$f(x) = \frac{2x^2 - 8}{x^2 - 16}$$

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Exercise 7 1

1. A point moves along a straight line in such a way that after t seconds its distance from the origin is $s = 2t^2 + 3t$ metres.

(i) Find the average velocity of the points between at $t = 3$ and $t = 6$ seconds.

(ii) Find the instantaneous velocities at $t = 3$ and $t = 6$ second .

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2. A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of $s = 16t^2$ in t seconds.

(i) How long does the camera fall before it hits the ground ?

(ii) What is the average velocity with which the camera falls during the last 2 seconds ?

(iii) What is the instantaneous velocity of the camera when it hits the ground ?

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3. A particle moves along a line according to the law $s(t) = 2t^3 - 9t^2 + 12t - 4$, where $t \geq 0$.

(i) At what times the particle changes direction ?

(ii) Find the total distance travelled by the particle in the first 4 seconds

(iii) Find the particle's acceleration each time the velocity is zero.

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4. If the voulme of a cube of side length x is v $= x^3$. Find the rate of change of the volume with respect to x when $x = 5$ units.

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5. If the mass m(x) (in kilograms) of a thin rod of length x (in metres) is given by, $m(x) = \sqrt{3x}$ then what is the rate of change of mass with respect to the length when it is $x = 3$ and $x = 27$ metres.

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6. A stone is dropped into a pond causing ripple in the form of concentric circles. The radius r of the outer ripple is increasing at a costant rate at 2cm per second. When the radius is 5 cm find the rate of changing of the total area of the disturbed water ?

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7. A beacon makes one revolution every 10 seconds. It is located on a ship which is anchored 5 km from a straight shore line. How fast is the beam moving along the shore line when it makes an angle of 45° with the shore ?

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8. A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m.min, how fast is the depth of the water increases when the water is 8 metres deep ?

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9. A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5m/s. When the base of the ladder is 8 metres from the wall.

(i) How fast is the top of the ladder moving down the wall ?

(ii) What rate, the area of the triangle formed by the ladder, wall, and the floor, is changing ?

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10. A police jeep, approaching an othogonal intersection from the northern direction , is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is $0.8 km$ to the east. The police determine with a radar that the distance between them and the car is increasing at 20km/hr. If the jeep is moving at 60 km/hr at the instant of measurement, what is the speed of the car ?

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Exercise 7 2

1. Find the slop of the tangent to the curves at the respective given points.

$(i) y = x^4 + 2x^2 - x$ at $x = 1$ (ii) $x = a\cos^3 t$, $y = b\sin^3 t$ at $t = \frac{\pi}{2}$

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2. Find the point on the curve $y = x^2 - 5x + 4$ at which the tangent is parallel to the line $3x + y = 7$

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3. Find the points on the curve $y = x^3 - 6x^2 + x + 3$ where the normal is parallel to the line $x + y = 1729$.

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4. Find the points on the curve $y^2 - 4xy = x^2 + 5$ for which the tangent is horizontal.

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5. Find the tangent and normal to the following curves at the given points on the curve

$(i) y = x^2 - x^4$ at $(1, 0)$ $(ii) y = x^4 + 2e^x$ at $(0, 2)$

$(iii) y = x \sin x$ at $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ $(iv) x = \cos t$, $y = 2\sin^2 t$ at $t = \frac{\pi}{3}$

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6. Find the equations of the tangents to the curve $y = 1 + x^3$ for which the tangent is orthogonal with the line $x + 12y = 12$.

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7. Find the equations of the tangents to the curve $y = \frac{x+1}{x-1}$ which are parallel to the line $x + 2y = 6$.

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8. Find the equation of tangent and normal to the curve given by $x = 7\cos tt$ and $y = 2\sin t$, $t \in \mathbb{R}$ at any point on the curve.

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9. Find the angle between the rectangular hyperbola $xy = 2$ and the parabola $x^2 + 4y = 0$.

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Exercise 7 3

1. Explain why Rolle's theorem is not applicable to the following functions in the respective intervals.

$(i) f(x) = \left|\frac{1}{x}\right|, x \in [-1, 1]$ (ii) $f(x) = \tan x$, $x \in [0, \pi]$

(iii) $f(x) = x - 2\log x, x \in [2, 7]$

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2. Using the Rolle's theorem , determine the values of x at which the tangent is parallel to the x-axis for the following functions :

$(i) f(x) = x^2 - x, \quad x \in [0, 1]$ (ii) $f(x) = \frac{x^2 - 2x}{x + 2}, x \in [-1, 6]$

$(iii) f(x) = \sqrt{x} - \frac{x}{3}, x \in [0, 9]$

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3. Explain why Lagrange's mean value theorem is not applicable to the following functions in the respective intervals :

$(i) f(x) = \frac{x+1}{x}, \qquad x \in [-1, 2]$

$(ii) f(x) = |3x+1|, x \in [-1, 3]$

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4. Using the Lagrange's mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval :

(i) $f(x) = x^3 - 3x + 2, \quad x \in [-2, 2]$ (ii) $f(x) = (x-2)(x-7), x \in [3, 1]$

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5. A race car driver is racing at 20^{th}km. If his speed never exceeds 150 km/hr, what is the maximum distance he can cover in the next two hours.

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6. Does there exist a differentiable function f(x) such that $f(0) = -1, \ f(2) = 4$ and $f'(x) \leq 2$ for all x . Justify your answer.

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Exercise 7 4

1. Write the Maclauring series expansion of the following functions

$(i) e^{x}$ $(ii) \sin x$ (iii) $\cos x$

$(iv) \log(1-x), \ -1 \leq x < 1$ (v) $\tan^{-1}(x)$: $-1 \leq x \leq 1$ $(vi) \cos^{2} x$

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2. Write down the Taylor series expansion, of the function logx about x=1 upto three non-zero terms for $x > 0$.

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3. Expand sinx in ascending $x - \frac{\pi}{4}$ upto three non-zero terms.

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4. Expand the polynomial $f(x)=x^2-3x+2$

in powers of x-1.

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Exercise 7 5

1. Evaluate the following limits, if necessary

use 1 'Hopital Rule:

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}$$

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2. Evaluate the following limits, if necessary use 1 'Hopital Rule:

$$\lim_{x \to \infty} \frac{2x^2 - 3}{x^2 - 5x + 3}$$

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3. Evaluate the following limits, if necessary use 1 'Hopital Rule:

$$\lim_{x \to \infty} \frac{x}{\log x}$$

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4. Evaluate the following limits, if necessary use 1 'Hopital Rule:

$$\lim_{x \to \frac{\pi}{2}} \frac{\sec x}{\tan x}$$

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5. Evaluate the following limits, if necessary use 1 'Hopital Rule:

$$\lim_{x \to \infty} e^{-x}\sqrt{x}$$

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6. Evaluate the following limits, if necessary use 1 'Hopital Rule:

$$\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$$

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7. Evaluate the following limits, if necessary use 1 'Hopital Rule:

$$\lim_{x \to 1^{+}} \left(\frac{2}{x^2 - 1} - \frac{x}{x - 1} \right)$$

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8. Evaluate the following limits, if necessary use 1 'Hopital Rule:

$$\lim_{x \to 0^+} x^x$$

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9. Evaluate the following limits, if necessary use 1 'Hopital Rule:

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$$

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10. Evaluate the following limits, if necessary use 1 'Hopital Rule:

$$\lim_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}$$

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11. Evaluate the following limits, if necessary use 1 'Hopital Rule:

$$\lim_{x \to 0^{+}} (\cos x)^{\frac{1}{x^2}}$$

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Exercise 7 6

1. Find the absolute extrema of the following functions on the given closed intervals

(i) $f(x) = x^2 - 12x + 10, [1, 2]$ (ii) $f(x) = 3x^4 - 4x^3, [-1, 2]$

$(iii) f(x) = 6x^{\frac{4}{3}} - 3x^{\frac{1}{3}}, [-1, 1]$

$(iv) f(x) = 2\cos x + \sin 2x$

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2. Find the intervals of monotonicities and hence find the local extremum for the following functions

$(i)\, f(x) = 2x^3 + 3x^2 - 12x$

$(ii)\, f(x) = \frac{x}{x-5}$

$(iii)\, f(x) = \frac{e^x}{1-e^x}$

$(iv)\, f(x) = (x^3)\,/\,(3) - \log x$

(v) $f(x) = \sin x \cos x + 5, x \in (0, 2\pi)$

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1. Find the intervls of concavity and points of inflexion for the following functions :

$(i) f(x) = x(x-4)^3$

$(ii) f(x) = \sin x + \cos x, 0 < x < 2\pi$

$(iii) f(x) = \frac{1}{2}\left(e^x - e^{-x}\right)$

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2. Find the local extrema for the following functions using second derivative test

$(i) f(x) = -3x^5 + 5x^3$ $(ii) f(x) = x \log x$

$(iii) f(x) = x^2 e^{-2x}$

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3. For the function $f(x) = 4x^3 + 3x^2 - 6x + 1$ find the intervals of monotonicity, local extrema intervals of concavity and points of inflection.

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Exercise 7 8

1. Find two positive number whose sum is 12 and their product is maximum.

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2. Find two positive number whose product is 20 and their product is minimum.

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3. Find the smallest possible value of $x^2 + y^2$ given that $x + y = 10$

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4. A garden is to be laid out in a rectangular area and protected by wire fence. What is the largest possible area of the fenced garden with 40 metres of wire.

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5. A rectangular page is to contain $24cm^2$ of point. The margins at the top and bottom of the page are $1.5cm$ and the margins at the other sides of the page is 1cm. What should be the dimensions of the page so that the area of the paper used is minimum.

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6. A farmer plans to fence a rectangular pasture adjacent to a river . The pasture must contain 180000 sq.mtrs in order to provide enough grass for herds. No fencing is needed along the river. What is the length of the minimum needed fencing material ?

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7. Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10cm.

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8. Find the dimensions of the largest rectangle that can be inscribed in a semi circle of radius r cm.

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9. A manufacturer wants to design an open box having a square base and a surface area of 108 sq cm. Determine the dimensions of the box for the maximum volume

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10. The volumme of a cylinder is given by the formula $V = \pi r^2 h$. Find the greatest and least values of V if $r + h = 6.$

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Exercise 7 9

1. Find the asymptotes of the following curves :

$(i) f(x) = \frac{x^2}{x^2 - 1}$ $\qquad$ $(ii) f(x) = \frac{x^2}{x + 1}$

$$(iii)\, f(x) = \frac{3x}{\sqrt{x^2+2}}$$

$$(iv)\, f(x) = \frac{x^2 - 6x - 1}{x+3}$$

$$(v)\, f(x) = \frac{x^2 + 6x - 4}{3x - 6}$$

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Exercise 7 10

1. The volume of a sphere is increasing in volume at the rate of $3\pi cm^3 / \text{sec}$. The rate of change of its radius when radius is $\frac{1}{2}$ cm

A. 3cm/s

B. 2cm/s

C. 1 cm/s

D. $\frac{1}{2}$cm/s

Answer: A

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2. A ballon rises straight up at $10m/s$. An observer is 40m away from the spot where the balloon left the ground. Find the rate of change of the balloon's angle of elevation in radian per second when the balloon is 30 metres above the ground.

A. $\frac{3}{25}$ radians/sec

B. $\frac{4}{25}$ radians/sec

C. $\frac{1}{5}$ radians/sec

D. $\frac{1}{3}$ radians/sec

Answer: B

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3. The position of a particle moving along a horizontal line of any time t is given by $s(t) = 3t^2 - 2t - 8$. The time at which the particle is at rest is

A. $t = 0$

B. $t = \frac{1}{3}$

C. $t = 1$

D. $t = 3$

Answer: B

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4. A stone is thrown up vertically. The height it reaches at time t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in time t seconds is given by

A. 2

B. 2.5

C. 3

D. 3.5

Answer: B

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5. Find the point on the curve $6y = x^3 + 2$ at which y-coordinate changes 8 times as fact as x-coordinate is

A. $(4, 11)$

B. $(4, -11)$

C. $(-4, 11)$

D. $(-4, -11)$

Answer: A

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6. The absicssa of the point on the curve $f(x) = \sqrt{8 - 2x}$ at which the slope of the tangent is -0.25 ?

A. -8

B. -4

C. -2

D. 0

Answer: B

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7. The slope of the line normal to the curve $f(x) = 2\cos 4x$ at $x = \frac{\pi}{12}$ is

A. $-4\sqrt{3}$

B. -4

C. $\frac{\sqrt{3}}{12}$

D. $4\sqrt{3}$

Answer: C

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8. The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when

A. $y = 0$

B. $y = \pm\sqrt{3}$

C. $y = \frac{1}{2}$

D. $y = \pm 3$

Answer: D

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9. Angle between $y^2 = x$ and $x^2 = y$ at the origin is

A. $\tan^{-1}. \frac{3}{4}$

B. $\tan^{-1}\left(\frac{4}{3}\right)$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{4}$

Answer: C

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10. What is the value of the limit $\lim_{x \to 0} \left(\cot x - \frac{1}{x} \right)$?

A. 0

B. 1

C. 2

D. $\leq$

Answer: D

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11. The function $\sin^4 x + \cos^4 x$ is increasing in the interval

A. $\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$

B. $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$

C. $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

D. $\left[0, \frac{\pi}{4}\right]$

Answer: C

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12. The number given by the Rolle's theorem for the function $x^3 - 3x^2, x \in [0, 3]$ is

A. 1

B. $\sqrt{2}$

C. $\frac{3}{2}$

D. 2

Answer: D

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13. The number given by the Mean value theorem for the function $\frac{1}{x}, x \in [1, 9]$ is

A. 2

B. 2.5

C. 3

D. 3.5

Answer: C

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14. The minimum value of the function $|3 - x| + 9$ is

A. 0

B. 3

C. 6

D. 9

Answer: D

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15. The maximum slope of the tangent to the curve $y = e^x \sin x, x \in [0, 2\pi]$ is at

A. $x = \frac{\pi}{4}$

B. $x = \frac{\pi}{2}$

C. $x = \pi$

D. $x = \frac{3\pi}{2}$

Answer: B

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16. The maximum value of the function $x^2 e^{-2x}$

, $x > 0$ is

A. $\frac{1}{e}$

B. $\frac{1}{2e}$

C. $\frac{1}{e^2}$

D. $\frac{4}{e^4}$

Answer: C

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17. One of the closest points on the curve $x^2 - y^2 = 4$ to the point $(6, 0)$ is

A. $(2, 0)$

B. $(\sqrt{5}, 1)$

C. $(3, \sqrt{5})$

D. $(\sqrt{13}, -\sqrt{3})$

Answer: C

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18. The maximum product of two positive numbers, when their sum of the squares is 200,is

A. 100

B. $25\sqrt{7}$

C. 28

D. $24\sqrt{14}$

Answer: A

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19. The curve $y = ax^4 + bx^2$ with $ab > 0$

A. has no horizontal tangent

B. is concave up

C. is concave down

D. has no points of inflection

Answer: D

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20. The point of inflection of the curve $y = (x - 1)^3$ is

A. $(0, 0)$

B. $(0, 1)$

C. $(1, 0)$

D. $(1, 1)$

Answer: C

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