MATHS

NCERT - FULL MARKS MATHS(TAMIL)

DIFFERENTIALS AND PARTIAL DERIVATIVES

Example

1. Find the linear approximation for $f(x)=\sqrt{1+x}, x \geq -1$, at $x_0=3$. Use the linear approximation to estimate f (3.2).

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2. Let us assume that the the the shape of a soap bubble is a sphere . Use linear approximation to approximate the increase in the surface area of a soap bubble as its radius increases from 5 cm to 5.2 cm also calculate the percentage error.

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3. A right circular cylinder has radius r = 10 cm and height h = 20 cm suppose that the radius of the cylinder is increased from 10 cm to 10.1 cm and the height does not change. Estimate the change in the volume of the cylinder . Also calculate the relative error and percentage error .

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4. Let f , g : $(a, b) \rightarrow R$ be differentiable functions. Show that d(fg) = fdg + gdf.

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5. Let $g(x) = x^2 + \sin x$. Calculate the differential dg.

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6. If the radius of a sphere , with radius 10 cm , has to decrease by 0.1 cm approximately how much will its volume decrease ?

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7. Let $F(x, y) = \frac{2x - 3y + 4}{x^2 + y^2 + 4}$ for all $(x, y) \in R^2$, Show that f is continuous on R^2.

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8. Consider $f(x, y) = \frac{xy}{x^2 + y^2}$ if $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$. Show that f is not continuous at (0,0) and continuous at all other points of R^2.

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9. Let $g(x, y) = \dfrac{x^2 y}{x^4 + y^2}$ for (x,y) $\neq$ (0,0) and f(0,0)=0.

(i) Show that $\lim_{(x,y) \to (0,0)} g(x, y) = 0$ along every line $y = mx, m \in R$.

(ii) Show that $\lim_{(x,y) \to (0,0)} g(x, y) = \dfrac{k}{1+k^2}$, along every parabola $y = kx^2, k \in R\{0\}$.

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10. Let f(x,y) = 0 if $xy \neq 0$ and $f(x, y)$ = 1 if xy = 0 .

(i) Calculate : $\dfrac{\partial f}{\partial x}(0, 0), \dfrac{\partial f}{\partial y}(0, 0)$

(ii) Show that f is not continuous at (0,0)

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(ii) Show that f is not continuous at (0,0)

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12. Let F (x,y) = $x^3 y + y^2 x + 7$ for all $(x, y) \in R^2$. Calculate $\frac{\partial F}{\partial x}(-1, 3)$ and $\frac{\partial F}{\partial y}(-2, 1)$

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13. Let $w(x, y) = xy + \frac{e^y}{y^2 + 1}$ for all (x,y) $\in \mathbb{R}^2$. Calculate $\frac{\partial^2 w}{\partial y \partial x}$ and $\frac{\partial^2 w}{\partial x \partial y}$

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14. Let $w(x, y) = xy + \frac{e^y}{y^2 + 1}$ for all (x,y) $\in \mathbb{R}^2$. Calculate $\frac{\partial^2 w}{\partial y \partial x}$ and $\frac{\partial^2 w}{\partial x \partial y}$

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15. Let $F(x, y) = \frac{2x - 3y + 4}{x^2 + y^2 + 4}$ for all $(x, y) \in R^2$, Show that f is continuous on R^2.

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16. If $W = x^2 + y^2 + z^2, x, y, z \in R$ find the differential dW.

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17. Let W(x,y,z) $= x^2 - xy + 3 \sin z, x, y, z \in R$, Find the linear approximation at (2,-1,0).

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18. Verify the above theorem for $F(x, y) = x^2 - 2y^2 + 2xy$ and $x(t) = \cos t, y(t) = \sin t, t \in [0, 2\pi]$

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19. Let g $(x, y) = x^2 - yx + \sin(x + y), x(t) = e^{3t}, y(t) = t^2, t \in R.$

Find $\frac{dg}{dt}$.

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20. Let $g(x, y) = 2y + x^2, x = 2r - s, y = r^2 + 2s, r, s \in \mathbb{R}.$

Find $\frac{\partial g}{\partial r}, \frac{\partial g}{\partial s}$

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21. Show that $F(x, y) = \frac{x^2 + 5xy - 10y^2}{3x + 7y}$ is a homogeneous function of degree 1.

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22. If $u = \sin^{-1}\left(\frac{x + y}{\sqrt{x} + \sqrt{y}}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u.$

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Exercise 8 1

1. Let $f(x) = \sqrt[3]{x}$. Find the linear approximation at x = 27. Use the linear approximation to approximate $\sqrt[3]{27.2}$

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2. Using the approximation to find approximate value of

$(123)^{\frac{2}{3}}$

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3. Use the linear approximation to find approximate values of

$4\sqrt{15}$

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4. Use the linear approximation to find approximate values of

$3\sqrt{26}$

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5. Find a linear approximation for the following functions at the indicated points.

$f(x) = x^3 - 5x + 12, x_0 = 2$

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6. Find a linear approximation for the functions at the indicated point

$g(x) = \sqrt{x^2 + 9}, x_9 = -4$

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7. Find a linear approximation for the following function at the indicated points.

$h(x) = \frac{x}{x+1}, x_0 = 1$

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8. The radius of a circular plate is measured as 12.65 cm instead of the actual length 12.5 cm. Find the following is calculating the area of the circular plate:

(i) Absolute error

(ii) Relative error

(iii) Percentage error

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11. A sphere is made of ice having radius 10 cm. Its radius decreases from 10 cm to 9.8. cm. Find approximations for the following: change in the volume

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12. A sphere is made of ice having radius 10 cm. Its radius decreases from 10 cm to 9.8. cm. Find approximations for the following:

change in the surface area

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13. The time T , taken for a complete oscillation of a single pendulum with length l , is given by the equation $T = 2\pi\sqrt{\frac{l}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of l.

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Exercise 8 2

1. Find differential dy for each of the following functions :

$$y = \frac{(1 - 2x)^3}{3 - 4x}$$

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2. Find differential dy for each of the following functions :

$$y = (3 + \sin(2x))^{2/3}$$

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3. Find differential dy for each of the following functions :

$$y = e^{x^2 - 5x + 7} \cos(x^2 - 1)$$

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4. Find df for $f(x) = x^2 + 3x$ and evaluate it for

x = 2 and dx = 0.1

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5. Find df for $f(x) = x^2 + 3x$ and evaluate it for

x = 3 and dx = 0.02

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6. Find Δf and df for the function f for the indicated values of x, Δx and compare

$f(x) = x^3 - 2x^2, x = 2, \Delta x = dx = 0.5$

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7. Find Δf and df for the function f for the indicated values of x, Δx and compare

$f(x) = x^2 + 2x + 3, x = -0.5, \Delta x = dx = 0.1$

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8. Assuming $\log_{10} = 0.4343$, find an approximate value of $\log_{10} 1003$.

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9. The trunk of a tree has diameter 30 cm. During the following year, the circumference grew 6 cm.

Approximately, how much did the tree's diameter grow?

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10. The trunk of a tree has diameter 30 cm. During the following year, the circumference grew 6 cm.

What is the percentage increase in area of the tree's cross-section?

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11. An egg of a particular bird is very nearly spherical. If the radius to the inside of the shell is 5 mm and radiusto the outside ofthe shell is 5.3 mm, find the volume ofthe shell approximately

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12. Assume that the cross section of the artery of human is circular. A drug is given to a patient to dilate his arteries. If the radius of an artery is increased from 2 mm to 2.1 mm, how much is cross-sectional area increased approximately?

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13. In a newly developed city, it is estimated that the voting population (in thousands) will increase according to $V(t) = 30 + 12t^2 - t^3, 0 \leq t \leq 8$ where t is the time in years. Find the approximate change in voters for the time change from 4 to $44\frac{1}{6}$ year.

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14. The relation between the number of words y a person learns in x hours is given by $y = 52\sqrt{x}, 0 \leq x \leq 9$. What is the approximate number of words learned when x changes from 1 to 1.1 hour ?

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15. The relation between the number of words y a person learns in x hours is given by $y = 52\sqrt{x}, 0 \le x \le 9$. What is the approximate number of words learned when x changes from

4 to 4.1 hour ?

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16. A circular plate expands uniformly under the influence of heat. If it's radius increases from 10.5 cm to 10.75 cm, then find an approximate change in the area and the approximate percentage change in the area.

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17. A coat of paint of thickness 0.2 cm is applied to the faces of a cube whose edge is 10 cm. Use the differentials to find approximately how many cubic centimeters of paint is used to paint this cube. Also calculate the exact amount of paint used to paint this cube.

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Exercise 8 3

1. Evaluate $\lim_{(x,y)\to(1,2)} g(x, y)$, if the limit exists, where $g(x, y) = \frac{3x^2 - xy}{x^2 + y^2 + 3}$.

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2. Evaluate $\lim_{(x,y)\to(0,0)} \cos\left(\frac{x^3 + y^2}{x + y + 2}\right)$. If the limit exists.

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3. Evaluate $\lim_{(x,y)\to(0,0)} \cos\left(\frac{e^x \sin y}{y}\right)$, if the limit exists.

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Exercise 8 4

1. Find the partial derivatives of the following functions at the indicated points.

$f(x, y) = 3x^2 - 2xy + y^2 + 5x + 2, (2, -5)$

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2. Find the partial derivatives of the following functions at the indicated points.

$g(x, y) = 3x^2 + y^2 + 5x + 2, (1, -2)$

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3. Find the partial derivatives of the following functions at the indicated points.

$h(x, y) = x \sin(xy) + z^2 x, \left(2, \frac{\pi}{4}, 1\right)$

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4. Find the partial derivatives of the following functions at the indicated points.

$G(x, y) = e^{x + 3y} \log\left(x^2 + y^2\right), (-1, 1)$

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5. If $U(xy, z) = \frac{x^2 + y^2}{xy} + 3z^2 y$, find $\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}$ and $\frac{\partial U}{\partial z}$.

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6. If $U(x, y, z) = \log(x^3 + y^3 + z^3)$, find $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z}$.

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7. For each of the following functions find the g_{xy}, g_{xx}, g_{yy} and g_{yx}.

$g(x, y) = xe^y + 3x^2y$

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8. For each of the following functions find the g_{xy}, g_{xx}, g_{yy} and g_{yx}.

$g(x, y) = \log(5x + 3y)$

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9. For each of the following functions find the g_{xy}, g_{xx}, g_{yy} and g_{yx}.

$g(x, y) = x^2 + 3xy - 7y + \cos(5x)$

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10. A firm produces two types of calculators each week, x number of type A and y number of type B . The weekly revenue and cost functions (in rupees) are $R(x, y) = 80x + 90y + 0.04xy - 0.05x^2 - 0.05y^2$ and $C(x, y) = 8x + 6y + 2000$ respectively.

Find the profit function P (x, y)

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11. A firm produces two types of calculators each week, x number of type A and y number of type B . The weekly revenue and cost functions (in rupees) are $R(x, y) = 80x + 90y + 0.04xy - 0.05x^2 - 0.05y^2$ and $C(x, y) = 8x + 6y + 2000$ respectively.

Find $\frac{\partial P}{\partial x}(1200, 1800)$ and $\frac{\partial p}{\partial y}(1200, 1800)$ and interpret these results.

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Exercise 8 5

1. If $w(x, y) = x^3 - 3xy + 2y^2, x, y \in \mathbb{R}$, find the linear approximation for w at (1, -1).

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2. Let $z(x, y) = x^2y + 3xy^4, x, y \in \mathbb{R}$. Find the linear approximation for z at (2, -1).

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3. If $v(x, y) = x^2 - xy + \frac{1}{4}y^2 + 7, x, y \in R$, find the differential dv.

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4. If $W(x, y, z) = x^2 - xy + 3\sin z, x, y, z \in \mathbb{R}$. Find the linear approximation at (2, -1, 0).

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5. Let $V(x, y, z) = xy + yz + zx, x, y, z \in \mathbb{R}$. Find the differential dV.

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Exercise 8 6

1. If $u(x, y) = x^2y + 3xy^4$, $x = e^t$ and $y = \sin t$, find $\frac{du}{dt}$ and evaluate it at t = 0.

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2. If $w(x, y, z) = x^2 + y^2 + z^2$, $x = e^t$, $y = e^t \sin t$ and $z = e^t \cos t$, find $\frac{dw}{dt}$.

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3. Let $U(x, y, z) = xyz$, $x = e^{-t}$, $y = e^{-t} \cos t$, $z = \sin t$, $t \in \mathbb{R}$. Find $\frac{dU}{dt}$.

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4. If $w(x, y) = 6x^3 - 3xy + 2y^2, x = e^x, y = \cos s, s \in \mathbb{R},$ find $\frac{dw}{ds}$, and evaluate at s = 0,

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5. If $z(x, y) = x \tan^{-1}(xy), x = t^2, y = se^t, s, t \in \mathbb{R}$. Find $\frac{\partial z}{\partial ts}$ and $\frac{\partial z}{\partial t}$ at s = t = 1.

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6. Let $z(x, y) = x^3 - 3x^2y^3$, where $x = se^t, y = se^{-t}, s, t \in \mathbb{R}$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

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7. $W(x, y, z) = xy + yz + zx$, $x = u - v$, $y = uv$, $u, v \in \mathbb{R}$. Find $\frac{\partial W}{\partial u}$, $\frac{\partial W}{\partial v}$, and evaluate them at $\left(\frac{1}{2}, 1\right)$.

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Exercise 8 7

1. In each of the following cases, determine whether the following function is homogeneous or not. If it is so, find the degree.

$f(x, y) = x^2y + 6x^3 + 7$

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2. In each of the following cases, determine whether the following function is homogeneous or not. If it is so, find the degree.

$$h(x, y) = \frac{6x^2y^3 - \pi y^5 + 9x^4y}{2020x^2 + 2019y^2}$$

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3. In each of the following cases, determine whether the following function is homogeneous or not. If it is so, find the degree.

$$g(x, y, z) = \frac{\sqrt{3x^2 + 5y^2 + z^2}}{4x + 7y}$$

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4. In each of the following cases, determine whether the following function is homogeneous or not. If it is so, find the degree.

$$U = (x, y, z) = xy + \sin\left(\frac{y^2 - 2z^2}{xy}\right)$$

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5. If $w(x, y, z) = \log\left(\frac{5x^3y^4 + 7y^2xz^4 - 75y^3z^4}{x^2 + y^2}\right)$, find $x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z}$.

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Exercise 8 8

1. A circular template has a radius of 10 cm. The measurement of radius has an approximate error of 0.02 cm. Then the percentage error in calculating area of this template is

A. $0.2\,\%$

B. $0.4\,\%$

C. $0.04\,\%$

D. $0.08\,\%$

Answer: B

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2. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?

A. $\frac{1}{31}$

B. $\frac{1}{5}$

C. 5

D. 31

Answer: B

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3. If $u(x, y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to

A. $e^{x^2+y^2}$

B. 2x u

C. x^2u

D. y^2u

Answer: B

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4. If $v(x, y) = \log(e^x + e^y)$, then $\frac{\partial v}{dex} + \frac{\partial v}{\partial y}$ is equal to

A. $e^x + e^y$

B. $\frac{1}{e^x + e^y}$

C. 2

D. 1

Answer: D

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5. If $w(x, y) = x^y, x > 0$, then $\frac{\partial w}{\partial x}$ is equal to

A. $x^y \log x$

B. y log x

C. yx^{y-1}

D. x log y

Answer: C

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6. If $f(x, y) = e^{xy}$, then $\frac{\partial^2 f}{\partial x\, \partial y}$ is equal to

A. xye^{xy}

B. $(1 + xy)e^{xy}$

C. $(1 + y)e^{xy}$

D. $(1 + x)e^{xy}$

Answer: B

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7. If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is

A. 0.4 cu.cm

B. 0.45 cu.cm

C. 2 cu.cm

D. 4.8 cu.cm

Answer: D

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8. The change in the surface area $S = 6x^2$ of a cube when the edge length varies from x_0 to $x_0 + dx$ is

A. $12x_0 + dx$

B. $12x_0 dx$

C. $6x_0 dx$

D. $6x_0 + dx$

Answer: B

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9. The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is

A. $0.3xdxm^3$

B. $0.03xm^3$

C. $0.03x^2m^3$

D. $0.03x^3m^3$

Answer: C

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10. If $g(x, y) = 3x^2 - 5y + 2y^2$, $x(t) = e^t$ and $y(t) = \cos t$, then $\frac{dg}{dt}$ is equal to

A. $6e^{2t} + 5\sin t - 4\cos t \sin t$

B. $6e^{2t} - 5\sin t + 4\cos t \sin t$

C. $3e^{2t} + 5\sin t + 4\cos t \sin t$

D. $3e^{2t} - 5\sin t + 4\cos t \sin t$

Answer: A

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11. If $f(x) = \frac{x}{x+1}$, then its differential is given by

A. $\frac{-1}{(x+1)^2}dx$

B. $\frac{1}{(x+1)^2}dx$

C. $\frac{1}{x+1}dx$

D. $\frac{-1}{x+1}dx$

Answer: B

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12. If $u(x, y) = x^2 + 3xy + y - 2019$, then $\left.\frac{\partial u}{\partial x}\right|_{(4, 5)}$ is equal to

A. -4

B. -3

C. -7

D. 13

Answer: C

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13. Linear approximation for g(x) = cos x at $x = \frac{\pi}{2}$ is

A. $x + \frac{\pi}{2}$

B. $-x + \frac{\pi}{2}$

C. $x - \frac{\pi}{2}$

D. $-x - \frac{\pi}{2}$

Answer: B

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14. If $w(x, y, z) = x^2(y - z) + y^2(z - x) + z^2(x - y)$, then $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}$ is

A. $xy + yz + zx$

B. $x(y + z)$

C. $y(z + x)$

D. 0

Answer: D

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15. If $f(x, y, z) = xy + yz + zx$, then $f(x) - f_z$ is equal to

A. z - x

B. y - z

C. x - z

D. y - x

Answer: A

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