



MATHS

BOOKS - UNITED BOOK HOUSE

SET 13

Exercise

1. Let Z be the set of integers and the mapping $f: Z \rightarrow Z$ be defined by, $f(x) : x^2$. State which of the following is equal to $f^{-1}(-4)$?

A. $\{2\}$

B. $\{-2\}$

C. $\{2,-1\}$

D. ϕ

Answer:



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2. Solve: $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{\sqrt{3}}{2}$

Answer:



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3. Matrices A and B will be inverse of each other only if

A. $AB = BA \neq 1$

B. $AB = BA = 0$

C. $AB = 0, BA = 1$

D. $AB = BA = 1.$

Answer:



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4. If $y = \log_{10} x$, then $\frac{dy}{dx}$ is equal to

A. $\frac{1}{x} \log_{10}^e$

B. $\frac{1}{x} \log_e^{10}$

C. $\frac{1}{x} \log_{10}^e$

D. $\frac{1}{10} x$

Answer:



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5. If $\int_a^b f(x) dx = \int_a^b \phi(x) dx$, then

A. $f(x) = \phi(x)$

B. $f(x) - \phi(x) = c$

C. $f(x) + \phi(x) = c$

D. none of these

Answer:



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6. In a given interval a function

A. can have two consecutive maxima

B. can have two consecutive minima

C. possesses maximum and minimum

values alternately

D. cannot have more than two extreme

values

Answer:



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7. If $\vec{OA} = \hat{i} - 2\hat{k}$ and $\vec{OB} = 3\hat{i} - 2\hat{j}$ then the direction cosines of the vectore \vec{AB} are

A. $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}$

B. 2,2,2

C. $\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

D. $-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}$

Answer:



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8. CHOOSE the correct answer from the following alternative : $P(A) = \frac{3}{7}, P(B) = \frac{4}{7}$

and $P(A \cap B) = \frac{2}{9}$, then the value of $P(A/B)$

is equal to-

A. $\frac{7}{18}$

B. $\frac{14}{21}$

C. $\frac{5}{18}$

D. $\frac{4}{9}$

Answer:



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9. If X follows a binomial distribution with parameter $n = 101$ and $p = \frac{1}{3}$ then $P(x=r)$ is maximum if r equal to

A. 34

B. 30

C. 32

D. 31

Answer:



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10.

Prove

that,

$$\{\cos(\sin^{-1} x)\}^2 = \{\sin(\cos^{-1} x)\}^2.$$



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11. Find the real values of K for which the following system of linear equations has non-trivial solutions:

$$x - ky + z = 0, \quad kx + 3y - kz = 0, \quad 3x + y - z = 0$$



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12. If $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$ then prove that, $AA' = I$. Hence find A^{-1} .



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13. Examine whether $f(x) = |x|$ has a derivative at $x = 0$.



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14. State Lagrange's mean value theorem.





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15. Evaluate: $\int \frac{\sin x dx}{\cos 2x}$



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16. Find the integrating factor of the differential equation $(x + y + 1) \frac{dy}{dx} = 1$.



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17. Prove that the function $\frac{\sin(x + \alpha)}{\sin(x + \beta)}$ has neither a maximum nor a minimum value.



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18. Using the method of differentail find the approximate value of $\sqrt{0.24}$.



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19. A variable plane moves in such a way that the sum of the reciprocals of its intercepts on the three coordinate axes is constant. Prove that the plane passes through a fixed point.



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20. If A and B are two independent events, prove that A^C and B are also independent events.



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21. Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that all the five cards are spades?



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22. If $\sin(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$,

find the value of $\tan 2\alpha$.



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23. If A and B are two matrices such that $AB = O$, can we deduce that either A or B is a zero matrix? Illustrate by an example.



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24. $(AB)^{-1} = B^{-1}A^{-1}$ where A and B are invertible matrices satisfying commutative property with respect to multiplication. Write true or false.



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25. Answer the foll. Question : 2.show that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right), (abc \neq 0)$$



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26. Evaluate: $\int \frac{dx}{\sqrt{\sin^3 x \sin(x + \alpha)}}$



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27. Evaluate: $\int \left(\log(\log x) + \frac{1}{(\log x)^2} \right) dx$



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28.

Solve:

$$\cos^2 x \frac{dy}{dx} + y = \tan x \left(0 \leq x \leq \frac{\pi}{2} \right).$$



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29. Prove, by vector method or otherwise, that the point of intersection of the diagonals of a

trapezium lies on the line passing through the midpoint of the parallel sides (you may assume that the trapezium is not a parallelogram).



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30. Find the value of λ if three vectors

$$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 3\hat{k} \quad \text{and}$$

$$\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k} \text{ are coplanar.}$$



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31. A candidate is selected for interview for the three posts. For the first post there are 3 candidates, for the second there are 4 and for the third there are 2. what is the probability that the candidate getting at least one post?



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32. For a random variable X , it is given, $E(x) = 10$ and $\text{var}(x) = 25$. Find the positive values of a and b such that $Y = aX - b$ have expectation 0 and variance 1.



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33. A stone is dropped into a quiet lake and waves moves in circles at a speed of 4 cm/sec. At the instant when the radius of the circular wave is 10 cm, how fast is the enclosed area increasing?



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34. If the sum of the lengths of the hypotenuse and another side of a right-angled triangle is given, show that the area of the triangle is maximum when the angle between these sides is $\frac{\pi}{3}$.



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35. Find the equations of the tangents to the ellipse $2x^2 + 3y^2 = 30$, which are parallel to the straight line $x + y + 18 = 0$.





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- 36.** Find the foot of the perpendicular drawn from the point $(2\hat{i} - \hat{j} + 5\hat{k})$ to the line $\vec{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + t(10\hat{i} - 4\hat{j} - 11\hat{k})$. Find also the length of the perpendicular.



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- 37.** Show that the equation of the plane passing through the point (1,2,3) and parallel

to the plane. $3x + 4y - 5z = 3$ is given by $3x + 4y - 5z = -4$.



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