MATHS

NCERT - FULL MARKS MATHS(TAMIL)

APPLICATIONS OF VECTOR ALGEBRA

Example

1. In triangle, ABC the points D,E,F are the midpoints of the sides , BC CA, and AB respectively. Using vector method, show that the area of ΔDEF is equal to $\frac{1}{4}(\text{area of}\Delta ABC)$.

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2. A straight line passes through the point (1,2,-3) and parallel to $4\hat{i}+5\hat{i}-7\hat{k}$. Find (i) vector equation in parametric form (ii) vector equation in non-parametric form (iii) Cartesian equation of the straight line.

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3. Find the acute angle between the lines $\vec{r} = \left(\hat{i} + 2\hat{j} + 4\hat{k}\right) + t\left(2\hat{i} + 2\hat{j} + \hat{k}\right)$ and the straight line passing through the points (5,4,1) and (9,2,12).

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4. Find the point of intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2}$ = z.

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5. Find the shortest distance between the two given straight lines $\vec{r} = \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) + t\left(-2\hat{i} + \hat{j} - 2\hat{k}\right)$ and $\frac{x-3}{2} = \frac{y}{-1} = \frac{z+2}{2}$

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6. Find the coordinates of the foot of the perpendicular drawn from the point (1,2,3) – to the straight line $\overrightarrow{r}=\left(\hat{i}-4\hat{j}+3\hat{k}\right)+t\left(2\hat{i}+3\hat{j}+\hat{k}\right)$. Also find the shortest distance from the given point to the straight line.

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7. Find the direction cosines of the normal to the plane and length of the perpendicular from the origin to the plane $\overrightarrow{r}.\left(3\hat{i}-4\hat{j}+12\hat{k}\right)=5.$

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8. Find the vector and Cartesian equations of the plane passing through the point with position vectork $4\hat{i}+2\hat{j}-3\hat{k}$ and normal to vector

$2\hat{i} - \hat{j} + \hat{k}.$

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9. Find the distance of the point (5,-5,-10) from the point of intersection of a straight line passing through the points A(4,1,2) and B (7,5,4) with the plane x - y + z = 5.

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10. Find the distance between the parallel planes x + 2y - 2z + 1 = 0 and 2x + 4y - 4z + 5 = 0.

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11. Find the distance between the planes $\vec{r}.\left(2\hat{i} - \hat{j} - 2\hat{k}\right) = 6 \text{ and } \vec{r}.\left(6\hat{i} - 3\hat{j} - 6\hat{k}\right) = 27$

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Exercise 6 1

1. A particle acted on by constant forces $8\hat{i}+2\hat{j}-6\hat{k}$ and $6\hat{i}+2\hat{j}-2\hat{k}$ is displaced from the point, (1,2,3) to the point (5,4,1). Find the total work done by the forces.

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2. Forces of magnitude $5\sqrt{2}$ and $10\sqrt{2}$ units acting in the directions $3\hat{i}+4\hat{j}+5\hat{k}$ and $10\hat{i}+6\hat{j}-8\hat{k}$, respectively, act on a particle which is displaced from the point with position vector $4\hat{i}-3\hat{j}-2\hat{k}$ to the point with position vector $6\hat{i}+\hat{j}-3\hat{k}$. Find the work done by the forces.

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3. Find the magnitude and direction cosines of the torque of a force represented by $3\hat{i}+4\hat{j}-5\hat{k}$ about the point with psition vector

$2\hat{i} - 3\hat{j} + 4\hat{k}$ acting through a point whose position vector is $4\hat{i} + 2\hat{j} - 3\hat{k}$.

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4. Find the torque of the resultant of the three forces represented by $-3\hat{i} + 6\hat{j} - 3\hat{k}, 4\hat{i} - 10\hat{j} + 12\hat{k}$ and $4\hat{i} + 7\hat{j}$ acting at the point with position vector $8\hat{i} - 6\hat{j} - 4\hat{k}$, about the point with position vector $18\hat{i} + 3\hat{j} - 9\hat{k}$.

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Exercise 6 2

1. If $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}, \vec{c} = 3\hat{i} + 2\hat{j} + \hat{k}$ find $\vec{a}.\left(\vec{a} \times \vec{c}\right)$.

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2. Find the volume of the parallelepiped whose coterminous edges are represented by the vectors $-6\hat{i}+14\hat{j}+10\hat{k}, 14\hat{i}-10\hat{j}-6\hat{k}$ and $2\hat{i}+4\hat{j}-2\hat{k}$.

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3. The volume of the parallelepiped whose coterminus edges are $7\hat{i}+\lambda\hat{j}-3\hat{k}, \hat{i}+2\hat{j}-\hat{k}-3\hat{i}+7\hat{j}+5\hat{k}$ is 90 cubic units. Find the value of λ.

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4. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors represented by concurrent edges of a parallelepiped of volume 4 cubic units, find the value of $\left(\vec{a}+\vec{b}\right).\left(\vec{b}\times\vec{c}\right)+\left(\vec{b}+\vec{c}\right).\left(\vec{c}\times\vec{a}\right)+\left(\vec{c}+\vec{a}\right).\left(\vec{a}\times\vec{b}\right)$.

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5. Find the altitude of a parallelepiped determined by the vectors $\vec{a} = -2\hat{i} + 5\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{c} = -3\hat{i} + \hat{j} + 4\hat{k}$ if the base is taken as the parallelogram determined by $\vec{b}$ and $\vec{c}$.

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6. Determine whether the three vectors $2\hat{i} + 3\hat{j} + \hat{k}$, $\hat{i} - 2\hat{j} + 2\hat{k}$ and $3\hat{i} + \hat{j} + 3\hat{k}$ are coplanar.

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7. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. If c_1 and 1 and $c_2 = 2$, find c_3 such that $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar.

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Exercise 6 3

1. If $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}, \vec{c} = 3\hat{i} + 2\hat{j} + \hat{k},$ find (i) $\left(\vec{a} \times \vec{b}\right) \times \vec{c}\,(ii)\,\vec{a}\left(\vec{b} \times \vec{c}\right).$

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2. $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}, \vec{b} = -\hat{i} + 2\hat{j} - 4\hat{k}, \vec{c} = -\hat{i} - 2\hat{j} + 3\hat{k}$ the find the value of $\left(\vec{a} \times \vec{b}\right).\left(\vec{a} \times \vec{c}\right).$

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3. If $\vec{a} = \hat{i} + 2\hat{i} + 3\hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k}, \vec{c} = 3\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{a} \times \left(\vec{b} \times \vec{c}\right.$

, find the values of l,m,n.

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4. If $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that $\vec{b}$ and $\vec{c}$ are non-parallel and $\vec{a} \times \left(\hat{b} \times \hat{c}\right) = \frac{1}{2}\hat{b}$ find the angle between $\widehat{a}$ and $\vec{c}$.

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Exercise 6 4

1. Find the non-parametric form of vector equation and Cartesian of the straight line passing through the point with position vector $4\hat{i} + 3\hat{j} - 7\hat{k}$ and parallel to the vector $2\hat{i} - 6\hat{j} + 7\hat{k}$.

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2. Find the parametric form of vector equation and Cartesian equation of the straight line passing through the point (-2,3,4) and parallel to the straight line $\frac{x-1}{-4} = \frac{y+3}{5} = \frac{8-z}{6}$.

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3. Find the points where the straight line passes through (6,7,4) and (8,4,9) cuts the xz and yz planes.

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4. Find the direction cosines of the straight line passing through the points (5,6,7) and (7,9,13). Also, find the parametric form of vector equation and Cartesian equation of the straight line passing through two given points.

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5. Find the acute angle between the following lines.

$\vec{r} = \left(4\hat{i} - \hat{j}\right) + t\left(\hat{i} + 2\hat{j} - 2\hat{k}\right), \hat{r} = \left(\hat{i} - 2\hat{j} + 4\hat{k}\right) + s\left(- \hat{i} - 2\hat{j} + 2\hat{k}$

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6. Find the acute angle between the following lines.

$$\frac{x+4}{3}=\frac{y-7}{4}=\frac{z+5}{5}, \vec{r}=4\hat{k}+\left(2\hat{i}+\hat{j}+\hat{k}\right)$$

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7. Find the acute angle between the following lines.

2x + 3y = - z and 6x = - y = - 4z.

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8. The vertices of ΔABC are A(7,2,1),B(6,0,3) and C(4,2,4). Find $\angle ABC$.

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9. If the straight line joining the points (2,1,4) and (a-1,4,-1) is parallel to the line joining the points (0,2,b,-1) and (5,3,-2) find the values of a and b.

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10. If the straight lines $\frac{x-5}{5m+2}=\frac{2-y}{5}=\frac{1-z}{-1}$ and $\frac{2y+1}{4m}=\frac{1-z}{-3}$ are perpendicular to each other, find the value of m.

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Exercise 6 5

1. Find the parametric form of vector equation and Cartesian equations of a straight line passing through $(5, 2, 8)$ and $isperpendic\underline{a}r \rightarrow thestraightl \in es$vecr=(hati+hatj-hatk)+s(2hati-2hatj+hatk)andvecr=(2hati-hatj-3hatk)+t(hati+2hatj+2hatk).`

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2. Show that the lines $\vec{r}=\left(6\hat{i}+\hat{j}+2\hat{k}\right)+s\left(\hat{i}+2\hat{j}-3\hat{k}\right)$ and $\overrightarrow{3\hat{i}+2\hat{j}-2\hat{k}}+t\left(2\hat{i}+4\hat{j}-$

are skew lines and hence find the shortest distance between them.

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3. If the two lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-m}{2}=z$ intersect at a point, find the value of m.

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4. Show that the lines $\frac{x-3}{3}=\frac{y-3}{-1}, z-1=0$ and $\frac{x-6}{2}=\frac{z-1}{3}, y-2=0$ intersect. Also find the point of intersection.

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5. Show that the straight lines $x+1=2y=-12z$ and $x=y+2=6z-6$ are skew and hence find the shortest distance between them.

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6. Find the parametric form of vector equation of the straight line passing through (-1,2,1) and parallel to the straight line $\overrightarrow{r} = \left(2\hat{i} + 3\hat{j} - \hat{k}\right) + t\left(\hat{i} - 2\hat{j} + \hat{k}\right)$ and hence find the shortest distance between the lines.

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7. Find the foot of the perpendicualr drawn from the point (5,4,2) to the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}$. Also, find the equation of the perpendicular.

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Exercise 6 6

1. Find a parametric form of vector equation of a plane which is at a distance of 7 units from the origin having 3,-4,5 as direction ratios of a normal to it.

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2. Find the direction cosines of the normal of the plane 12x + 3y - 4z = 65. Also. Find the non-parametric form of vector equation of a plane and the length of the perpendicular to the plane from the origin.

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3. Find the vector and Cartesian equaitions, of the plane passing through the point with position vector $2\hat{i}+6\hat{j}+3\hat{k}$ and normal to the vector $\hat{i}+3\hat{j}+5\hat{k}$.

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4. A plane passes through the point (-1,1,2) and the normal to the plane of magnitude $3\sqrt{3}$ makes equal acute angles with the coordinate axes. Find the equation of the plane.

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5. Find the intercepts cut off by the plane $\overrightarrow{r}.\left(6\hat{i}+4\hat{j}-3\hat{k}\right)=12$ on the coordinate axes.

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6. If a plane meets the coordinate axes at A,B,C such that the centriod of the triangle ABC is the point (u,v,w), find the equation of the plane.

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Exercise 6 10

1. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are parallel vectors, then $\left[\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}\right]$ is equal to

A. 2

B. -1

C. 1

D. 0

Answer: D

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2. If a vector $\overrightarrow{\alpha}$ lies in the plane of $\overrightarrow{\beta}$ and $\overrightarrow{\gamma}$, then

A. $\left[\overrightarrow{\alpha}, \overrightarrow{\beta}, \overrightarrow{\gamma}\right] = 1$

B. $\left[\overrightarrow{\alpha}, \overrightarrow{\beta}, \overrightarrow{\gamma}\right] = 1$

C. $\left[\overrightarrow{\alpha}, \overrightarrow{\beta}, \overrightarrow{\gamma}\right] = 0$

D. $\left[\overrightarrow{\alpha}, \overrightarrow{\beta}, \overrightarrow{\gamma}\right] = 2$

Answer: C

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3. If $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$ then the value of $\left[\vec{a},\vec{b},\vec{c}\right]$ is

A. $|\vec{a}||\vec{b}||\vec{c}|$

B. $\frac{1}{3}|\vec{a}||\vec{b}||\vec{c}|$

C. 1

D. -1

Answer: A

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4. If $\vec{a},\vec{b},\vec{c}$ are three unit vectors such that $\vec{a}$ is perpendicular to $\vec{b}$, and is parallel to $\vec{c}$ then $\vec{a}\times\left(\vec{b}\times\vec{c}\right)$ is equal to

A. $\vec{a}$

B. $\vec{b}$

C. $\vec{c}$

D. $\vec{0}$

Answer: B

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5. If, $\left[\vec{a}, \vec{b}, \vec{c}\right] = 1$ then the value of $\dfrac{\vec{a}.\left(\vec{b}\times\vec{c}\right)}{\left(\vec{c}\times\vec{a}\right)} + \dfrac{\vec{b}.\left(\vec{c}\times\vec{a}\right)}{\left(\vec{a}\times\vec{b}\right).\vec{c}} + \dfrac{\vec{c}.\left(\vec{a}\times\rightarrow\right)}{\left(\vec{c}\times\vec{b}\right).\vec{a}}$ is

A. 1

B. -1

C. 2

D. 3

Answer: C

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6. The volume of the parallelepiped with its edges represented by the vectors $\hat{i}+\hat{j}, \hat{i}+2\hat{j}, \hat{i}+\hat{j}+\pi\hat{k}$ is

A. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

C. π

D. $\frac{\pi}{4}$

Answer: C

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7. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $\left[\vec{a}, \vec{b}, \vec{a}\times\vec{b}\right]=\frac{\pi}{4}$, then the angle between $\vec{a}$ and $\vec{b}$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: A

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8. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + \hat{j}, \vec{c} = \hat{i}$ and $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \lambda \vec{a} = \mu \vec{b}$ is

A. 0

B. 1

C. 6

D. 3

Answer: A

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9. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, non-zero vectors such that $\left[\vec{a}, \vec{b}, \vec{c}\right[= 3$ then $\left\{\left[\vec{a} \times \vec{b} \quad \vec{b} \times \vec{c} \quad \vec{c} \times \vec{a}\right]\right\}^2$ is equal to

A. 81

B. 9

C. 27

D. 18

Answer: A

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10. If $\vec{a}, \vec{,} \vec{b},$ are three non-coplanar vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{\vec{b} + \vec{c}}{\sqrt{2}}$, then the angle between $\vec{a} \vec{b}$ is

A. $\frac{\pi}{2}$

B. $\frac{3\pi}{4}$

C. $\frac{\pi}{4}$

D. π

Answer: B

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11. If the volume of the parallelepiped with $\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}$ as coterminous edges is 8 cubic units then the volume of the parallelepiped with

$$\left(\vec{a} \times \vec{b}\right) \times \left(\vec{b} \times \vec{c}\right), \left(\vec{b} \times \vec{c}\right) \times \left(\vec{c} \times \vec{a}\right) \text{ and } \left(\vec{c} \times \vec{a}\right) \times \left(\vec{a}\right.$$

as coterminous edges is,

A. 8 cubis units

B. 512 cubic units

C. 64 cubic units

D. 24 cubic units

Answer: C

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12. Consider the vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ such that $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) = \vec{0}$. Let p_1 and p_2 be the planes determined by the paris of vectors, $\vec{a}, \vec{b}$ and $\vec{c}, \vec{d}$ respectively. Then the angle between P_1 and P_2 is

A. 0°

B. $45(\circ)$

C. 60°

D. 90°

Answer: A

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13. If $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \left(\vec{a} \times \vec{b}\right) \times \vec{c}$, where $\vec{a}, \vec{b}, \vec{c}$ are any three vectors such that $\vec{b} . \vec{c} \neq 0$ and $\vec{a} . \vec{b} \neq 0$, then $\vec{a}$ and $\vec{c}$ are

A. perpendicular

B. parallel

C. incilined at an angle $\frac{\pi}{3}$

D. inclined at an angle $\frac{\pi}{6}$

Answer: B

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14. If $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 5\hat{j}, \vec{c} = 3\hat{i} + 5\hat{j} - \hat{k},$ then a vector perpendicualr to $\vec{a}$ and lines in the containing $\vec{b}$ and $\vec{c}$ is \

A. $-17\hat{i} + 21\hat{j} - 97\hat{k}$

B. $17\hat{i} + 21\hat{j} - 123\hat{k}$

C. $-17\hat{i} - 21\hat{j} + 97\hat{k}$

D. $-17\hat{i}-21\hat{j}-97\hat{k}$

Answer: D

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15. The angle between the lines $\frac{x-2}{3}=\frac{y+1}{-2}, z=2$ and $\frac{x-1}{1}=\frac{2y=3}{3}=\frac{z+5}{2}$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: D

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16. If the line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lies in the plane x + 3y - $\alpha x+\beta=0$, then (α, β) is

A. (-5,5)

B. (-6,7)

C. (5,-5)

D. (6,-7)

Answer: B

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17. The angle between the line $\vec{r}=\left(\hat{i}+2\hat{j}-3\hat{k}\right)+t\left(2\hat{i}+\hat{j}-2\hat{k}\right)$ and the plane $\vec{r}\,.\left(\hat{i}+\hat{j}\right)+4=0$

A. 0°

B. 30°

C. 45°

D. 90°

Answer: C

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18. The coordinates of the point where the line $\vec{r} = \left(6\hat{i} - \hat{j} - 3\hat{k}\right) + t\left(-\hat{i} + 4\hat{k}\right)$ meets the plane $\vec{r}.\left(\hat{i} + \hat{j} - \hat{k}\right) = 3$ are

A. (2,1,0)

B. (7,-1,-7)

C. (1,2-6)

D. (5,-1,1)

Answer: D

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19. Distance from the origin to the plane 3x - 6y + 2z + 7 = 0 is

A. 0

B. 1

C. 2

D. 3

Answer: B

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20. The distance between the planes x + 2y + 3z + 7 = 0 and 2x + 4y + 6z +

7 0 is

A. $\frac{\sqrt{7}}{2\sqrt{2}}$

B. $\frac{7}{2}$

C. $\frac{\sqrt{7}}{2}$

D. $\frac{7}{2\sqrt{2}}$

Answer: A

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21. If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$, then

A. $c = \pm 3$

B. $c = \pm \sqrt{3}$

C. $c > 0$

D. $0 < \quad < 1$

Answer: B

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22. The vector equation $\vec{r} = \left(\hat{i} - 2\hat{j} - \hat{k}\right) + t\left(6\hat{i} - \hat{k}\right)$ represents a straight line passing through the points

A. (0,6,-1) and (1,-2,-1)

B. (0,6,-1) and (-1,-1,-2)

C. (1,-2,-1) and (1,4,-2)

D. (1,-2,1) and (0,-6,1)

Answer: C

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23. If the distance of the point (1,1,1) from the origin is half of its distance from the plane x + y + z + k = 0, then the values of k are

A. ± 3

B. ± 6

C. $-3, 9$

D. $3, \ -9$

Answer: D

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24. If the planes $\vec{r}\left(2\hat{i}-\lambda\hat{j}+\hat{k}\right)=3$ and $\vec{r}.\left(4\hat{i}+\hat{j}-\mu\hat{k}\right)=5$ are parallel then the value of λ and μ are

A. $\frac{1}{2},\ -2$

B. $-\frac{1}{2}, 2$

C. $-\frac{1}{2},\ -2$

D. $\frac{1}{2}, 2$

Answer: C

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25. If the length of the perpendicular from the origin to the plane $2x+3y+\lambda z=1, \lambda>0$, is $\frac{1}{5}$, then the value of λ is

A. $2\sqrt{3}$

B. $3\sqrt{2}$

C. 0

D. 1

Answer: A

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