MATHS

BOOKS - UNITED BOOK HOUSE

MISCELLANEOUS EXERCISE

Exercise

1. Find the number of quivalance relations on the set A = {a,b,c} containing elements (b,c) and (c,b).

Watch Video Solution

2. If f(x) = $\sin x, g(x) = x^2$ and $h(x), = \log x$, find the composite function[ho(gof)](x).

Watch Video Solution

3. Prove that $\tan^{-1}\left(\frac{3\sin 2x}{5+3\cos 2x}\right)+\tan^{-1}\left(\frac{1}{4}\tan x=x.\right.$

Watch Video Solution

4. If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$ and x + y + z = $\frac{3}{2}$, then show that x = y = z.

Watch Video Solution

5. Solve : $\cos^{-1}x-\sin^{-1}x=\cos^{-1}\left(x\sqrt{3}\right)$.

Watch Video Solution

6. If $\phi=\frac{\tan^{-1}\left(x\sqrt{3}\right)}{2k-x}$ and $\theta\frac{2x-k}{k\sqrt{3}}$, then show that one value of $(\phi-\theta)is$30^@`

Watch Video Solution

7. By using properties of determinats. Prove that-

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(x-4)^2$$

Watch Video Solution

8. By using properties of determinats. Prove that-

$$|(1, 1+x, 1+x+y), (2, 3+2x, 1+3x+2y), (3, 6+3x, 1+6x+3y):|$$

Watch Video Solution

9. By using properties of determinats. Prove that-

$$\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & \sin\left(2\theta+\frac{4\pi}{3}\right) \\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right) \end{vmatrix} = 0$$

Watch Video Solution

10. Let X = $\begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & -7 \end{bmatrix}$ express X as sum of two matrics such that one is symmetric and other is skew-symmetric.

Watch Video Solution

11. If $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} \times P = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$, find the matrix P.

Watch Video Solution

12. Prove that: $\lim_{x \to \infty} \sqrt{x}\left(\sqrt{x+3} - \sqrt{x}\right) = \frac{3}{2}$

Watch Video Solution

13. Prove that: $\lim_{x \to 0} \left(\frac{x - 1 + \cos x}{x}\right)^{\frac{1}{x}} = e^{-\frac{1}{2}}$

Watch Video Solution

14. Prove that: $\lim_{x \to \infty} \left(\frac{x+5}{x+1}\right)^x = e^4$

Watch Video Solution

15. Prove that: $\lim_{x \to \frac{\pi}{2}} (1 + \cos x)^{3 \sec x} = e^3$

Watch Video Solution

16. Prove that: $\lim_{x \to 0} \left([1 + 3x]^{\frac{x+3}{x}}\right) = e^9$

Watch Video Solution

17. Evaluate the following limits: $\lim_{x \to 0} \frac{xe^x - \log(x+1)}{x^2}$

Watch Video Solution

18. Evaluate the following limits: $\lim_{x \to 0} \left[\tan\left(\frac{\pi}{4} + x\right)\right]^{\frac{1}{x}}$

Watch Video Solution

19. Evaluate the following limits: $\lim_{x \to 0} (\sin x + \cos x)^{\frac{1}{x}}$

Watch Video Solution

20. Evaluate the following limits: $\lim_{x \to 0} \frac{\sin \log(1 + x)}{\log(\sin x + 1)}$

Watch Video Solution

21. It g(x) is the inverse of f(x) and f(X) = $\left(1 + x^3\right)^{-1}$, show that g'(x) = $\frac{1}{1 + \{g(x)\}^3}$.

Watch Video Solution

22. Find the differential coefficients of the folloiwng functions:

x^x

Watch Video Solution

23. Find form 1st principle the differential coefficients of the folloiwng functions:

$\tan x$

Watch Video Solution

24. Find form 1st principle the differential coefficients of the folloiwng functions:

$\log(\sin x)$

Watch Video Solution

25. Find $\frac{dy}{dx}$ when

$x^{\sin y} + y^{\sin x} = 1$

Watch Video Solution

26. Find $\frac{dy}{dx}$ when

$x^{\log x} + (\sin x)^{x} + 15x$

Watch Video Solution

27. Find $\frac{d^2y}{dx^2}$ when

$x = e^{-t} \text{ and } y = te^{-1}$

Watch Video Solution

28. Find $\frac{d^2y}{dx^2}$ when

$x = a \sin^3 t$ and $y = a \cos^3 t$ at $t = \frac{\pi}{4}$

Watch Video Solution

29. Find $\frac{d^2y}{dx^2}$ when

$x = a(t - \sin t)$ and $y = a(1 + \cos t)$ at $t = \frac{\pi}{2}$

Watch Video Solution

30. If $y^2(1 - x^2) = x^2 + 1$, show that $(1 - x^4)\left(\frac{dy}{dx}\right)^2 = y^4 - 1.$

Watch Video Solution

31. If x = $\sec\theta - \cos\theta, y = \sec^n\theta - \cos^n\theta$, then show that $(x^2 + 4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2 + 4)$

Watch Video Solution

32. If $y = x^{y^x}$, prove that $y_1 = \left(y \log y \frac{1 + x \log x \log y}{x \log x(1 - x \log y)}\right.$

Watch Video Solution

33. or,find the derivatives of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at x=0.

Watch Video Solution

34. If $y=\left(\sin^{-1}x\right)^2+\left(\cos^{-1}x\right)^2$, then show that $\left(1-x^2\right)y_2-xy_1=4$

Watch Video Solution

35. If $y=x\log[x/(a+bx)]$, show that $x^3y_2-\left(y-xy_1\right)^2=0$

Watch Video Solution

36. If $x^3 + y^3 = 3ax^2$, show that $y_2 + \frac{2a^2x^2}{y^5} = 0$

Watch Video Solution

37. Evaluate:

$$\int \frac{x^2+1}{x^4+x^2+1} dx$$

Watch Video Solution

38. Evaluate:

$$\int \frac{dx}{\sin^4 x + \cos^4 x}$$

Watch Video Solution

39. Evaluate:

$$\int dx \left(\left(2x - x^2\right)^3 / 2 \right)$$

Watch Video Solution

40. Evaluate:

$$\int x^{\frac{13}{2}}\left(1+x^{5/2}\right)^{\frac{1}{2}}dx$$

Watch Video Solution

41. Evaluate:

$$\int (\log(1+x^2)\,dx$$

Watch Video Solution

42. answer the foll. Questions: (ii)evaluate : $\int \frac{x^4+1}{x^6+1}dx$

Watch Video Solution

43. Evaluate:

$$\int \frac{dx}{\sqrt{\sin^3 x \cos(x-\alpha)}}$$

Watch Video Solution

44. Evaluate:

$$\int_0^{\pi} |\cos x| dx$$

Watch Video Solution

45. Evaluate: $\int \frac{\sin x + \cos x}{9 + 16\sin 2x} dx$

Watch Video Solution

46. Evaluate:

$$\int_0^{\pi/4} \frac{\sec\theta d\theta}{1 + 2\sin^2\theta}$$

Watch Video Solution

47. Evaluate:

$$\int_1^e \frac{(x+1)^3}{x^2} dx$$

Watch Video Solution

48. Evaluate:

$$\int_0^\pi x \sin x \cos^2 x dx$$

Watch Video Solution

49. Show that: $\int_0^1 \left(\frac{\log(1+x)}{1+x^2}\right) dx = \frac{\pi}{8} \log 2.$

Watch Video Solution

50. Evaluate: $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$

Watch Video Solution

51. Evaluate :

$$\lim_{n \to \infty} \left[\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \ldots. + \frac{1}{4n}\right]$$

Watch Video Solution

52. Solve: xdy-ydx= $\sqrt{x^2 + y^2}dx.$

Watch Video Solution

53. Solve $xdy + ydx = \frac{ydx - xdy}{x^2 + y^2}$

Watch Video Solution

54. Solve $x \cos x \frac{dy}{dx} + y(\sin x + \cos x) = 1$

Watch Video Solution

55. Solve(cos y + y cos x) dx + sin (x - x sin y) dy = 0`

Watch Video Solution

56. Solve $\frac{d^2y}{dt^2} = \tan y \sec^2 y$, given $\frac{dy}{dt}$ = 0 when y = 0

Watch Video Solution

57. Prove that cos (sinx) > sin (cos x) for all x in $0 \leq x \leq \pi/2$

Watch Video Solution

58. If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$ then

Watch Video Solution

59. Prove that all points of the curve $y^2 = 4a\left[x + a\frac{\sin x}{a}\right]$ at which the langent is parallel to the axis of x, lie on a parabola.

Watch Video Solution

60. Find the point on the curve $4x^2 + a2y^2 = 4a^2, 4 < a^2 < 8$ that is farthest from the point (0,-2)`

Watch Video Solution

61. The area bounded by the parabola $y = x - x^2$ and the line y = mx equals $\frac{9}{2}$, find m.

Watch Video Solution

62. Prove that f(x)= sin x+$\sqrt{3}$ cosx has maximum value at x=$\frac{\pi}{6}$

Watch Video Solution

63. The length of the hypotenuse of a right angled triangle is 3 ft. Find the volume of the greatest cone that can e generated by revolving the triangle about a side.

Watch Video Solution

64. The angle between the tangents to the curves y=sin x and y = cos x at their point of intersection is

Watch Video Solution

65. Show that $f(x) = (x-2)e^x + x + 2$ is positive for all positive values of x.

Watch Video Solution

66. Vectors $\vec{m}, \vec{n}, \vec{r}$ are such that $\vec{m} + \vec{n} + \vec{r} = 0$, prove that $\vec{m} \times \vec{n} = \vec{n} \times \vec{r} = \vec{r} \times \vec{m}$

Watch Video Solution

67. If $\vec{p} \times \vec{q} = \vec{m} \times \vec{n}$ and $\vec{p} \times \vec{m} = \vec{q} \times \vec{n}$, show that $\vec{p} - \vec{n}$ is parallel to $\vec{q} - \vec{m}$ where $\vec{p} \neq \vec{n}$ and $\vec{q} \neq \vec{m}$.

Watch Video Solution

68. If $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 0$. Show that the vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar.

Watch Video Solution

69. The vector $\vec{r}$ is collinear with vector $\vec{n} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{r} . \vec{n} = 16$, show that $\vec{r} = \frac{1}{7}\left(16\hat{i} + 8\hat{j} + 24\hat{k}\right)$

Watch Video Solution

70. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors of equal magnitude, show that $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$. Also find the angle.

Watch Video Solution

71. Find the equation of the plane passing through the point(1,2,1) and perpendicular to the line joining the points (2,3,5) and (1,4,2).

Watch Video Solution

72. Find the vector equation of the following plane in scalar product from $\vec{r}. = \left(\hat{i}-\hat{j}\right)+\lambda\left(\hat{i}+2\hat{j}-\hat{k}\right)+\mu\left(-\hat{i}+\hat{j}+2\hat{k}\right),$

Watch Video Solution

73. two numbers are selected at random from 1,2,3,......, 100 and are multiplied. Find the probability that the product thus obtained is divisible by 3.

Watch Video Solution

74. In a class, 5% of the boys and 10% of the girls have an IQ more than 150. In the class 60% of the students are boys and rest girls. If a student is selected at random and found to have an IQ of more than 150, find the probability that the student is a boy.

Watch Video Solution

75. If $P(A/C) \geq P(B/C)$ $P\left(A/\overline{C}\right) \geq P\left(B/\overline{C}\right)$, then prove that, $P(A) \geq P(B)$.

Watch Video Solution