MATHS

BOOKS - UNITED BOOK HOUSE

SET 16

Exercise

1. Prove that,

$$\tan^{-1}\left(\frac{4}{3}\right)+\tan^{-1}\left(\frac{12}{5}\right)=\pi-\tan^{-1}\left(\frac{56}{33}\right)$$

.

Watch Video Solution

2. Without expanding, prove that $\begin{bmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{bmatrix}$ =0

Watch Video Solution

3. Verify that the matrix equation $A^2 - 4a + 3I$=0 is satisfied by the matrix

$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$,

Watch Video Solution

4. Evaluate $\lim_{x \to 0} \frac{e^{px} - e^{-qx}}{x}$

Watch Video Solution

5. Find $\frac{dy}{dx}$, whne $y = \log\left(x + \sqrt{x^2 - a^2}\right)$

Watch Video Solution

6. Examine whether Rolle's theorem is applicable to the following function in the given intervals:

$$f(x) = \cos x \in -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

Watch Video Solution

7. Evaluate: $\int \frac{x dx}{\sqrt{3x^2 + 1}}$

Watch Video Solution

8. Find the intervals in which the function $f(x) = \frac{x}{x^2+1}$ is decreasing.

Watch Video Solution

9. Using differentials, find the approximate value of $(82)^{\frac{1}{4}}$.

Watch Video Solution

10. If the vectors $3\hat{i} - 2\hat{j} + m\hat{k}$ and $-2\hat{i} + \hat{j} + 4\hat{k}$ are perpendicular to each other, find the value of m.

Watch Video Solution

11. Can the numbers $\frac{1}{2}, -\frac{1}{\sqrt{2}}, -\frac{1}{2}$ be the direction cosines of a straight line? Give reason.

Watch Video Solution

12. Prove that, if $P(A/B) = P(A)$ then $P(A^c/B) = P(A^c)$.

Watch Video Solution

13. If the probability of success in a single trials is 0.05, how many Bernoulli trials must be performed in, order that the probability of at least one success is $\frac{2}{3}$ or more.

Watch Video Solution

14. Prove that, $\begin{vmatrix} \sin A & \cos A & \sin(A+\theta) \\ \sin B & \cos B & \sin(B+\theta) \\ \sin C & \cos c & \sin(C+\theta) \end{vmatrix}$ =0

Watch Video Solution

15. If A,B,C be the angles of a triangle, then prove that , $\begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix}$ =0

Watch Video Solution

16. Solve: $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix}$ =0

Watch Video Solution

17. If A = $\begin{vmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{vmatrix}$, find A^{-1}

Watch Video Solution

18. If f(x) = $x\sqrt{x^2+a^2} + a^2 \log\left(x\sqrt{x^2+a^2}\right)$

then find the value of f(0).

Watch Video Solution

19. If $y = a\cos(\log x) + b\sin(\log x)$, show that, $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Watch Video Solution

20. Integrate: $\int \frac{2x^2 - 3x + 9}{x^2 + 4x - 5} dx$

Watch Video Solution

21. Evaluate: $\int \frac{(x - l)(x - m)}{(x - a)(x - b)} dx$

Watch Video Solution

22. Solve: $\left(1+x^2\right)\frac{dy}{dx}+2xy=\sqrt{x^2+4}$

Watch Video Solution

23. Solve: $\log\frac{dy}{dx}$ = 4x - 2y-2`: given y = 1, when x = 1.

Watch Video Solution

24. (d) answer any one question : (i) if three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ of magnitudes 3,4and5 are such that each vector is perpendicular to the sum of the other two vectors,then prove that $\left|\vec{a}+\vec{b}+\vec{c}\right| = 5\sqrt{2}$.

Watch Video Solution

25. The position vectors of the points A,B,C and D are $6\hat{i}-7\hat{j}$, $16\hat{i}-29\hat{j}-4\hat{k}$, $3\hat{i}-6\hat{k}$

and $2\hat{i} + 5\hat{j} + 10\hat{k}$ respectively. Show that the points A,B,C and D are non coplanar.

Watch Video Solution

26. Show that,

$$\int_0^{\frac{\pi}{2}} \frac{\sin 2x dx}{\sin^4 x + \cos^4 x} = \frac{\pi}{2}$$

Watch Video Solution

27. Evaluate: $\int_0^{\frac{\pi}{4}} \log(1 + \tan\theta) d\theta.$

Watch Video Solution

28. The probability that three is at least one error in an accounts statements prepared by A is 0.2 and for B and C they are 0.25 and 0.4 respectively A,B and C prepared 10,16,20 startment respectively. Find the expected number of correct statements in all.

Watch Video Solution

29. A normal to the parabola $y^2 = 5x$ makes an angle 45° with line x-axis.

Find the equation of the normal and the coordinates of its foot.

Watch Video Solution

30. Show that, the function $f(x) = |x - 1|$ is not differentiable at x = 1

Watch Video Solution

31. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

Watch Video Solution

32. Find the distance of the point (1,2,3) from the line

$$\frac{x-6}{2}=\frac{y-7}{2}=\frac{z-7}{-3}.$$

Watch Video Solution

33. ©answer any one question: (i) find the vector equation of the plane at a distance $\frac{6}{\sqrt{29}}$ unit from the origin and perpendicular to the vector $2\hat{i} - 3\hat{j} + 4\hat{k}$. Also convert this equation in cartesian form.

Watch Video Solution