

# MATHS

# NCERT - FULL MARKS MATHS(TAMIL)

# DIFFERENTIALS AND PARTIAL DERIVATIVES

### Example

1. Find the linear approximation for  $f(x)=\sqrt{1+x}, x\geq -1$ , at

 $x_0 = 3$ . Use the linear approximation to estimate f (3.2).



**2.** Let us assume that the the the shape of a soap bubble is a sphere . Use linear approximation to approximate the increase in

the surface area of a soap bubble as its radius increases from 5 cm

to 5.2 cm also calculate the percentage error.

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**3.** A right circular cylinder has radius r = 10 cm and height h = 20 cm suppose that the radius of the cylinder is increased from 10 cm to 10.1 cm and the height does not change. Estimate the change in the volume of the cylinder . Also calculate the relative error and percentage error .

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**4.** Let f , g  $: (a, b) \to R$  be differentiable functions. Show that d(fg) = fdg + gdf.

5. Let  $g(x) = x^2 + \sin x$ . Calculate the differential dg.

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6. If the radius of a sphere , with radius 10 cm , has to decrease by

0.1 cm approximately how much will its volume decrease ?



7. Let 
$$F(x,y)=rac{2x-3y+4}{x^2+y^2+4}$$
 for all  $(x,y)\in R^2,\,$  Show that f is

continuous on  $R^2$ .

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**8.** Consider  $f(x,y)=rac{xy}{x^2+y^2}$  if (x,y)
eq (0,0) and f(0,0)=0

. Show that f is not continuous at (0,0) and continuous at all other

## points of $R^2$ .

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9. Let 
$$g(x, y) = \frac{x^2 y}{x^4 + y^2}$$
 for (x,y)  $\neq$  (0,0) and f(0,0)=0.  
(i) Show that  $\lim_{(x,y) \to (0,0)} g(x, y) = 0$  along every line  $y = mx, m \in R$ .  
(ii) Show that  $\lim_{(x,y) \to (0,0)} g(x, y) = \frac{k}{1+k^2}$ , along every parabola  $y = kx^2, k \in R\{0\}$ .

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**10.** Let f(x,y) = 0 if  $xy \neq 0$  and f(x, y) = 1 if xy = 0.

(i) Calculate : 
$$\frac{\partial f}{\partial x}(0,0), \frac{\partial f}{\partial y}(0,0)$$

(ii) Show that f is not continuous at (0,0)

- **11.** Let f(x,y) = 0 if  $xy \neq 0$  and f(x, y) = 1 if xy = 0.
- (i) Calculate :  $\frac{\partial f}{\partial x}(0,0), \frac{\partial f}{\partial y}(0,0)$

(ii) Show that f is not continuous at (0,0)

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12. Let F (x,y) = 
$$x^3y + y^2x + 7$$
 for all  $(x, y) \in R^2$ . Calculate $rac{\partial F}{\partial x}(-1,3)$  and  $rac{\partial F}{\partial y}(-2,1)$ 

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**13.** Let 
$$w(x,y) = xy + \frac{e^y}{y^2 + 1}$$
 for all (x,y)  $\in \mathbb{R}^2$ . Calculate  $\frac{\partial^2 w}{\partial y \partial x}$  and  $\frac{\partial^2 w}{\partial x \partial y}$ 

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14. Let  $w(x,y) = xy + \frac{e^y}{y^2 + 1}$  for all (x,y)  $\in \mathbb{R}^2$ . Calculate  $\frac{\partial^2 w}{\partial y \partial x}$  and  $\frac{\partial^2 w}{\partial x \partial y}$ 

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15. Let 
$$F(x,y)=rac{2x-3y+4}{x^2+y^2+4}$$
 for all  $(x,y)\in R^2,\,$  Show that f is continuous on  $R^2.$ 

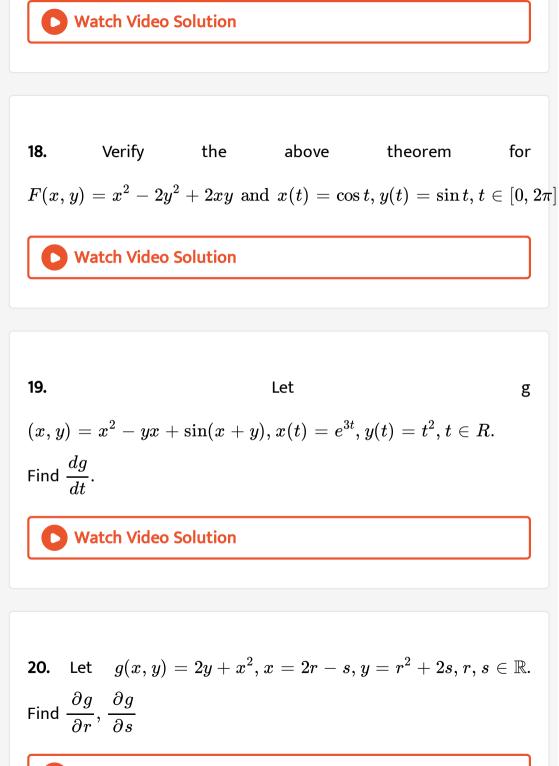
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16. If  $W=x^2+y^2+z^2, x, y, z\in R$  find the differential dW.

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17. Let W(x,y,z)  $=x^2-xy+3\sin z, x, y, z\in R$ , Find the linear

approximation at (2,-1,0).



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**21.** Show that 
$$F(x,y) = rac{x^2+5xy-10y^2}{3x+7y}$$
 is a homogeneous

function of degree 1.

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22. If 
$$u = \sin^{-1} \left( \frac{x+y}{\sqrt{x}+\sqrt{y}} \right)$$
 show that

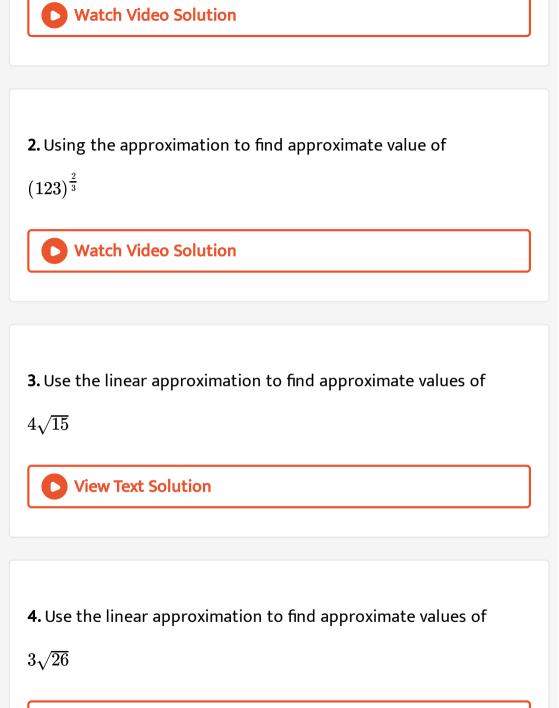
$$xrac{\partial u}{\partial x}+yrac{\partial u}{\partial y}=rac{1}{2}{
m tan}\,u.$$

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## Exercise 81

**1.** Let  $f(x) = \sqrt[3]{x}$ . Find the linear approximation at x = 27. Use the

linear approximation to approximate  $\sqrt[3]{27.2}$ 



5. Find a linear approximation for the following functions at the

indicated points.

$$f(x)=x^3-5x+12, x_0=2$$

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6. Find a linear approximation for the functions at the indicated

point

$$g(x)=\sqrt{x^2+9}, x_9=\,-\,4$$

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**7.** Find a linear approximation for the following function at the indicated points.

$$h(x)=rac{x}{x+1}, x_0=1$$

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**8.** The radius of a circular plate is measured as 12.65 cm instead of the actual length 12.5 cm. Find the following is calculating the area of the circular plate:

(i) Absolute error

(ii) Relative error

(iii) Percentage error

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9. The radius of a circular plate is measured as 12.65 cm instead of

the actual length 12.5 cm. Find the following is calculating the area

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**11.** A sphere is made of ice having radius 10 cm. Its radius decreases

from 10 cm to 9.8. cm. Find approximations for the following:

change in the volume



**12.** A sphere is made of ice having radius 10 cm. Its radius decreases from 10 cm to 9.8. cm. Find approximations for the following: change in the surface area

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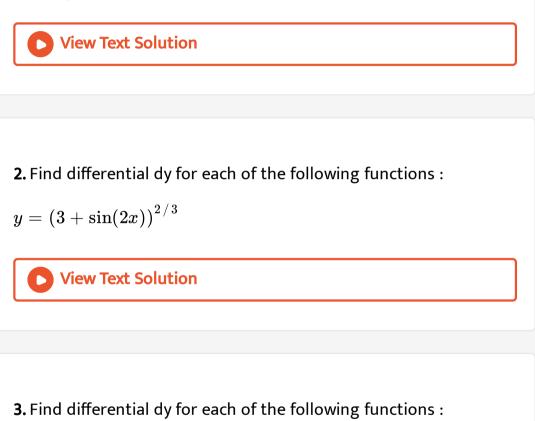
**13.** The time T , taken for a complete oscillation of a single pendulum with length I , is given by the equation  $T = 2\pi \sqrt{\frac{l}{g}}$ , where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of I.





**1.** Find differential dy for each of the following functions :

$$y=rac{\left( 1-2x
ight) ^{3}}{3-4x}$$



$$y=e^{x^2-5x+7}\cosig(x^2-1ig)$$

**4.** Find df for  $f(x) = x^2 + 3x$  and evaluate it for

x = 2 and dx = 0.1



**5.** Find df for 
$$f(x) = x^2 + 3x$$
 and evaluate it for

x = 3 and dx = 0.02

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**6.** Find  $\Delta f$  and df for the function f for the indicated values of x,

 $\Delta x$  and compare

$$f(x) = x^3 - 2x^2, x = 2, \Delta x = dx = 0.5$$

7. Find  $\Delta f$  and df for the function f for the indicated values of x,

 $\Delta x$  and compare

$$f(x)=x^2+2x+3, x=-0.5, \Delta x=dx=0.1$$

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8. Assuming  $\log_{10} = 0.4343$ , find an approximate value of  $\log_{10} 1003$ .

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9. The trunk of a tree has diameter 30 cm. During the following

year, the circumference grew 6 cm.

Approximately, how much did the tree's diameter grow?

10. The trunk of a tree has diameter 30 cm. During the following

year, the circumference grew 6 cm.

What is the percentage increase in area of the tree's cross-section?



**11.** An egg of a particular bird is very nearly spherical. If the radius to the inside of the shell is 5 mm and radiusto the outside of the shell is 5.3 mm, find the volume of the shell approximately



**12.** Assume that the cross section of the artery of human is circular. A drug is given to a patient to dilate his arteries. If the radius of an artery is increased from 2 mm to 2.1 mm, how much is crosssectional area increased approximately? **13.** In a newly developed city, it is estimated that the voting population (in thousands) will increase according to  $V(t) = 30 + 12t^2 - t^3, 0 \le t \le 8$  where t is the time in years. Find the approximate change in voters for the time change from 4 to  $44\frac{1}{6}$  year.

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14. The relation between the number of words y a person learns in

x hours is given by  $y=52\sqrt{x}, 0\leq x\leq 9$ . What is the approximate number of words learned when x changes from 1 to 1.1 hour ?



15. The relation between the number of words y a person learns in x hours is given by  $y = 52\sqrt{x}, 0 \le x \le 9$ . What is the approximate number of words learned when x changes from 4 to 4.1 hour ?

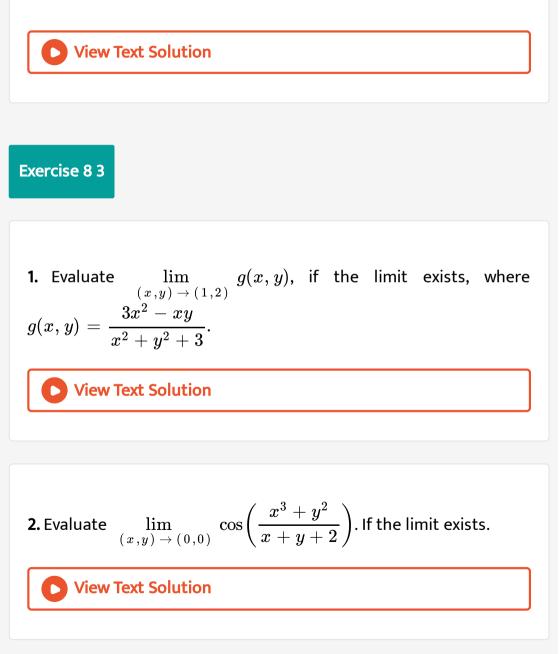
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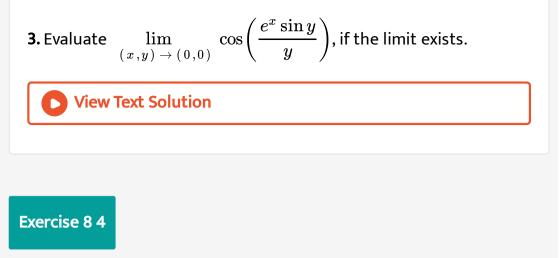
**16.** A circular plate expands uniformly under the influence of heat. If it's radius increases from 10.5 cm to 10.75 cm, then find an approximate change in the area and the approximate percentage change in the area.



**17.** A coat of paint of thickness 0.2 cm is applied to the faces of a cube whose edge is 10 cm. Use the differentials to find

approximately how many cubic centimeters of paint is used to paint this cube. Also calculate the exact amount of paint used to paint this cube.





**1.** Find the partial derivatives of the following functions at the indicated points.

$$f(x,y)=3x^2-2xy+y^2+5x+2,\,(2,\,-5)$$

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**2.** Find the partial derivatives of the following functions at the indicated points.

$$g(x,y)=3x^2+y^2+5x+2,\,(1,\ -2)$$

**3.** Find the partial derivatives of the following functions at the indicated points.

$$h(x,y)=x\sin(xy)+z^2x,\left(2,rac{\pi}{4},1
ight),$$

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**4.** Find the partial derivatives of the following functions at the indicated points.

$$G(x,y) = e^{x+3y} \log ig(x^2+y^2ig), (\,-1,1)$$

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5. If 
$$U(xy,z) = rac{x^2+y^2}{xy} + 3z^2y$$
, find  $rac{\partial U}{\partial x}, rac{\partial U}{\partial y}$  and  $rac{\partial U}{\partial z}$ .

6. If 
$$U(x, y, z) = \log(x^3 + y^3 + z^3)$$
, find  $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z}$ .  
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7. For each of the following functions find the  $g_{xy}, g_{xx}, g_{yy}$  and  $g_{yx}$ .  
 $g(x, y) = xe^y + 3x^2y$ 

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**8.** For each of the following functions find the  $g_{xy}, g_{xx}, g_{yy}$  and  $g_{yx}$ .

 $g(x,y) = \log(5x+3y)$ 

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**9.** For each of the following functions find the  $g_{xy}, g_{xx}, g_{yy}$  and  $g_{yx}$ .

$$g(x,y)=x^2+3xy-7y+\cos(5x)$$

10. A firm produces two types of calculators each week, x number of type A and y number of type B. The weekly revenue and cost functions (in rupees) are  $R(x, y) = 80x + 90y + 0.04xy - 0.05x^2 - 0.05y^2$  and C(x, y) = 8x + 6y + 2000 respectively.

Find the profit function P (x, y)

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11. A firm produces two types of calculators each week, x number of type A and y number of type B. The weekly revenue and cost functions (in rupees) are  $R(x, y) = 80x + 90y + 0.04xy - 0.05x^2 - 0.05y^2$  and C(x, y) = 8x + 6y + 2000 respectively.

Find 
$$\frac{\partial P}{\partial x}(1200, 1800)$$
 and  $\frac{\partial p}{\partial y}(1200, 1800)$  and interpret these

results.

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1. If 
$$w(x,y)=x^3-3xy+2y^2, x,y\in \mathbb{R}$$
, find the linear

approximation for w at (1, -1).

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2. Let 
$$z(x,y)=x^2y+3xy^4, x,y\in\mathbb{R}.$$
 Find the linear

approximation for z at (2, -1).

**3.** If 
$$v(x,y) = x^2 - xy + rac{1}{4}y^2 + 7, x, y \in R$$
, find the differential

dv.



4. If  $W(x,y,z)=x^2-xy+3\sin z, x,y,z\in\mathbb{R}.$  Find the linear approximation at (2, -1, 0).

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5. Let  $V(x,y,z)=xy+yz+zx, x, y, z\in \mathbb{R}.$  Find the differential

dV.



1. If 
$$u(x,y)=x^2y+3xy^4, x=e^t$$
 and  $y=\sin t$ , find  $\displaystyle rac{du}{dt}$  and

evaluate it at t = 0.



2. If 
$$w(x, y, z) = x^2 + y^2 + z^2, x = e^t, y = e^t \sin t$$
 and  $z = e^t \cos t$ , find  $\frac{dw}{dt}$ .  
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3. Let  $U(x, y, z) = xyz, x = e^{-t}, y = e^{-t} \cos t, z = \sin t, t \in \mathbb{R}.$ Find  $\frac{dU}{dt}$ .

4. If 
$$w(x,y) = 6x^3 - 3xy + 2y^2, x = e^x, y = \cos s, s \in \mathbb{R}$$
, find  $\frac{dw}{ds}$ , and evaluate at s = 0,



5. If 
$$z(x, y) = x \tan^{-1}(xy), x = t^2, y = se^t, s, t \in \mathbb{R}$$
. Find  $\frac{\partial z}{\partial ts}$   
and  $\frac{\partial z}{\partial t}$  at s = t = 1.

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6. Let  $z(x, y) = x^3 - 3x^2y^3$ , where  $x = se^t$ ,  $y = se^{-t}$ ,  $s, t \in \mathbb{R}$ . Find  $\frac{\partial z}{\partial s}$  and  $\frac{\partial z}{\partial t}$ .

7. 
$$W(x, y, z) = xy + yz + zx, x = u - v, y = uv, u, v \in \mathbb{R}$$
. Find  
 $\frac{\partial W}{\partial u}, \frac{\partial W}{\partial v}$ , and evaluate them at  $\left(\frac{1}{2}, 1\right)$ .

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## Exercise 8 7

1. In each of the following cases, determine whether the following

function is homogeneous or not. If it is so, find the degree.

$$f(x,y) = x^2y + 6x^3 + 7$$

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2. In each of the following cases, determine whether the following

function is homogeneous or not. If it is so, find the degree.

$$h(x,y)=rac{6x^2y^3-\pi y^5+9x^4y}{2020x^2+2019y^2}$$

**3.** In each of the following cases, determine whether the following function is homogeneous or not. If it is so, find the degree.

$$g(x,y,z) = rac{\sqrt{3x^2+5y^2+z^2}}{4x+7y}$$

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4. In each of the following cases, determine whether the following

function is homogeneous or not. If it is so, find the degree.

$$U=(x,y,z)=xy+\siniggl(rac{y^2-2z^2}{xy}iggr)$$

5. If 
$$w(x, y, z) = \log\left(\frac{5x^3y^4 + 7y^2xz^4 - 75y^3z^4}{x^2 + y^2}\right)$$
, find  
 $x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z}$ .  
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## Exercise 88

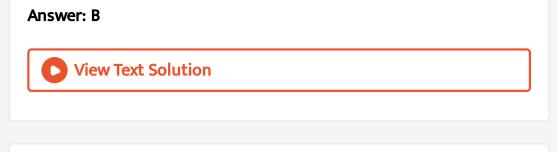
**1.** A circular template has a radius of 10 cm. The measurement of radius has an approximate error of 0.02 cm. Then the percentage error in calculating area of this template is

A. 0.2~%

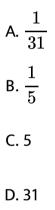
 $\mathsf{B.}\,0.4\,\%$ 

 $\mathsf{C}.\,0.04\,\%$ 

D. 0.08~%



**2.** The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?



#### Answer: B



3. If 
$$u(x,y)=e^{x^2+y^2}$$
, then  $rac{\partial u}{\partial x}$  is equal to  
A.  $e^{x^2+y^2}$   
B. 2x u  
C.  $x^2u$ 

D. 
$$y^2 u$$

## Answer: B

**Niew Text Solution** 

**4.** If 
$$v(x,y) = \log(e^x + e^y)$$
, then  $rac{\partial v}{dex} + rac{\partial v}{\partial y}$  is equal to

A. 
$$e^x + e^y$$

$$\mathsf{B.} \ \frac{1}{e^x + e^y}$$

C. 2

#### Answer: D



5. If 
$$w(x,y)=x^y, x>0$$
, then  $\displaystyle rac{\partial w}{\partial x}$  is equal to

A.  $x^y \log x$ 

B. y log x

C.  $yx^{y-1}$ 

D. x log y

Answer: C

**6.** If 
$$f(x,y) = e^{xy}$$
, then  $\frac{\partial^2 f}{\partial x \, \partial y}$  is equal to

A.  $xye^{xy}$ 

- $\mathsf{B.}\,(1+xy)e^{xy}$
- $\mathsf{C.}\,(1+y)e^{xy}$
- D.  $(1+x)e^{xy}$

#### Answer: B



7. If we measure the side of a cube to be 4 cm with an error of 0.1

cm, then the error in our calculation of the volume is

A. 0.4 cu.cm

B. 0.45 cu.cm

C. 2 cu.cm

D. 4.8 cu.cm

Answer: D

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8. The change in the surface area  $S=6x^2$  of a cube when the

edge length varies from  $x_0$  to  $x_0 + dx$  is

A.  $12x_0+dx$ 

B.  $12x_0dx$ 

 $\mathsf{C.}\,6x_0dx$ 

D.  $6x_0 + dx$ 

Answer: B

**9.** The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is

A.  $0.3 x dxm^3$ 

 $\mathrm{B.}\,0.03 xm^3$ 

 $\mathsf{C.}\, 0.03 x^2 m^3$ 

D.  $0.03x^3m^3$ 

#### Answer: C

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10. If  $g(x,y) = 3x^2 - 5y + 2y^2, x(t) = e^t$  and  $y(t) = \cos t$ , then  $rac{dg}{dt}$  is equal to

A.  $6e^{2t} + 5\sin t - 4\cos t\sin t$ 

 $\mathsf{B.}\,6e^{2t}-5\sin t+4\cos t\sin t$ 

 $\mathsf{C.}\, 3e^{2t} + 5\sin t + 4\cos t\sin t$ 

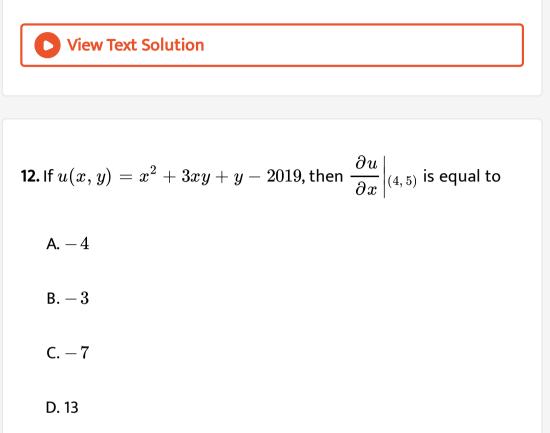
D.  $3e^{2t} - 5\sin t + 4\cos t\sin t$ 

#### Answer: A

11. If 
$$f(x) = rac{x}{x+1}$$
, then its differential is given by

A. 
$$\frac{-1}{\left(x+1\right)^{2}}dx$$
B. 
$$\frac{1}{\left(x+1\right)^{2}}dx$$
C. 
$$\frac{1}{x+1}dx$$
D. 
$$\frac{-1}{x+1}dx$$

#### Answer: B



Answer: C



**13.** Linear approximation for g(x) = cos x at 
$$x=rac{\pi}{2}$$
 is

A. 
$$x+rac{\pi}{2}$$
  
B.  $-x+rac{\pi}{2}$   
C.  $x-rac{\pi}{2}$   
D.  $-x-rac{\pi}{2}$ 

#### Answer: B



14. If 
$$w(x, y, z) = x^2(y - z) + y^2(z - x) + z^2(x - y)$$
, then  
 $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}$  is

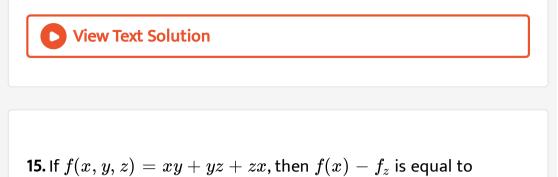
A. xy + yz + zx

 $\mathsf{B.}\,x(y+z)$ 

 $\mathsf{C}.\,y(z+x)$ 

D. 0

#### Answer: D



A. z - x

B. y - z

C. x - z

D. y - x

Answer: A

