MATHS

NCERT - FULL MARKS MATHS(TAMIL)

COMPLEX NUMBERS

Example

1. Simplify the following

i^7

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2. Simplify the following

i^{1729}

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3. Simplify the following

$\sum_{n=1}^{102} i^n$

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4. Simplify the following

$Ii^2 i^3 \ldots . i^{40}$

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5. Write $\dfrac{3+4i}{5-12i}$ in the x + iy form, hence find its real and imaginary parts.

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6. Simplify $\left(\dfrac{1+i}{1-i}\right)^3-\left(\dfrac{1-i}{1+i}\right)^3$ into rectangular form

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7. If $\dfrac{z+3}{z-5i}=\dfrac{1+4i}{2}$, find the complex number z in the rectangular form

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8. If $z_1 = 3 - 2i$ and $z_2 = 6 + 4i$, find $\frac{z_1}{z_2}$ in the rectangular form

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9. Find z^{-1}, if $z = (2 + 3i)(1 - i)$

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10. Show that $\left(2 + I\sqrt{3}\right)^{10} + \left(2 - I\sqrt{3}\right)^{10}$ is real

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11. $\left(\frac{19 + 9i}{5 - 3i}\right)^{15} - \left(\frac{8 + i}{1 + 2i}\right)^{15}$ is purely imaginary.

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12. If $z_1 = 3 + 4i,\ z_2 = 5 - 12i,$ and $z_3 = 6 + 8i,$ find $|z_1|, |z_2|, |z_3|, |z_1 + z_2|, |z_2 - z_3|,$ and $|z_1 + z_3|.$

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13. Find the following

$$\left|\frac{2+i}{-1+2i}\right|$$

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14. Find the following

$$\left|\overline{(1+i)}(2+3i)(4i-3)\right|$$

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15. Find the following

$$\left|\frac{i(2+i)^3}{(1+i)^2}\right|$$

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16. Which one of the points i, – +2 i , and 3 is farthest from the origin?

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17. If $z_1, z_2,$ and z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = z_1 + z_2 + z_3 \mid \; = 1$. Find the value of

$$\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|$$

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18. If $|z| = 2$ show that $3 \leq |z+3+4i| \leq 7$

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19. Show that the points $1, \frac{-1}{2} + i\frac{\sqrt{3}}{2}$, and $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

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20. Let $z_1, z_2,$ and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1 + z_2 + z_3 \neq 0$

Prove that $\left|\frac{z_1z_2 + z_2z_3 + z_3z_1}{z_1 + z_2 + z_3}\right| = r$

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21. Show that the equation $z^2 = \bar{z}$has four solutions.

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22. Find the square root of $6 - 8i$

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23. Given the complex number $z = 3 + 2i$ represent the complex numbers $z, iz,$ and and $z + iz$ in one Argand diagram. Show that these complex numbers form the vertices of an isosceles right triangle.

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24. Show that $|3z - 5 + i| = 4$ represents a circle, and, find its centre and radius.

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25. Show that $|z + 2 - i| < 2$ represents interior points of a circle. Find its centre and radius.

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26. Obtain the Cartesian form of the locus of z in each of the following cases.

$|z| = |z - i|$

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27. Obtain the Cartesian form of the locus of z in each of the following cases.

$|2z - 3 - i| = 3$

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28. Find the modulus and principal argument of the following complex numbers.

$\sqrt{3} + i$

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29. Find the modulus and principal argument of the following complex numbers.

$-\sqrt{3}+i$

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30. Find the modulus and principal argument of the following complex numbers.

$-\sqrt{3}-i$

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31. Find the modulus and principal argument of the following complex numbers.

$\sqrt{3}-i$

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32. Represent the complex number

$-1-i$

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33. Find the principal argument Argz, when $z=\frac{-2}{1+i\sqrt{3}}$

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34. Find the product $\frac{3}{2}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right).6\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$ in rectangular form.

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35. Find the quotient $\dfrac{2\left(\cos\frac{9\pi}{4}+i\sin\frac{9\pi}{4}\right)}{4\left(\cos\left(\frac{-3\pi}{2}\right)+i\sin\left(\frac{-3\pi}{2}\right)\right)}$ in rectangular form.

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36. If z=x +iy and arg $\left(\dfrac{z-1}{z+1}\right)=\dfrac{\pi}{2}$, show that $x^2+y^2=1$.

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37. If $z=(\cos\theta+I\sin\theta)$, show that $z^n+\dfrac{1}{z^n}=2\cos n\theta$ and $z^n-\dfrac{1}{z^n}=2I\sin n\theta$

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38. Simplify $\left(\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right)^{18}$

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39. $\left(\frac{1+\cos 2\theta + i\sin 2\theta}{1+\cos 2\theta - i\sin 2\theta}\right)^{30}$

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40. Simplify

$(1+i)^{18}$

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41. Simplify

$$\left(-\sqrt{3}+3i\right)^{31}$$

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42. Find the cube roots of unity.

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43. Find the fourth roots of unity

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44. Solve the equation $z^3 + 8i = 0$, where $z \in C$.

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45. Find all cube roots of $\sqrt{3}+i$

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46. Suppose $z_1, z_2,$ and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z|=2$. If $z_{1\square}=1+i\sqrt{3}$, then find z_2 and z_3.

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Exercise 2 1

1. Simplify the following:

$i^{1947} + i^{1950}$

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2. Simplify the following:

$i^{1948} - i^{1869}$

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3. Simplify the following:

$ii^2i^3.....i^{2000}$

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4. Simplify the following:

$$\sum_{n=1}^{10} i^{n+50}$$

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Exercise 2 2

1. Evaluate the following if $z = -2i$ and $w = -1 + 3i$

$z + w$

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2. Evaluate the following if $z = -2i$ and $w = -1 + 3i$

$z - iw$

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3. Evaluate the following if $z = -2i$ and $w = -1 + 3i$

$2z + 3w$

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4. Evaluate the following if $z = -2i$ and $w = -1 + 3i$

zw

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5. Evaluate the following if $z = -2i$ and $w = -1 + 3i$

$z^2 + 2zw + w^2$

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6. Evaluate the following if $z = -2i$ and $w = -1 + 3i$

$(z + w)^2$

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7. Find the values of the real numbers x and y, if the complex numbers

$(3 - i)x - (2 - i)y + 2i + 5$ and $2x + (-1) + 2i)y + 3 + 2i$

are equal.

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Exercise 2 4

1. Write the following in the rectangular form:

$$\overline{(5+9i)+(2-4i)}$$

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2. Write the following in the rectangular form:

$$\frac{10-5i}{6+2i}$$

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3. Write the following in the rectangular form:

$$\overline{3i}+\frac{1}{2-i}$$

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4. If z= x+ iy, find the following in rectangular form.

$$\text{Re}\left(\frac{1}{z}\right)$$

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5. If z= x+ iy, find the following in rectangular form.

`"Re "(i barz)

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6. If z= x+ iy, find the following in rectangular form.

Im $(3z + 4\bar{z} + \; - 4i)$

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7. If $z_1 = 2 - I$ and $z_2 = -4 + 3i$, find the inverse of $z_1 z_2$ and $\frac{z_1}{z_2}$

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8. The complex numbers u,v, and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$. If $v = 3 - 4i$ and $w = 4 + 3i$, find u in rectangular form.

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9. Find the least value of the positive integer n for which $(\sqrt{3} + i)^n$

real

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10. Find the least value of the positive integer n for which

$$\left(\sqrt{3}+i\right)^{n}$$

purely imaginary

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Exercise 2 5

1. Find the modulus of the following complex numbers

$$\frac{2i}{3+4i}$$

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2. Find the modulus of the following complex numbers

$$\frac{2-i}{1+i}+\frac{1-2i}{1-i}$$

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3. Find the modulus of the following complex numbers

$$(1-i)^{10}$$

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4. Find the modulus of the following complex numbers

$$2i(3-4i)(4-3i)$$

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5. For any two complex numbers z_1 and z_2, such that $|z_1| = |z_2| = 1$ and $z_1 z_2 \neq = -1$, then show that $\dfrac{z_1 + z_2}{1 + z_1 z_2}$ is a real number.

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6. If $|z| = 3, \quad \text{show that} \quad 7 \leq |z + 6 - 8i| \leq 13$

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7. If $z_1, z_2,$ and z_3 are three complex numbers such that $|z_1| = 1, |z_2| = 2, |z_3| = 3$ and $|z_1 + z_2 + z_3| = 1,$ show that $|9z_1 z_2 + 4z_1 z_3 + z_2 z_3| = 6$

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8. Show that the equation $z^3 = 2\bar{z} = 0$ has five solutions.

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Exercise 2 6

1. Obtain the Cartesian form of the locus of z= x+iy in each of the following cases:

$[\text{Re} \quad (iz)]^2 = 3$

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2. Obtain the Cartesian form of the locus of z= x+iy in each of the following cases:

Im [(1-z)z+1]=0`

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3. Obtain the Cartesian form of the locus of z= x+iy in each of the following cases:

$|z+i| = |z-1|$

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4. Obtain the Cartesian form of the locus of z= x+iy in each of the following cases:

$\bar{z} = z^{-1}$

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5. Show that the following equations represent a circle, and, find its centre and radius.

$|z-2-i|=3$

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6. Show that the following equations represent a circle, and, find its centre and radius.

$|2z+2-4i|=2$

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7. Show that the following equations represent a circle, and, find its centre and radius.

$|3z-6+12i|=8$

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8. Obtain the Cartesian equation for the locus of $z = x + iy$ in each of the following cases:

$|z - 4| = 16$

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9. Obtain the Cartesian equation for the locus of $z = x + iy$ in each of the following cases:

$|z - 4|^2 - |z - 1|^2 = 16$

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Exercise 2 7

1. Write in polar form of the following complex numbers

$2 + i2\sqrt{3}$

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2. Write in polar form of the following complex numbers

$3 - I\sqrt{3}$

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3. Write in polar form of the following complex numbers

$-2 - i2$

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4. Write in polar form of the following complex numbers

$$\frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$$

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5. Find the rectangular form of the complex numbers

$$\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$$

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6. Find the rectangular form of the complex numbers

$$\frac{\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}}{2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)}$$

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Exercise 2 8

1. Find the value of $\left(\frac{1+\sin\frac{\pi}{10}+i\cos\frac{\pi}{10}}{1+\sin\frac{\pi}{10}-i\cos\frac{\pi}{10}}\right)^{10}$

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2. Solve the equation $z^3+27=0$

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3. Find the value of $\sum_{k=1}^{8}\left(\cos\frac{2k\pi}{9}+i\sin\frac{2k\pi}{9}\right)$

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4. If $z = 2 - 2i$, find the rotation of z by θ radians in the counter clockwise direction about the origin when

$$\theta = \frac{\pi}{3}$$

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5. If $z = 2 - 2i$, find the rotation of z by θ radians in the counter clockwise direction about the origin when

$$\theta = \frac{2\pi}{3}$$

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6. If $z = 2 - 2i$, find the rotation of z by θ radians in the counter clockwise direction about the origin when

$$\theta = \frac{3\pi}{2}$$

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Exercise 2 9

1. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is

A. 0

B. 1

C. -1

D. i

Answer: A

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2. The value of $\sum_{i=1}^{13}\left(i^{n}+i^{n-1}\right)$ is

A. 1+i

B. i

C. 1

D. 0

Answer: A

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3. The area of the triangle formed by the complex numbers $z, iz, \text{ and } z+iz$ in the Argand's diagram is

A. $\frac{1}{2}|z|^{2}$

B. $|z|^2$

C. $\frac{3}{2}|z|^2$

D. $2|z|^2$

Answer: A

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4. The conjugate of a complex number is $\frac{1}{i-2}$. Then, the complex number is

A. $\frac{1}{i+2}$

B. $\frac{-1}{i+2}$

C. $\frac{-1}{i-2}$

D. $\frac{1}{i-2}$

Answer: B

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5. If $z=\frac{\left(\sqrt{3}+i\right)^{3}(3i+4)^{2}}{(8+6i)^{2}}$, then |z| is equal to

A. 0

B. 1

C. 2

D. 3

Answer: C

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6. If z is a non zero complex number, such that $2iz^2 = \bar{z}$ then

|Z| is

A. $\frac{1}{2}$

B. 1

C. 2

D. 3

Answer: A

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7. If $|z - 2 + i| \leq 2$, then the greatest value of $|z|$ is

A. $\sqrt{3} - 2$

B. $\sqrt{3} + 2$

C. $\sqrt{5}-2$

D. $\sqrt{5}+2$

Answer: D

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8. If $\left|z-\frac{3}{z}\right|=2$, then the least value of $|z|$ is

A. 1

B. 2

C. 3

D. 5

Answer: A

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9. If $|z|=1$, then the value of $\frac{1+z}{1+\bar{z}}$ is

A. z

B. $\bar{z}$

C. $\frac{1}{z}$

D. 1

Answer: A

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10. The solution of the equation $|z|-z=1+2i$ is

A. $\frac{3}{2}-2i$

B. $-\frac{3}{2}+2i$

C. $2-\frac{3}{2}i$

D. $2+\frac{3}{2}i$

Answer: A

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11. If $|z_1|=1, |z_2|=2, |z_3|=3$ and $|9z_1z_2+z_2z_3|=12,$ then the value of $|z_1+z_2+z_3|$ is

A. 1

B. 2

C. 3

D. 4

Answer: B

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12. If z is a complex number such that $z \in C / R$ and $z + \frac{1}{z} \in R,$ then |z| is

A. 0

B. 1

C. 2

D. 3

Answer: B

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13. z_1, z_3 and z_3 are complex numbers such that $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$ then $z_1^2 + z_2^2 + z_3^2$ is

A. 3

B. 2

C. 1

D. 0

Answer: D

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14. If $\dfrac{z-1}{z+1}$ is purely imaginary, then |z| is

A. $\dfrac{1}{2}$

B. 1

C. 2

D. 3

Answer: B

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15. If z= x + iy is a complex number such that $|z+2| = |z-2|$, then the locus of z is

A. real axis

B. imaginary axis

C. ellipse

D. circle

Answer: B

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16. The principal argument of $\frac{3}{-1+i}$ is

A. $\frac{-5\pi}{6}$

B. $\frac{-2\pi}{3}$

C. $\frac{-3\pi}{4}$

D. $\frac{-\pi}{2}$

Answer: C

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17. The principal argument of $(\sin 40^\circ + I\cos 40^\circ)$ is

A. -110°

B. -70°

C. 70°

D. 110°

Answer: A

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18. If (1+i) (1+2i) (1+3i)....(1+ni) = x +iy, then $2 \cdot 5 \cdot 10 \ldots. (1+n^2)$ is

A. 1

B. i

C. $x^2 + y^2$

D. $1 + n^2$

Answer: C

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19. If $\omega \neq 1$ is a cubic root of unity and $(1+\omega)^7 = A + B\omega$, then (A,B) equals

A. $(1, 0)$

B. $(-1, 1)$

C. $(0, 1)$

D. $(1, 1)$

Answer: D

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20. The principal argument of the complex number $\frac{\left(1+i\sqrt{3}\right)^2}{4i\left(1-i\sqrt{3}\right)}$ is

A. $\frac{2\pi}{3}$

B. $\frac{\pi}{6}$

C. $\frac{5\pi}{6}$0

D. $\frac{\pi}{2}$

Answer: D

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21. If α and β are the roots of $x^2 + x + 1 = 0$, then $\alpha^{2020} + \beta^{2020}$ is

A. -2

B. -1

C. 1

D. 2

Answer: B

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22. The product of all four values of $\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is

A. -2

B. -1

C. 1

D. 2

Answer: C

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23. If $\omega \neq 1$ is a cubic root of unity and

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k,$$ then k is equal to

A. 1

B. -1

C. $\sqrt{3}i$

D. $-\sqrt{3}i$

Answer: D

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24. The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

A. cis $\frac{2\pi}{3}$

B. cis $\frac{4\pi}{3}$

C. $-$ cis $\frac{2\pi}{3}$

D. $-$ cis $\frac{4\pi}{3}$

Answer: A

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25. If $\omega = cis\frac{2\pi}{3}$, then the number of distinct roots of

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$$

A. 1

B. 2

C. 3

D. 4

Answer: A

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