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## MATHS

## BOOKS - UNITED BOOK HOUSE

## QUESTION PAPER 2017

Exercise

1. An unbiased die is thrown 3 times. If the first
thrown is a 5, the conditional probability of
A. $\frac{1}{9}$
B. $\frac{1}{18}$
C. $\frac{1}{108}$
D. $\frac{5}{16}$

Answer:

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2. If $\vec{\alpha}=2 \hat{i}+3 \hat{j}-6 \hat{k} \quad$ and
$\vec{\beta}=p \hat{i}-\hat{j}+2 \hat{k}$ are two parallel vectors,
then the value of $p$ is
A. $-\frac{1}{3}$
B. $\frac{2}{3}$
C. $-\frac{2}{3}$
D. $-\frac{3}{2}$

## Answer:

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3. The value of $\sin \left(\cos ^{-1} x\right)-\cos \left(\sin ^{-1} x\right)$ is-
A. 1
B. $x$
C. $-\frac{1}{x}$
D. 0

## Answer:

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4. The degree of the differential equation
$\frac{d^{3} y}{d x^{3}}+y=\sqrt[3]{1+\frac{d y}{d x}}$ is
A. 1
B. 2
C. 4
D. 3

## Answer:

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5. Let IR be the set of real numbers and the
mapping $f: I R \rightarrow I R$ and $g: I R \rightarrow I R$ be define by $f(x)=5-x^{2}$ and $g(x)=3 x-4$, then the value of $(f \circ g)(-1)$ is
A. 8
B. -44
C. 54
D. 16

## Answer:

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6. If $f(x)$ is the probability distribution function of a random variable $X$ and $X$ can assume only
two values $x_{1}$ and $x_{2}$ then the value of $f\left(x_{1}\right)+f\left(x_{2}\right)$ is
A. $>1$
B. $<1$
C. $=\frac{1}{2}$
D. $=2$

Answer:
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7. If the direction ratios of a striaght line are proportional, to $0,1,-1$ then its inclination with
z -axis is

> A. $\frac{\pi}{2}$
> B. $\pi$
> C. $\frac{3 \pi}{2}$
> D. $\frac{3 \pi}{4}$

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8. If $f(x)=|x|+|x-1|$ then $\mathrm{f}(\mathrm{x})$ is-
A. continuous at $\mathrm{x}=0$ and $\mathrm{x}=1$
B. continuous at $x=0$ bur discontinuous at

$$
x=1
$$

C. continuous at $x=1$ but discontinuous at
$x=0$
D. none of these.

## Answer:

9. The value of $\int_{-a}^{a} \frac{x e^{x^{4}}}{1+x^{2}} d x$ is
A. 0
B. 1
C. a
D. 2 a

Answer:
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10. If the inverse of the matrix $A$ exists, then
the value of $\operatorname{det}\left(A^{\wedge}-1\right)$
A. 0
B. 1
C. $\frac{1}{\operatorname{det} A}$
D. $\operatorname{det} A$

Answer:

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11. If $R$ is a relation defined as $R=\{$ $(x, y): x, y \in N$ and $\mathrm{x}+3 \mathrm{y}=12\}$. Then find the domain and range of $R$.

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12. 

Evaluate
$\sec ^{2}\left(\tan ^{-1} 2\right)+\operatorname{cosec}\left(\cot ^{-1} 3\right)$.

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13. Show that , $A=\frac{1}{3}\left[\begin{array}{ccc}-1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1\end{array}\right]$ is proper orthogonal matrix.

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14. Prove that $\left|\begin{array}{ccc}1 & \log _{x} y & \log _{x} z \\ \log _{y} x & 1 & \log _{y} z \\ \log _{z} x & \log _{z} y & 1\end{array}\right|=0$

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15. If $x=\sqrt{a^{\sin ^{-1} t}}$ and $y=\sqrt{a^{\cos ^{-1} t}}$ show that $\frac{d y}{d x}=-\frac{y}{x}$.

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16. Verify Rolle' theorem for the function

$$
f(x)=x^{2}-4 x+3 \text { in } 1 \leq x \leq 3
$$

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17. Evaluate: $\int \frac{\cos x-\cos 2 x}{1-\cos x} d x$.

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18. A particle starts with the velocity $u$ and moves in a straight line, its acceleration being always equal to its displacement. If $v$ be the velocity when its displacement is $x$, then show that $v^{2}=u^{2}+x^{2}$.
19. Find the interval where $f(x)=\frac{1}{1+x^{2}}$ decrease.

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20. If $f(x)=f(a+x)$ then prove that the value of $\int_{a}^{a+t} f(x) d x$ is independent of a.
21. If $\vec{a}=3 \hat{i}+\hat{j}+9 \hat{k}$
$\vec{b}=\hat{i}+\lambda \hat{j}+3 \hat{k}$ then find the value of $\lambda$ for which the vector $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are perpendicular to each other.

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22. Find the vector equation of a plane
through the point $\hat{i}+\hat{j}+\hat{k}$ and parallel to the plane $\vec{r} \cdot(2 \hat{i}-\hat{j}+2 \hat{k})=5$.
23. If $P(A)=\frac{2}{3}, P(B)=\frac{1}{2} \quad$ and
$P(A \cap B)=\frac{1}{6}$ then find the value of $P\left(A \cap B^{C}\right)$.

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24. 

Solve:
$\tan ^{-1}(x+1)+\tan ^{-1}(x-1)=\tan ^{-1}\left(\frac{8}{31}\right)$
25. If $A=\left(\begin{array}{cc}2 & -1 \\ 1 & 3\end{array}\right)$ then show that
$A^{2}-5 A+7 I_{2}=0$ hence find $A^{-1}$

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26. Solve by Cramer's rule: $x+3 y=4, y+3 z=7$, $4 x+z=6$.
27. 

Prove
$2 a b \quad a^{2} \quad b^{2}$
$a^{2} \quad b^{2} \quad 2 a b=-\left(a^{3}+b^{3}\right)^{2}$.
$b^{2} \quad 2 a b \quad a^{2}$

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28. If $f(x)=\left(\frac{a+x}{b+x}\right)^{a+b+2 x}$ then prove
that
$f^{\prime}(0)=\left[2 \log \left(\frac{a}{b}\right)+\frac{b^{2}-a^{2}}{a b}\right]\left(\frac{a}{b}\right)^{a+b}$

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29. Evaluate: $\int \frac{x d x}{x^{4}-x^{2}+1}$

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30. Solve: $y d x-\left(x+2 y^{2}\right) d y=0$.

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31. In a certain culture the rate of increment of
bacterial at any instant is proportional to the
cube root of the number of bacteria present
at that instant. IF the number of bacteria becomes 8 times in 3 hours, in how much time that number becomes 64 times?

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32. The position vectors of four points $A, B, C$ and D
are
$4 \hat{i}+8 \hat{j}+12 \hat{k}, 2 \hat{i}+4 \hat{j}+6 \hat{k}, 3 \hat{i}+5 \hat{j}+4 \hat{k}$
and $5 \hat{i}+8 \hat{j}+5 \hat{k}$ respectively. Using vector method prove that the four points $A, B, C$ and $D$ are coplanar.

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33. If a vector $2 i^{+} 3 j+8 k$ is perpendicular to the vector $4 i-4 j^{+} \alpha k$ then the value of $\alpha$ is

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34. Evaluate: $\int_{0}^{\frac{\pi}{4}} \log (1+\tan \theta) d \theta$.

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35. find the value of $\int_{1}^{2} 5 x^{2} d x$.

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36. A man is known to speak the truth 3 out of

4 times. HE throws an unbiased die and reports that it is a six. Find the probability
that it is actually six.

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37. If the sum of the mean and variance of a binomial distribution for 5 random trials is 1.8 then find the binomial distribution.

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38. Solve the following linear programming problem by graphical method and find the maximum value of $Z$. (Graph sheet is not required): $Z=x+y$

Subject to the constraints
$5 x+10 y \leq 50, x+y \geq 1, y \leq 4, x, y \geq 0$.

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39. Using calculus, show that the maximum
value of the function $\left(\frac{1}{x}\right)^{x}$ is $e^{\frac{1}{e}}$.

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40. Using integration, prove that the area of the closed region bounded by the curves $y^{2}=4 x$ and $x^{2}=4 y$ is $\frac{16}{3}$ sq. unit.
41. Solve: $x d y-y d x=\sqrt{x^{2}+y^{2}} d x$.

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42. Find the shortest distance between the straight lines

$$
\begin{aligned}
& \vec{r}=-4 \hat{i}+4 \hat{j}+\hat{k}+\lambda_{1}(\hat{i}+\hat{j}-\hat{k}) \quad \text { and } \\
& \vec{r}=-3 \hat{i}-8 \hat{j}+3 \hat{k}+\lambda_{2}(\hat{i}+3 \hat{j}-3 \hat{k}) .
\end{aligned}
$$

43. A variable plane is at a constant distance $p$
from the origin and meets the coordinate axes
in $A, B$ and $C$, show that the locus of the centroid of the tetrahedron $O A B C$ os
$\frac{1}{x^{2}}+\frac{1}{y^{2}}+\frac{1}{z^{2}}=\frac{16}{p^{2}}$

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