



MATHS

BOOKS - MODERN PUBLISHERS MATHS (HINGLISH)

COMPLEX NUMBERS

Illustrative Examples

1. Evaluate the following

(i) i^9

(ii) i^{342}

(iii) i^{998}

(iv) i^{-63}

(v) $\left(i^3 + \frac{1}{i^3}\right)$



Watch Video Solution

2. Evaluate the following:

(i) $\sqrt{-25} \times (-81)$

(ii) $\sqrt{-36} \times \sqrt{16}$

(iii) $4\sqrt{-4} + 5\sqrt{-9} - 3\sqrt{-16}$



[Watch Video Solution](#)

3. Prove that:

(i) $1 + i^{10} + i^{100} - i^{1000} = 0$

(ii) $i^{107} + i^{112} + i^{117} + i^{122} = 0$

(iii) $(1 + i^{14} + i^{18} + i^{22})$ is real number.



[Watch Video Solution](#)

4. Prove that $i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0$ for all $n \in \mathbb{N}$



[Watch Video Solution](#)

5. Write the following as complex numbers:

(i) $\sqrt{-16}$

(ii) $-1 - \sqrt{-5}$

(iii) $-b + \sqrt{-4ac}$, ($a, c > 0$)



Watch Video Solution

6. Write the real and imaginary parts of the following complex numbers:

(i) $7 + 3i$

(ii) $0 + 7i$

(iii) $\sqrt{5} + 0i$



Watch Video Solution

7. Given: $a + ib = 2 - 3i$, find 'a' and 'b'.



Watch Video Solution

8. Find 'x' and 'y' such that $2x+3iy$ and $2+9i$ represent the same complex number.

 [Watch Video Solution](#)

9. Find 'x' and 'y' when:

$$(2y + 7) + (4 - 3x)i = 0.$$

 [Watch Video Solution](#)

10. Represent the following complex numbers in the complex plane:

(i) $1 + 2i$

(ii) $5 + 7i$

(iii) $0 + 0i$

(iv) i .

 [Watch Video Solution](#)

11. Write the complex numbers that represent the following points in the plane:

(i) (2,3)

(ii) (0,2)

(iii) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

 [Watch Video Solution](#)

12. Can two different points in the complex plane represent the same complex number? (Give reasons for your answer).

 [Watch Video Solution](#)

13. If $z_1 = 2 + 3i$ and $z_2 = 3 + i$, plot the number $z_1 + z_2$. Also show that:

$$|z_1| + |z_2| > |z_1 + z_2|.$$

 [Watch Video Solution](#)

 Watch Video Solution

14. Perform the indicated operations and write the result in the form $x+iy$:

(i) $(-3 + 2i) + (-6 + 3i)$

(ii) $\left(\frac{1}{2} + \frac{7}{2}i\right) - \left(4 + \frac{5}{2}i\right)$

(iii) $(1 - 2i) - i + (4 - 7i) - 2i + (5i + 3)$.

 Watch Video Solution

15. Prove that $|z| = |-z|$

 Watch Video Solution

16. Find the modulus of the complex number:

$$\frac{1}{2 - 3i}$$

 Watch Video Solution

17. if $z_1 = 3 + 4i$ and $z_2 = 12 - 5i$, verify:

(i) $|-z_1| = |z_1|$

(ii) $|z_1 + z_2| < |z_1| + |z_2|$

(iii) $|z_1 z_2| = |z_1| |z_2|$.

 [Watch Video Solution](#)

18. If z_1, z_2 are complex number such that $\frac{2z_1}{3z_2}$ is purely imaginary number, then find $\left| \frac{z_1 - z_2}{z_1 + z_2} \right|$.

 [Watch Video Solution](#)

19. If $z = x + iy$ and $w = \frac{1 - iz}{z - i}$, show that $|w| = 1$ is purely real.

 [Watch Video Solution](#)

20. If $(1 + i)(1 + 2i)(1 + 3i)(1 + ni) = (x + iy)$, show tht
2. 5. $10(1 + n^2) = x^2 + y^2$.

 [Watch Video Solution](#)

21. Express the following in polar form,

(i) $\sqrt{3} - i$

(ii) $-4 + 4\sqrt{3}i$.

 [Watch Video Solution](#)

22. Convert the following in the polar form:

$$\frac{1 + 7i}{(2 - i)^2}$$

 [Watch Video Solution](#)

23. Express $\sin 50^\circ + i\cos 50^\circ$ in the polar form. Also, find r and θ .



Watch Video Solution

24. Convert $4(\cos 300^\circ + i\sin 300^\circ)$ into Cartesian form.



Watch Video Solution

25. Give the following products in polar form:

$$[2(\cos 0^\circ + i\sin 0^\circ)][4\cos 90^\circ + i\sin 90^\circ]$$



Watch Video Solution

26. Write down the following complex number in polar form: Quotient of $12(\cos 150^\circ + i\sin 150^\circ)$ and $3(\cos 60^\circ + i\sin 60^\circ)$



Watch Video Solution

27. Show that $\arg. \bar{z} = 2\pi - \arg. z.$



Watch Video Solution

28. Solve the equation : $27x^2 - 10x + 1 = 0$



Watch Video Solution

29. Solve: $\sqrt{5}x^2 + x + \sqrt{5} = 0$



Watch Video Solution

30. Solve the following equations by using the general expression for a quadratic equation: $x^2 - 7ix - 12 = 0$



Watch Video Solution

31. Solve the following quadratic equation: $ix^2 - x + 12i = 0$



Watch Video Solution

[Watch Video Solution](#)

32. Solve the following equation by factorization method:

$$x^2(3\sqrt{2} + 2i)x + 6\sqrt{2}i = 0$$

[Watch Video Solution](#)

33. Solve the equation:

$$x^2 - (5 + i)x + (18 - i) = 0$$

[Watch Video Solution](#)

Exercise 5 A

1. Evaluate the following :

(i) i^7

(ii) i^{51}

(iii) $\frac{1}{i}$

(iv) i^{-71}

(v) $\left(i^{37} + \frac{1}{i^{67}}\right)$



Watch Video Solution

2. Evaluate the following:

(i) $\sqrt{-81}$

(ii) $(\sqrt{-2})^6$

(iii) $\sqrt{-25} \times \sqrt{-49}$

(iv) $\frac{2}{3} \times \sqrt{\frac{-9}{16}}$

(v) $\sqrt{\frac{-49}{25}} \times \sqrt{\frac{-1}{9}}$



Watch Video Solution

3. Simplify: $\sqrt{\frac{-x}{16}} + \sqrt{\frac{-x}{25}} - \sqrt{\frac{-x}{36}}$, where 'x' is a positive real number.



Watch Video Solution

4. Prove that:

$$(i) 1 + i^2 + i^4 + i^6 = 0$$

$$(ii) 1 + i^{10} + i^{100} + i^{1000} = 2$$

$$(iii) i^{104} + i^{109} + i^{114} + i^{119} = 0$$

$$(iv) 6i^{54} + 5i^{37} - 2i^{11} + 6i^{68} = 7i$$

$$(v) \frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} = -1$$



Watch Video Solution

5. Prove that:

$$(i) (1 - i)^2 = -2i$$

$$(ii) (1 + i)^4 \times \left(1 + \frac{1}{i}\right)^4 = 16$$

$$(iii) \left\{ i^{19} + \left(\frac{1}{i}\right)^{25} \right\}^2 = -4$$

$$(iv) i^{4n} + i^{4n+1} + i^{4n+2} + i^{4n+3} = 0$$

$$(v) 2i^2 + 6i^3 + 3i^{16} - 6i^{19} + 4i^{25} = 1 + 4i.$$



Watch Video Solution

6. Show that $1 + i^{10} + i^{20} + i^{30}$ is a real number.

 [Watch Video Solution](#)

7. Show that $(-\sqrt{-1})^{4n+3} = i$, where n is a positive integer.

 [Watch Video Solution](#)

Exercise 5 B

1. Write the following as complex numbers:

(i) $\sqrt{-27}$

(ii) \sqrt{x} , ($x > 0$)

 [Watch Video Solution](#)

2. Write the following as complex numbers:

(i) $1 + \sqrt{-1}$

(ii) $4 - \sqrt{-5}$

(iii) $\frac{\sqrt{3}}{2} - \frac{\sqrt{-2}}{7}$



[Watch Video Solution](#)

3. Write the real and imaginary parts of the following

(i) 7

(ii) $3i$

(iii) $\frac{\sqrt{5}}{7}i$



[Watch Video Solution](#)

4. Write the real and imaginary parts of the following

(i) $-2\sqrt{2}i$

$$(ii) \frac{-1}{5} + \frac{i}{5}$$

$$(iii) \sqrt{3} + \frac{\sqrt{2}}{76}i$$



Watch Video Solution

5. Write the real and imaginary parts of the following

$$(i) \sqrt{37} + \sqrt{-19}$$

$$(ii) \frac{\sqrt{17}}{2} + \frac{2}{\sqrt{70}}i.$$



Watch Video Solution

6. Find real number 'x' and 'y' such that

$$3x + 2iy - ix + 5y = 7 + 5i.$$



Watch Video Solution

7. Find a and b such that $2a + 14b$ and $2i$ represent the same complex number.

 [Watch Video Solution](#)

8. Find x and y if: $(3y - 2) + i(7 - 2x) = 0$

 [Watch Video Solution](#)

9. Find the values of ' x ' and ' y ' if:

(i) $3x + (2x - y)i = 6 - 3i$

(ii) $3x + 5iy = 5i$

(iii) $4x + i(3x - y) = 3 + i(-6)$

(iv) $\left(\frac{3}{\sqrt{5}}x - 5\right) + 2\sqrt{5}yi = \sqrt{2}$.

 [Watch Video Solution](#)

Exercise 5 C

1. (i) If $z=3+4i$, then find modulus of z i.e., $|z|$.

(ii) If $z=4+3i$, then find modulus of z i.e., $|z|$.



Watch Video Solution

2. Find the modulus and amplitude of the following:

(i) $\frac{1+i}{1-i}$

(ii) $\frac{1}{1+i}$.



Watch Video Solution

3. If two complex numbers z_1, z_2 are such that

$$|z_1| = |z_2|,$$

is it then necessary that $z_1 = z_2$?



Watch Video Solution

4. Write the complex numbers that represent the following points in the plane:

(i) (0,0)

(ii) (3,0)

(iii) (-1,0)

(iv) (0,-1)

(v) (1,-2)

(vi) (4,-1)

(vii) $\left(-\frac{1}{3}, \frac{1}{5}\right)$



[Watch Video Solution](#)

5. Plot the following numbers on a complex number plane and find their absolute values:

(a) 5

(b) $2i$

(c) (i) $4 - 3i$

(ii) $\frac{\sqrt{3}}{2} + \frac{1}{2}i$



Watch Video Solution

6. Plot all the complex numbers in the complex number plane whose absolute value is 4.



Watch Video Solution

7. For any complex number z , prove that:

(i) $-|z| \leq \operatorname{Re}(z) \leq |z|$

(ii) $-|z| \leq \operatorname{Im}(z) \leq |z|$.



Watch Video Solution

8. Let $z_1 = 2 - 3i$ and $z_2 = 5 + 12i$, verify that:

(i) $|z_1 + z_2| < |z_1| + |z_2|$

$$(ii) |z_2 - z_1| > |z_2| - |z_1|$$



Watch Video Solution

Exercise 5 D

1. In the following, perform the indicated operations and write the result in the form $x+iy$:

$$(i) (5 + 4i) + (5 - 4i)$$

$$(ii) -2i + \left(\frac{3}{2} - 4i\right)$$

$$(iii) \left(\frac{1}{5} + \frac{2}{5}i\right) - \left(4 + \frac{5}{2}i\right)$$

$$(iv) 3(7 + 7i) + i(7 + 7i).$$



Watch Video Solution

2. In the following, perform the indicated operations and write the result in the form $x+iy$:

$$(i) (7 - 2i) - (3 + 2i) + (7 + 8i)$$

$$(ii) 3 - 4i + 2i - (8 + 7i)$$

$$(iii) (7 - 2i) - (4 + i) + (-3 + 5i)$$



[Watch Video Solution](#)

3. In the following, perform the indicated operations and write the result in the form $x+iy$:

$$\sqrt{3} + (\sqrt{3} - 2i) - (3 - 2i)$$



[Watch Video Solution](#)

4. In the following, perform the indicated operations and write the result in the form $x+iy$:

$$i^3 + (6 + 3i) - (20 + 5i) + (14 + 3i).$$



[Watch Video Solution](#)

5. Find the sum of: $\frac{2}{3} + \frac{5}{3}i$, and $\frac{-5}{3} - i$.



Watch Video Solution

Exercise 5 E Short Answer Type Questions

1. Write the following in the form $x+iy$:

(i) $(2i)^3$

(ii) i^{-35}

(iii) $(-i)(2i)\left(-\frac{1}{8}i\right)^3$.



Watch Video Solution

2. Express the following in the form $a + ib$: $(-5i)\left(\frac{1}{8}i\right)$



Watch Video Solution

3. Write the following in the form $x+iy$:

(i) $i + i^2 + i^3 + i^4$

$$(ii) i^4 + i^8 + i^{12} + i^{16}$$

$$(iii) i + i^5 + i^9 + i^{13}$$

$$(iv) i^9 + i^{10} + i^{11} + i^{12}.$$



Watch Video Solution

4. Express the result in the form $x+iy$, where x,y are real number

$$i = \sqrt{-1}:$$

$$(i) (-5 + 3i)(8 - 7i)$$

$$(ii) (-\sqrt{3} + \sqrt{-2})(2\sqrt{3} - i)$$

$$(iii) (\sqrt{2} - \sqrt{3}i)^2$$



Watch Video Solution

5. Express the result in the form $x+iy$, where x,y are real number

$$i = \sqrt{-1}:$$

$$(i) 3i^3(15i^6)$$

$$(ii) (2 + 7i)^3$$

$$(iii) \left(\frac{1}{2} + 2i\right)^3$$



Watch Video Solution

6. Express the result in the form $x+iy$, where x,y are real number

$$i = \sqrt{-1}:$$

$$(i) \frac{2 - 3i}{4 - i}$$

$$(ii) \frac{2 + 3i}{-5 - 4i}$$

$$(iii) \frac{1 + i}{3 + i}$$

$$(iv) \frac{3 + 2i}{4 - 3i}$$



Watch Video Solution

7. Express the result in the form $x+iy$, where x,y are real number

$$i = \sqrt{-1}:$$

$$(i) (5 + 9i) \div (-3 + 4i)$$

$$(ii) \left[\left(\sqrt{5} + \frac{i}{2} \right) (\sqrt{5} - 2i) \right] \div (6 + 5i)$$

$$(iii) \frac{(1-i)(2-i)(3-i)}{1+i}$$

$$(iv) \frac{1+3i}{(1-2i)^2}$$



Watch Video Solution

8. Express the result in the form $x+iy$, where x,y are real number

$$i = \sqrt{-1}:$$

$$(i) \frac{5 + \sqrt{2}i}{1 - \sqrt{2}i}$$

$$(ii) \frac{2+i}{(1+i)(1-2i)}$$



Watch Video Solution

9. Perform the following by the indicated operations. Express the result

in the form $x+iy$, where x,y are real number $i = \sqrt{-1}$:

Simply:

$$(i) (\sqrt{5} + 7i)(\sqrt{5} - 7i)^3$$

$$(ii) (1+i)(2+3i)(3+4i)(4+5i)$$

$$(iii) (1+i^2) + i^4 + i^6.$$



Watch Video Solution

10. Perform the following by the indicated operations. Express the result in the form $x+iy$, where x,y are real number $i = \sqrt{-1}$:

Prove that: $\left[\frac{\sqrt{7} + i\sqrt{3}}{\sqrt{7} - i\sqrt{3}} + \frac{\sqrt{7} - i\sqrt{3}}{\sqrt{7} + i\sqrt{3}} \right]$ is real.



Watch Video Solution

11. Compute: (i) $\sqrt{-49}(2 + \sqrt{-9})$

(ii) $[2 + \sqrt{-25}] - [3 - \sqrt{-16}] + [1 - \sqrt{-9}]$.



Watch Video Solution

12. Find real q such that $\frac{3 + 2i \sin \theta}{1 - 2i \sin \theta}$ is purely real.



Watch Video Solution

13. (a) Write the conjugates of the following:

(i) $3+i$

(ii) $3-i$

(iii) $-\sqrt{5} - \sqrt{7}i$

(iv) $-\sqrt{5}i$

(v) $\frac{4}{5}$

(vi) $49 - \frac{i}{7}$

(vii) $\frac{1-i}{1+i}$

(viii) $(1+i)^2$

(ix) $(2+5i)^2$

(x) $\left(-2 - \frac{1}{3}i\right)^3$

(b) Find the real number x and y if:

(i) $(x - iy)(3 + 5i)$ is the conjugate of $-6-24i$

(ii) $-3 + ix^2y$ and $x^2 + y + 4i$ are conjugate of each other.



Watch Video Solution

14. Find the reciprocal of :

(i) $7 + \sqrt{7}i$

(ii) $i - 5$.



Watch Video Solution

15. Prove that:

(i) $z = \bar{z}$ iff z is real

(ii) $z = -\bar{z}$ iff z is either zero or purely imaginary.



Watch Video Solution

16. Prove that: $R_e z = \frac{z + \bar{z}}{2}$, $I_m z = \frac{z - \bar{z}}{2i}$



Watch Video Solution

17. Prove that: (i) the sum of a complex number and its conjugate is real
(ii) the product of a complex number and its conjugate is real.

 [Watch Video Solution](#)

18. Give an example to show that subtraction of complex number is not commutative.

 [Watch Video Solution](#)

19. Find the sum and product of the complex numbers $-\sqrt{3} + \sqrt{-2}$ and $2\sqrt{3} - i$

 [Watch Video Solution](#)

20. If z_1, z_2 are respectively $1 - i, -2 + 4i$, find $I_m \left\{ \frac{z_1 z_2}{z_1} \right\}$.

 [Watch Video Solution](#)

21. Find the values of x and y if:

$$(i) (x + iy)(1 + i) = 1 - i$$

$$(ii) \frac{(1 + i)x - 2i}{3 + i} + \frac{(2 - 3i)y + i}{3 - i} = i, \text{ where } i = \sqrt{-1}.$$



Watch Video Solution

22. If $1 + 4\sqrt{3}i = (1 + ib)^2$, prove that:

$$a^2 - b^2 = 1 \text{ and } ab = 2\sqrt{3}.$$



Watch Video Solution

23. Show that if $a, b, c, d \in R$,

$$\overline{(a + ib)(c + id)} = (a - ib)(c - id).$$



Watch Video Solution

Exercise 5 E Long Answer Type Questions

1. (i) If $x + iy = \frac{a + ib}{a - ib}$, prove that $x^2 + y^2 = 1$.

(ii) If $\frac{(a + i)^2}{2a - i} = p + iq$, prove that:

$$p^2 + q^2 = \frac{(a^2 + 1)^2}{4a^2 + 1}.$$



Watch Video Solution

2. If $x + iy = \sqrt{\frac{a + ib}{c + id}}$, prove that $(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$



Watch Video Solution

3. Define addition and multiplication of two complex numbers

z_1 and z_2 . Hence show that:

(i) $R_e(z_1 + z_2) = R_e(z_1) + R_e(z_2)$

(ii) $I_m(z_1 + z_2) = I_m(z_1) + I_m(z_2)$

$$(iii) R_e(z_1 z_2) = R_e(z_1)R_e(z_2) - I_m(z_1)I_m(z_2)$$

$$(iv) I_m(z_1 z_2) = R_e(z_1)I_m(z_2) + I_m(z_1)R_e(z_2).$$

 [Watch Video Solution](#)

Exercise 5 F Short Answer Type Questions

1. Find the additive inverse of the complex number:

(i) $-5 + 7i$

(ii) $(\sqrt{6} + 5i)(\sqrt{6} + 5i)$

 [Watch Video Solution](#)

2. find the difference of the complex numbers:

$$z_1 = -3 + 2i \text{ and } z_2 = 13 - i.$$

 [Watch Video Solution](#)

3. Find the multiplicative inverse of the following complex numbers:

(i) $4 - 3i$

(ii) $3 + 2i$

 [Watch Video Solution](#)

Exercise 5 F Long Answer Type Questions

1. Find the multiplicative inverse of the product of complex numbers:

$3 + 4i, 5 - 12i.$

 [Watch Video Solution](#)

2. If z is any non-zero complex number, prove that the multiplicative inverse of z is $\frac{\bar{z}}{|z|^2}$.

Hence, express $(4 - \sqrt{-9})^{-1}$ in the form $x+iy$, where $x, y \in \mathbb{R}$

 [Watch Video Solution](#)

Exercise 5 G Short Answer Type Questions

1. Find the square roots of the following

(i) -31

(ii) -32

(iii) -36

(iv) -144

(v) $-\frac{16}{25}$

(vi) $-\frac{8}{729}$.



[Watch Video Solution](#)

2. Explain the fallacy in the following:

$$-1 = i \cdot i = \sqrt{-1} \cdot \sqrt{-1} = \sqrt{(-1)(-1)} = \sqrt{1} = 1$$



[Watch Video Solution](#)

Exercise 5 G Long Answer Type Questions

1. find the square roots of the following:

(i) $2i$

(ii) $-i$



[Watch Video Solution](#)

2. find the square roots of the following:

(i) $7 + 24i$

(ii) $7 - 24i$

(iii) $-7 - 24i$

(iv) $-7 + 24i$.



[Watch Video Solution](#)

3. find the square roots of the following:

(i) $1 - i$

(ii) $1 + i$.

 [Watch Video Solution](#)

4. find the square roots of the following:

(i) $-8 - 6i$

(ii) $5 - 12i$.

 [Watch Video Solution](#)

5. Find the square roots of the following:

(i) $3 - 4\sqrt{7}i$

(ii) $4 + 6\sqrt{-5}$

(iii) $-2 + 2\sqrt{3}i$.

 [Watch Video Solution](#)

6. Evaluate:

$$\sqrt{7 - 30\sqrt{-2}}$$



Watch Video Solution

7. Evaluate:

(i) $x^2 + 4x + 7$ when $x = 2 + \sqrt{-3}$

(ii) $2x^3 - 9x^2 - 10x + 13$ when $x = 3 + \sqrt{-5}$



Watch Video Solution

Exercise 5 H Long Answer Type Questions

1. If $z_1 = 5 - 7i$ and $z_2 = 7 - 9i$, verify:

(i) $|-z_1| = |z_1|$

(ii) $|z_1 z_2| = |z_1| |z_2|$



Watch Video Solution

2. If z is a complex number, show that $|z|^2 = |z^2|$

 [Watch Video Solution](#)

3. Show that if $iz^3 + z^2 - z + i = 0$, then $|z| = 1$

 [Watch Video Solution](#)

4. Find non zero integral solutions of $|1 - i|^x = 2^x$.

 [Watch Video Solution](#)

5. If $|z_1| = |z_2| = \dots = |z_n| = 1$, prove that

$$|z_1 + z_2 + z_3 + \dots + z_n| = \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \dots + \frac{1}{z_n}.$$

 [Watch Video Solution](#)

6. If z is a complex number such that $|z| = 1$, prove that $\frac{z-1}{z+1}$ is purely imaginary, what will be your conclusion if $z = 1$?



Watch Video Solution

7. If z_1 and z_2 ($\neq 0$) are two complex numbers, prove that:

(i) $|z_1 z_2| = |z_1| |z_2|$

(ii) $\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}, z_2 \neq 0.$



Watch Video Solution

Exercise 5 | Short Answer Type Questions

1. Convert each of the following complex numbers in polar form:

(i) -3

(ii) $\sqrt{3} + i$

(iii) $i.$



Watch Video Solution

2. Convert the following in polar form:

$$\frac{1 + 3i}{1 - 2i}$$

 [Watch Video Solution](#)

3. Find the absolute values and arguments of the following complex numbers:

(i) $3(\cos 60^\circ + i\sin 60^\circ)$

(ii) $i\cos\frac{\pi}{6} + \sin\frac{\pi}{6}$

(iii) $1 - \cos\theta + i\sin\theta$.

 [Watch Video Solution](#)

4. If two complex numbers z_1 and z_2 are such that $|z_1| = |z_2|$, is it then necessary that $z_1 = z_2$?

 [Watch Video Solution](#)

Exercise 5 | Long Answer Type Questions

1. Give the following in polar form:

(i) $\sin 120^\circ - i\cos 120^\circ$

(ii) $[2\cos 210^\circ + i\sin 210^\circ][4(\cos 120^\circ + i\sin 120^\circ)]$

(iii) $[3(\cos 225^\circ + i\sin 225^\circ)][6(\cos 45^\circ + i\sin 45^\circ)]$.



[Watch Video Solution](#)

2. Give the following quotients in polar form:

(i) $\frac{9(\cos 90^\circ + i\sin 90^\circ)}{3(\cos 45^\circ + i\sin 45^\circ)}$

(ii) $\frac{7(\cos 135^\circ + i\sin 135^\circ)}{14(\cos 90^\circ + i\sin 90^\circ)}$.



[Watch Video Solution](#)

3. Express the following in the polar form:

(i) $1 + i$

(ii) $-1 - i$

(iii) $1 + \sqrt{3}i$

(iv) $\frac{-16}{1 + \sqrt{3}i}$



Watch Video Solution

4. Show that for $z \in \mathbb{C}$, $|z| = 0$ if and only if $z = 0$



Watch Video Solution

5. If $z = x + iy$ prove that $|x| + |y| \leq \sqrt{2}|z|$



Watch Video Solution

6. Prove that:

(i) $R_e(z_1 z_2) = R_e(z_1)R_e(z_2) - I_m(z_1)I_m(z_2)$

(ii) $I_m(z_1 z_2) = R_e(z_1)I_m(z_2) + R_e(z_2)I_m(z_1)$.



Watch Video Solution

 [Watch Video Solution](#)

7. If $z_1, z_2 \in \mathbb{C}$, show that $(z_1 + z_2)^2 = z_1^2 + 2z_1z_2 + z_2^2$.

 [Watch Video Solution](#)

8. Prove that $|z_1 + z_2|^2 = |z_1|^2$, if z_1/z_2 is purely imaginary.

 [Watch Video Solution](#)

Exercise 5 J Long Answer Type Questions

1. Solve $x^2 + 2 = 0$.

 [Watch Video Solution](#)

2. Solve $x^2 + x + 1 = 0$.



Watch Video Solution

3. Solve the Equation:

(i) $x^2 + 3x + 5 = 0$

(ii) $x^2 - x + 2 = 0$.



Watch Video Solution

4. Solve: $\sqrt{5}x^2 + x + \sqrt{5} = 0$



Watch Video Solution

5. Solve the Equation:

(i) $x^2 + x + \frac{1}{\sqrt{2}} = 0$

(ii) $x^2 + \frac{x}{\sqrt{2}} + 1 = 0$

(iii) $x^2 + \frac{x}{2} + 1 = 0$.



Watch Video Solution

6. Solve the Equation:

$$x^2 - x + 1 + i = 0.$$

 [Watch Video Solution](#)

7. Solve the Equation:

$$x^2 - (2 + i)x = 1 - 7i$$

 [Watch Video Solution](#)

8. Solve the equation $2x^2 + 3ix = 2 = 0$ using the general expression for a quadratic equation.

 [Watch Video Solution](#)

9. Solve the following quadratic equation: $x^2 + 4ix - 4 = 0$



Watch Video Solution

10. Solve the following equation by factorization method:

$$x^2(3\sqrt{2} + 2i)x + 6\sqrt{2}i = 0$$



Watch Video Solution

11. Solve

(i) $x^2 - (3\sqrt{2} - 2i)x - 6\sqrt{2}i = 0$



Watch Video Solution

12. Solve the following quadratic equation:

$$2x^2 - (3 + 7i)x + (9i - 3) = 0$$



Watch Video Solution

Objective Type Questions Multiple Choice Question A

1. Evaluate $1 + i^2 + i^4 + i^6 + \dots + i^{2n}$.

A. positive

B. negative

C. 0

D. cannot be evaluated.

Answer: D



[Watch Video Solution](#)

2. Number of solutions of the equation $z^2 + |z|^2 = 0$, where $z \in \mathbb{C}$, is

A. 1

B. 2

C. 3

D. infinitely many.

Answer: D

 [Watch Video Solution](#)

3. Find the amplitude of $\sin\left(\frac{\pi}{5}\right) + i\left(1 - \cos\left(\frac{\pi}{5}\right)\right)$

A. $\sqrt{5} + \frac{1}{2}$

B. $\sqrt{5} - \frac{1}{2}$

C. $\frac{\sqrt{5} + 1}{2}$

D. $\frac{\sqrt{5} - 1}{2}$

Answer: D

 [Watch Video Solution](#)

4. If $z = x + iy$ lies in III quadrant, then $\frac{\bar{z}}{z}$ also lies in III quadrant If:

A. $x > y > 0$

B. $x < y < 0$

C. $y < x < 0$

D. $y > x > 0$.

Answer: C



Watch Video Solution

5. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then

A. $x = 2n + 1$

B. $x = 4n$

C. $x = 2n$

D. $x = 4n + 1$.

Answer: B





Watch Video Solution

6. $i^9 + i^{19}$

A. 1

B. 2

C. 0

D. none of these

Answer: C



Watch Video Solution

7. For a positive integer n , find the value of $(1 - i)^n \left(1 - \frac{1}{i}\right)^n$

A. 2^n

B. $\frac{1}{2^n}$

C. 0

D. 1

Answer: A



[Watch Video Solution](#)

8. Simplify the following: i^{-35}

A. i

B. $-i$

C. 1

D. -1

Answer: A



[Watch Video Solution](#)

9. i^{-39}

A. i

B. $-i$

C. 1

D. -1

Answer: A



Watch Video Solution

10. Express of the complex number in the form $a + ib = (5i) \left(-\frac{3}{5}i \right)$

A. $2i$

B. i^7

C. 3

D. none of these

Answer: C



Watch Video Solution

11. Evaluate $1 + i + i^2 + i^3$

A. i

B. 0

C. $-i$

D. 1

Answer: B



Watch Video Solution

12. The complex conjugate of $-21+20i$ is:

A. $-21 - 20i$

B. $21 - 20i$

C. $-21 + 20i$

D. none of these

Answer: A

 [Watch Video Solution](#)

13. Solve the equation: $x^2 + 3 = 0$

A. -3

B. $-\sqrt{3}$

C. $\pm\sqrt{3}i$

D. none of these

Answer: C

 [Watch Video Solution](#)

14. Complex conjugate of $3i-4$ is:

A. $3i + 4$

B. $-3i - 4$

C. $-3i + 4$

D. none of these

Answer: B



[Watch Video Solution](#)

15. Modulus of complex number $3i-4$ is:

A. -1

B. 25

C. 5

D. -7

Answer: C



Watch Video Solution

16. Additive inverse of complex number $-4 - 7i$ is:

A. $4 + 7i$

B. $-4 + 7i$

C. $-4 - 7i$

D. none of these

Answer: A



Watch Video Solution

17. If $\sqrt{a + ib} = x + iy$, then value of $\sqrt{a - ib}$ is:

A. $x + iy$

B. $x - iy$

C. $\sqrt{x^2 - y^2}$

D. $\sqrt{x^2 + y^2}$.

Answer: B

 [Watch Video Solution](#)

18. If $i^2 = -1$, then the sum $i + i^2 + i^3 + \dots$ upto 1000 terms is equal to

A. -1

B. 0

C. 1

D. $-i$

Answer: B

 [Watch Video Solution](#)

19. The least positive integer n such that $\left(\frac{2i}{1+i}\right)^n$ is a positive integer is

- A. 4
- B. 8
- C. 12
- D. 16

Answer: B



[Watch Video Solution](#)

20. If $a = 1 + i$, then a^2 equals $1 - i$ b. $2i$ c. $(1 + i)(1 - i)$ d. $(i - 1)$

- A. $2i$
- B. $1 - i$

C. $1 + i$

D. $i - 1$

Answer: A



Watch Video Solution

21. If $x + iy = (1 + i)(1 + 2i)(1 + 3i)$, then $x^2 + y^2$ equals:

A. -1

B. 0

C. 1

D. 100

Answer: D



Watch Video Solution

22. If θ is the amplitude of $\frac{a + ib}{a - ib}$, then $\tan \theta$ equals:

A. $\frac{a}{a^2 + b^2}$

B. $\frac{b}{a^2 + b^2}$

C. $\frac{2ab}{a^2 - b^2}$

D. $\frac{2a}{a^2 + b^2}$

Answer: C



Watch Video Solution

23. For the equation $|x^2| + |x| - 6 = 0$, the sum of the real roots is

A. 0

B. 1

C. 2

D. 4

Answer: A



Watch Video Solution

24. $|2x - x^2 - 3| = 1$

A. 2

B. 4

C. 3

D. 0

Answer: B



Watch Video Solution

25. The number of solutions of $x^2 + |x - 1| = 1$ is

A. 1

B. 0

C. 2

D. 3

Answer: B

 [Watch Video Solution](#)

Objective Type Questions Fill In The Blanks B

1. Evaluate : $\left[i^{18} + \left(\frac{1}{i} \right)^{25} \right]^3$

 [Watch Video Solution](#)

2. Evaluate : $\sqrt{-16} + 3\sqrt{-25} + \sqrt{-36} - \sqrt{-625}$.

 [Watch Video Solution](#)

3. If $\frac{3 + 4i}{2 - 4i} = x + iy$, then $x = \dots$. And $y = \dots$

 [Watch Video Solution](#)

4. If $z_1 = 2 - i$ and $z_2 = -2 + i$, then $Re\left(\frac{1}{z_1 z_2}\right) = \dots$

 [Watch Video Solution](#)

5. The multiplicative inverse of $3+4i$ is ...

 [Watch Video Solution](#)

6. The modulus of the complex number $\frac{1}{2 - 3i}$ is ...

 [Watch Video Solution](#)

7. Show that if $iz^3 + z^2 - z + i = 0$, then $|z| = 1$

 [Watch Video Solution](#)

8. If $\sqrt{3} - i = r(\cos \theta + i \sin \theta)$, then $r = \dots$

 [Watch Video Solution](#)

9. The polar form of $\sin 50^\circ + i \cos 50^\circ$ is ...

 [Watch Video Solution](#)

10. The roots of $x^2 - 7ix - 12 = 0$ are ... And

 [Watch Video Solution](#)

1. $\left[i^{17} - \left(\frac{1}{i} \right)^{34} \right]^2 = 2i.$

 [Watch Video Solution](#)

2. Given $a+ib=2-3i$, then $a=2$ and $b=-3$.

 [Watch Video Solution](#)

3. If $z=2-3i$, then show that $z^2 - 4z + 13 = 0$.

 [Watch Video Solution](#)

4. For any complex number z , the product $z\bar{z}$ is always a positive real number.

 [Watch Video Solution](#)

5. If $z_1 = 5 + 7i$, then $|-z_1| = |z_1|$. is *True*

 [Watch Video Solution](#)

Objective Type Questions Very Short Answer Type Questions D

1. Find the value of $i^2 + (-i)^4 - i^6$.

 [Watch Video Solution](#)

2. Simplify: $\sqrt{\frac{-x}{16}} + \sqrt{\frac{-x}{25}} - \sqrt{\frac{-x}{36}}$, where 'x' is a positive real number.

 [Watch Video Solution](#)

3. For any positive integer n, find the value of

$$i^n + i^{n+1} + i^{n+2} + i^{n+3} + i^{n+4} + i^{n+5} + i^{n+6} + i^{n+7}.$$



Watch Video Solution

4. Find the multiplicative inverse of the complex number $2 + 9i$



Watch Video Solution

5. Write the following in the form $x+iy$:

$$i^9 + i^{10} + i^{11} + i^{12}.$$



Watch Video Solution

6. Write the square of $\frac{i}{1+i}$ in the form $x+iy$.



Watch Video Solution

7. Write the conjugate of $\left(-2 - \frac{1}{3}i\right)^3$.



[Watch Video Solution](#)

8. Write the additive inverse of $(\sqrt{6} + 5i) \left(\sqrt{6} - \frac{1}{5}i \right)$.

[Watch Video Solution](#)

9. Find the multiplicative inverse of the complex number $4 - 3i$

[Watch Video Solution](#)

10. If $x + iy = \sqrt{\frac{a + ib}{c + id}}$, then find the value of $x^2 + y^2$.

[Watch Video Solution](#)

11. If n is any positive integer, write the value of $\frac{i^{4n+1} - i^{4n-1}}{2}$.

[Watch Video Solution](#)

12. If $z=3+4i$, then find $|z|$ and z^{-1} .

 [Watch Video Solution](#)

13. Find the square roots of the following: i

 [Watch Video Solution](#)

14. Evaluate $x^2 + 4x + 7$ when $x = -2 + \sqrt{-3}$.

 [Watch Video Solution](#)

15. Write the argument of $-i$.

 [Watch Video Solution](#)

16. Find the principal argument of $(1 + i\sqrt{3})^2$.

 [Watch Video Solution](#)

17. Find the value of z , if $|z| = 4$ and $\arg(z) = \frac{5\pi}{6}$.

 [Watch Video Solution](#)

18. Write the value of $\arg(z) + \arg(z)$.

 [Watch Video Solution](#)

19. Write the argument of $(1 + \sqrt{3})(1 + i)(\cos \theta + i \sin \theta)$.

 [Watch Video Solution](#)

20. Write $-1 + \sqrt{3}$ in polar form.

 [Watch Video Solution](#)

21. If $\pi < \theta < 2\pi$ and $z = 1 + \cos \theta + i \sin \theta$, then find the value of $|z|$.

 [Watch Video Solution](#)

22. Write the least positive integral value of n for which $\left(\frac{1+i}{1-i}\right)^n$ is real.

 [Watch Video Solution](#)

23. Solve $x^2 + 2 = 0$.

 [Watch Video Solution](#)

24. Solve the following quadratic equation: $x^2 + 4ix - 4 = 0$

 [Watch Video Solution](#)

25. If $|z - 3i| = |z + 3i|$, then find the locus of z .

 [Watch Video Solution](#)

Ncert Exercise 5 1

1. Express of the complex number in the form $a + ib$. $(5i) \left(-\frac{3}{5}i \right)$

 [Watch Video Solution](#)

2. $i^9 + i^{19}$

 [Watch Video Solution](#)

3. Express of the complex number in the form $a + ib$.

 [Watch Video Solution](#)

4. Express of the complex number in the form $a + ib$.

$$3(7 + i7) + i(7 + i7)$$

 [Watch Video Solution](#)

5. Express of the complex number in the form $a + ib$.

$$(1 - i) - (-1 + i6)$$

 [Watch Video Solution](#)

6. Express of the complex number in the form $a + ib$.

$$\left(\frac{1}{5} + i\frac{2}{5}\right) - \left(4 + i\frac{5}{2}\right)$$

 [Watch Video Solution](#)

Watch video Solution

7. Express of the complex number in the form $a + ib$.

$$\left[\left(\frac{1}{3} + i \frac{7}{3} \right) + \left(4 + i \frac{1}{3} \right) \right] - \left(-\frac{4}{3} + i \right)$$

 Watch Video Solution

8. Express of the complex number in the form $a + ib.(1 - i)^4$

 Watch Video Solution

9. Express of the complex number in the form $a + ib.\left(\frac{1}{3} + 3i\right)^3$

 Watch Video Solution

10. Express of the complex number in the form $a + ib.\left(-2 - \frac{1}{3}i\right)^3$

 Watch Video Solution

11. Find the multiplicative inverse of the complex number $4 - 3i$

 [Watch Video Solution](#)

12. Find the multiplicative inverse of the complex number $\sqrt{5} + 3i$

 [Watch Video Solution](#)

13. Find multiplicative inverse of the following: $-i$

 [Watch Video Solution](#)

14. Express the following expression in the form of $a + ib$

$$\frac{(3 + i\sqrt{5})(3 - i\sqrt{5})}{(\sqrt{3} + \sqrt{2}i) - (\sqrt{3} - i\sqrt{2})}$$

 [Watch Video Solution](#)

Ncert Exercise 5 2

1. $z = -1 - i\sqrt{3}$



Watch Video Solution

2. Find the modulus and the arguments of the complex number

$$z = -\sqrt{3} + i$$



Watch Video Solution

3. Convert of the complex number in the polar form: $1 - i$



Watch Video Solution

4. Convert of the complex number in the polar form: $1 + i$

 [Watch Video Solution](#)

5. Convert of the complex number in the polar form: $1 - i$

 [Watch Video Solution](#)

6. Convert of the complex number in the polar form: 3

 [Watch Video Solution](#)

7. Convert of the complex number in the polar form: $\sqrt{2} + i$

 [Watch Video Solution](#)

8. Convert of the complex number in the polar form: i

 [Watch Video Solution](#)

Ncert Exercise 5.3

1. solve each of the following equations: $x^2 + 3 = 0$

 [Watch Video Solution](#)

2. Solve each of the following equations: $2x^2 + x + 1 = 0$

 [Watch Video Solution](#)

3. Solve the equation: $x^2 + 3x + 9 = 0$

 [Watch Video Solution](#)

4. Solve the following quadratic: $-x^2 + x - 2 = 0$

 [Watch Video Solution](#)

5. Solve the equation: $x^2 + 3x + 5 = 0$

 [Watch Video Solution](#)

6. Solve the equation: $x^2 - x + 2 = 0$

 [Watch Video Solution](#)

7. Solve the following quadratic: $\sqrt{2}x^2 + x + \sqrt{2} = 0$

 [Watch Video Solution](#)

8. Solve the equation: $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

 [Watch Video Solution](#)

9. Solve the following quadratic: $x^2 + x + \frac{1}{\sqrt{2}} = 0$

 [Watch Video Solution](#)

10. Solve the following quadratic: $x^2 + \frac{x}{\sqrt{2}} + 1 = 0$

 [Watch Video Solution](#)

Miscellaneous Exercise On Chapter

1. Evaluate : $\left[i^{18} + \left(\frac{1}{i} \right)^{25} \right]^3$

 [Watch Video Solution](#)

2. Prove that $Re(z_1 z_2) = Rez_1 Rez_2 - Imz_1 Imz_2$,

 [Watch Video Solution](#)

3. Reduce $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ to the standard form.

 [Watch Video Solution](#)

4. If $x - iy = \sqrt{\frac{a - ib}{c - id}}$ prove that $x^2 + y^2 = \frac{a^2 + b^2}{c^2 + d^2}$

 [Watch Video Solution](#)

5. Convert the following in the polar form : (i) $\frac{1+7i}{(2-i)^2}$ (ii) $\frac{1+3i}{1-2i}$

 [Watch Video Solution](#)

6. Solve the equation : $3x^2 - 4x + \frac{20}{3} = 0$

 [Watch Video Solution](#)

7. Solve the equation : $x^2 - 2x + \frac{3}{2} = 0$

 [Watch Video Solution](#)

8. Solve the equation : $27x^2 - 10x + 1 = 0$

 [Watch Video Solution](#)

9. Solve the equation : $21x^2 - 28x + 10 = 0$

 [Watch Video Solution](#)

10. If $z_1 = 2 - i$, $z_2 = 1 + i$, find $\left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + i} \right|$



Watch Video Solution

11. If $a + ib = \frac{(x + i)^2}{2x^2 + 1}$, prove that $a^2 + b^2 = \frac{(x^2 + 1)^2}{(2x^2 + 1)^2}$



Watch Video Solution

12. Let $z_1 = 2 - i$, $z_2 = -2 + i$. Find (i) $\operatorname{Re} \left(\frac{z_1 z_2}{\bar{z}_1} \right)$ (ii) $\operatorname{Im} \left(\frac{1}{z_1 \bar{z}_1} \right)$



Watch Video Solution

13. Find the modulus and argument of the complex number $\frac{1 + 2i}{1 - 3i}$.



Watch Video Solution

14. Find the real numbers x and y , if $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$.

 [Watch Video Solution](#)

15. Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$

 [Watch Video Solution](#)

16. If $(x + iy)^3 = u + iv$, then show that $\frac{u}{x} + \frac{v}{y} = (x^2 - y^2)$.

 [Watch Video Solution](#)

17. If α and β are different complex numbers with $|\beta| = 1$, then find

$$\left| \frac{\beta - \alpha}{1 - \bar{\alpha}\beta} \right|.$$

 [Watch Video Solution](#)

18. Find the number of non-zero integral solution of the equation

$$|1 - i|^x = 2^x$$



Watch Video Solution

19. If $(a + ib)(c + id)(e + if)(g + ih) = A + iB$, then show that $(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$



Watch Video Solution

20. If $\left(\frac{1+i}{1-i}\right)^m = 1$, then find the least integral value of m



Watch Video Solution

Questions From Ncert Exemplar

1. If $(x + iy)^{1/3} = a + ib$, where $x, y, a, b \in R$ show that

$$\frac{x}{a} - \frac{y}{b} = -2(a^2 + b^2).$$



Watch Video Solution

2. Solve the equation: $z^2 + z + 1 = 0$. Where $z=x+iy$.

 [Watch Video Solution](#)

3. If the imaginary part of $\frac{2z + 1}{iz + 1}$ is -2 , then show that the locus of the point representing z in the argand plane is a straight line.

 [Watch Video Solution](#)

4. Let z_1 and z_2 be two complex numbers such that $\bar{z}_1 + \overline{iz_2} = 0$ and $\arg(z_1 z_2) = \pi$ then find $\arg(z_1)$

 [Watch Video Solution](#)

5. If z_1 and z_2 , are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$ then $\arg(z_1) - \arg(z_2)$ is equal to (1) 0 (2) $-\frac{\pi}{2}$
(3) $\frac{\pi}{2}$ (4) $-\pi$



[Watch Video Solution](#)

6. If z_1, z_2, z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = \left| \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} \right| = 1$ then $|z_1 + z_2 + z_3|$ is equal to



[Watch Video Solution](#)

7. If a complex number z lies in the interior or on the boundary of a circle of radius 3 and center at $(-4, 0)$, then the greatest and least values of $|z + 1|$ are



[Watch Video Solution](#)

8. The value of p for which the difference between the roots of the equation $x^2 + px + 8 = 0$ is 2, are



[Watch Video Solution](#)

9. Find the value of 'k' if for the complex numbers z_1 and z_2 .

$$|1 - \overline{z_1}z_2|^2 - |z_1 - z_2|^2 = k(1 - |z_1|^2)(1 - |z_2|^2).$$

 [Watch Video Solution](#)

10. If z_1 and z_2 both satisfy the relation

$$z + \bar{z} = 2|z - 1| \text{ and } \arg(z_1 - z_2) = \frac{\pi}{4}, \text{ then } \operatorname{Im}(z_1 + z_2) \text{ equals}$$

 [Watch Video Solution](#)

Exercise

1. If $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = x + iy, f \in d(x, y)$

 [Watch Video Solution](#)

2. If $a = \cos \theta + is \int h \eta$, find the value of $\frac{1+a}{1-a}$.

 [Watch Video Solution](#)

3. If the real part of $\frac{z+2}{z-1}is4$, then show that the locus of the point representing z in the complex plane is a circle.

 [Watch Video Solution](#)

4. Show that the complex number z , satisfying $\frac{z-1}{z+1} = \frac{\pi}{4}$ lies on a circle.

 [Watch Video Solution](#)

5. Show that $\left| \frac{z-2}{z-3} \right| = 2$ represents a circle, find its centre and radius.

 [Watch Video Solution](#)

6. If $\frac{z-1}{z+1}$ is a purely imaginary number ($z \neq -1$), then find the value of $|z|$.

 [Watch Video Solution](#)

7. If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers then $\arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right) =$

 [Watch Video Solution](#)

8. If the complex number Z_1 and Z_2 , $\arg(Z_1) - \arg(Z_2) = 0$. then show that $|z_1 - z_2| = |z_1 - z_2|$.

 [Watch Video Solution](#)

9. Convert the complex number $z = \frac{i-1}{\frac{\cos \pi}{3} + i \frac{\sin \pi}{3}}$ in the polar form.

 [Watch Video Solution](#)

10. If z and w are two complex numbers such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$, then show that $zw = -i$.

 [Watch Video Solution](#)

Revision Exercise

1. If $z_1 = 2 - i$, $z_2 = 1 + i$, find $\left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + i} \right|$

 [Watch Video Solution](#)

2. If $a + ib = \frac{(x + i)^2}{2x^2 + 1}$, prove that $a^2 + b^2 = \frac{(x^2 + 1)^2}{(2x^2 + 1)^2}$

 [Watch Video Solution](#)

3. Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$

 [Watch Video Solution](#)

4. Find the modulus and argument of the complex number $\frac{1+2i}{1-3i}$.

 [Watch Video Solution](#)

5. Convert $z = \frac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$ in polar form.

 [Watch Video Solution](#)

6. If $z_1 = -3i$, $z_2 = 3 + 4i$ and $z_3 = 2 - 3i$, verify that $z_1(z_2 + z_3) = z_1z_2 + z_1z_3$.

 [Watch Video Solution](#)

7. If $\frac{a + ib}{c + id} = x + iy$, prove that (i) $\frac{a - ib}{c - id} = (x - iy)$ (ii) $\frac{a^2 + b^2}{c^2 + d^2} = (x^2 + y^2)$

 [Watch Video Solution](#)

8. If $a^2 + b^2 = 1$. Then $\frac{1 + b + ia}{1 + b - ia} =$

 [Watch Video Solution](#)

9. Solve: (i) $3x^2 - 4x + \frac{20}{3} = 0$.

(ii) $x^2 - 2x + \frac{2}{3} = 0$.

 [Watch Video Solution](#)

10. Solve: (i) $27x^2 - 10x + 1 = 0$.

(ii) $21x^2 - 28x + 10 = 0$.

 [Watch Video Solution](#)

11. Given that:

$$(a_1 + ib_1)(a_2 + ib_2) \dots (a_n + ib_n) = c + id,$$

show that:

$$\begin{aligned} \tan^{-1}\left(\frac{b_1}{a_1}\right) + \tan^{-1}\left(\frac{b_2}{a_2}\right) + \dots + \tan^{-1}\left(\frac{b_n}{a_n}\right) \\ = m\pi + \tan^{-1}\left(\frac{d}{c}\right). \end{aligned}$$



Watch Video Solution

12. Show that the area of the triangle on the Argand diagram formed by the complex number z , iz and $z + iz$ is $\frac{1}{2}|z|^2$



Watch Video Solution

Check Your Understanding

1. $\sqrt{a} \times \sqrt{b} \neq \sqrt{ab}$: if both a and b are negative. It is true?



Watch Video Solution

2. Find the real part of $\frac{1 - i}{1 + i}$.



Watch Video Solution

3. Find the real numbers x and y , if $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$.



Watch Video Solution

4. Give an example to show that the subtraction of complex numbers is not commutative.



Watch Video Solution

5. What is the additive inverse of $-2 + 3i$.

 [Watch Video Solution](#)

6. Find the multiplicative inverse of $3 + 4i$.

 [Watch Video Solution](#)

7. Find the square roots of -144.

 [Watch Video Solution](#)

8. Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$

 [Watch Video Solution](#)

9. Convert of the complex number in the polar form: i

 [Watch Video Solution](#)

10. Solve : $x^2 + 3 = 0$.



Watch Video Solution

Competition File

1. If the conjugate of a complex numbers is $\frac{1}{i-1}$, where $i = \sqrt{-1}$.

Then, the complex number is

A. $\frac{1}{i-1}$

B. $\frac{-1}{i-1}$

C. $\frac{1}{i+1}$

D. $\frac{-1}{i+1}$.

Answer: D



Watch Video Solution

2. If $\left|z - \frac{4}{z}\right| = 2$, then find the maximum value of $|z|$.

A. $\sqrt{3} + 1$

B. $\sqrt{5} + 1$

C. 2

D. $2 + \sqrt{2}$.

Answer: B



Watch Video Solution

3. The number of complex numbers z such that

$$|z - 1| = |z + 1| = |z - i| \text{ is}$$

A. 0

B. 1

C. 2

D. ∞

Answer: B



Watch Video Solution

4. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then

$$\alpha^{2009} + \beta^{2009} = -1 \text{ (b) } 1 \text{ (c) } 2 \text{ (d) } -2$$

A. -2

B. -1

C. 1

D. 2

Answer: C



Watch Video Solution

5. Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line $\operatorname{Re} z = 1$, then it is necessary that :

A. $\beta \in (0, 1)$

B. $\beta \in (-1, 0)$

C. $|\beta| = 1$

D. $\beta \in (1, \infty)$

Answer: D



Watch Video Solution

6. If $\omega (\neq 1)$ is a cube root of unity, and $(1 + \omega)^7 = A + B\omega$. Then (A,

B) equals

A. (0,1)

B. (1,1)

C. (1,0)

D. (-1,1)

Answer: B



[Watch Video Solution](#)

7. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, the point represented by the complex numbers z lies

- A. either on the real axis or on a circle passing through the origin.
- B. on a circle with centre at the origin
- C. either on the real axis or on a circle not passing through the origin
- D. on the imaginary axis.

Answer: A



[Watch Video Solution](#)

8. Let z be a complex number such that the imaginary part of z is nonzero and $a = z^2 + z + 1$ is real. Then a cannot take the value

A. -1

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{3}{4}$.

Answer: D



Watch Video Solution

9. If z is a complex number of unit modulus of modulus and argument θ ,

then $\arg\left(\frac{1+z}{1+\bar{z}}\right)$ equals to

A. $\frac{\pi}{2} - \theta$

B. θ

C. $\pi - \theta$

D. $-\theta$

Answer: B

 [Watch Video Solution](#)

10. If z is a complex number such that $|z| \geq 2$ then the minimum value of $\left|z + \frac{1}{2}\right|$ is

A. lies in the interval (1,2)

B. is strictly greater than $\frac{5}{2}$

C. is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$.

D. is equal to $\frac{5}{2}$

Answer: A

 [Watch Video Solution](#)

11. A complex number z is said to be uni-modular if $|z| = 1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1z_2}$ is uni-modular and z_2 is not uni-modular. Then the point z_1 lies on a:

- A. straight line parallel to x-axis
- B. straight line parallel to y-axis
- C. circle of radius 2
- D. circle of radius $\sqrt{2}$.

Answer: C



Watch Video Solution

12. A value of θ for which $\frac{2 + 3i \sin \theta}{1 - 2i \sin \theta}$ is purely imaginary, is

- A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}$

C. $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

D. $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

Answer: D



Watch Video Solution

13. If ratio of the roots of the quadratic equation $3m^2x^2 + m(m - 4)x + 2 = 0$ is λ such that $\lambda + \frac{1}{\lambda} = 1$ then least value of m is

A. $4 - 2\sqrt{3}$

B. $4 - 3\sqrt{2}$

C. $2 - \sqrt{3}$

D. $-2 + \sqrt{2}$

Answer: B



Watch Video Solution

14. Let $\frac{z - \alpha}{z + \alpha}$ is purely imaginary and $|z| = 2$, $\alpha \in \mathbb{R}$ then α is equal to

A. $\sqrt{2}$

B. 2

C. $\frac{1}{2}$

D. 1

Answer: B



Watch Video Solution

15. If one root of the quadratic equation $X^2 + px + q = 0$ is $2\sqrt{3}$

: where $P, Q \in \mathbb{Q}$. Then which of the following is true ?

A. $p^2 - 4q + 12 = 0$

B. $p^2 - 4q - 12 = 0$

C. $q^2 - 4p + 12 = 0$

D. $q^2 - 4p - 12 = 0$

Answer: B

 [Watch Video Solution](#)

16. If $S = \left\{ \frac{\alpha + i}{\alpha - i}, \alpha \in R \right\}$ then the S lies on

A. a circle with radius = $\sqrt{2}$

B. a straight line with slope=-1

C. a straight line with slope=1

D. a circle with radius=1.

Answer: D

 [Watch Video Solution](#)

17. Let $z = \frac{(1+i)^2}{a-i}$ ($a > a$) and $|z| = \sqrt{\frac{2}{5}}$, then \bar{z} is equal to

A. $-\frac{1}{5} - \frac{3i}{5}$

B. $\frac{1}{5} + \frac{3i}{5}$

C. $\frac{3}{5} - \frac{1}{5}i$

D. $-\frac{3}{5} - \frac{i}{5}$.

Answer: A



[Watch Video Solution](#)

Frequently Asked Questions

1. Find real and imaginary parts of $\frac{1-i}{1+i}$.



[Watch Video Solution](#)

2. Write the following in the form $x+iy$:

(i) $(3 + 2i)(2 - i)$

(ii) $2i^2 + 6i^3 + 3i^{16} - 6i^{19} + 4i^{25}$.

(iii) $\frac{(3 - 2i)(2 + 3i)}{(1 + 2i)(2 - i)}$.



Watch Video Solution

3. (a) Express $\frac{1 + 7i}{(2 - i)^2}$ in the form $a+ib$.

(b) Reduce $\frac{1}{1 - 4i} - \frac{2}{1 + i}$ in the form $a+ib$.



Watch Video Solution

4. Perform the indicated operation and give your answer in the form

$x + iy$, where x and y are real numbers and $i = \sqrt{-1}$:

(i) $\left(\frac{1}{2} + \frac{1}{4}i\right)\left(-\frac{2}{3} - \frac{1}{4}i\right)$

(ii) $\frac{5 + 2i}{-1 + \sqrt{3}i}$.

(iii) $(\sqrt{5} - 7i)(\sqrt{5} - 7i)^2 + (-2 + 7i)^2$.



Watch Video Solution

5. Express each one of the following in the standard form

$$a + ib: \left(\frac{1}{1 - 2i} + \frac{3}{1 + i} \right) \left(\frac{3 + 4i}{2 - 4i} \right)$$



Watch Video Solution

6. Prove that:

(i) $\left[\left(\frac{3 + 2i}{2 - 5i} \right) + \left(\frac{3 - 2i}{2 + 5i} \right) \right]$ is rational

(ii) $\left[\left(\frac{2 - 3i}{3 - 4i} \right) \left(\frac{2 + 3i}{3 + 4i} \right) \right]$ is real.



Watch Video Solution

7. If $z = 2 - 3i$ show that $z^2 = 4z + 13 = 0$ and hence find the value of $4z^3 - 3z^2 + 169$.



Watch Video Solution

8. Show that a real value of x will satisfy the equation

$$\frac{1 - ix}{1 + ix} = a - ib \quad \text{if} \quad a^2 + b^2 = 1, \text{ when } a, b \text{ are real.}$$

 [Watch Video Solution](#)

9. If $y = \sqrt{x^2 + 6x + 8}$, show that one value of

$$\sqrt{1 + iy} + \sqrt{1 - iy} [i = \sqrt{-1}] \text{ is } \sqrt{2x + 8}$$

 [Watch Video Solution](#)

10. If $a + ib = \frac{c + i}{c - i}$, where c is real, prove that:

$$a^2 + b^2 = 1 \text{ and } \frac{b}{a} = \frac{2c}{c^2 - 1}.$$

 [Watch Video Solution](#)

11. If $\frac{a + ib}{c + id} = x + iy$, show that:

$$x^2 + y^2 = \frac{a^2 + b^2}{c^2 + d^2}.$$



[Watch Video Solution](#)

12. Find the conjugate and modulus of complex number $7 - 24i$.



[Watch Video Solution](#)

13. Find the conjugate of $\frac{(3 - 2i)(2 + 3i)}{(1 + 2i)(2 - i)}$.



[Watch Video Solution](#)

14. Prove that for any complex number z , the product $z\bar{z}$ is always a non-negative real number.



[Watch Video Solution](#)

15. If $z = \left(\frac{\sqrt{5}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{5}}{2} - \frac{i}{2}\right)^5$, then prove that $Im(z) = 0$.

 [Watch Video Solution](#)

16. Write the additive inverse of the complex number $-2 + 3i$.

 [Watch Video Solution](#)

17. Find the additive inverse and reciprocal of complex number $3 - 4i$.

 [Watch Video Solution](#)

18. Find the multiplicative inverse of the following:

(i) $3 + 4i$

(ii) $(5 - 7i)^2$.

 [Watch Video Solution](#)

19. Inverse of $\frac{3 + 4i}{4 - 5i}$



[Watch Video Solution](#)

20. If $z = (1 - i)^6 + (1 - i)$, then find the modulus of z (i.e., $|z|$) and multiplicative inverse of z (i.e., z^{-1}).



[Watch Video Solution](#)

21. Find the value of $\sqrt{3 - 4i}$.



[Watch Video Solution](#)

22. Find the square roots of $-48 - 14i$.



[Watch Video Solution](#)

23. Find the square roots of :

$$\left(\frac{2 + 3i}{5 - 4i} + \frac{2 - 3i}{5 + 4i} \right).$$

 [Watch Video Solution](#)

24. Square root of $\left(x^2 + \frac{1}{x^2} \right) - \frac{4}{i} \left(x - \frac{1}{x} \right) - 6$ is

 [Watch Video Solution](#)

25. Evaluate $\sqrt{4 + 3\sqrt{-20}} + \sqrt{4 - 3\sqrt{-20}}$.

 [Watch Video Solution](#)

26. $2x^4 + 5x^3 + 7x^2 - x + 41$, when $x = -2 - \sqrt{3}i$

 [Watch Video Solution](#)

1. Number of solutions of the equation $z^2 + |z|^2 = 0$, where $z \in C$, is

A. 1

B. 2

C. 3

D. infinitely many.

Answer: D



[Watch Video Solution](#)

2. i^{-35} is equal to:

A. i

B. $-i$

C. 1

D. -1

Answer: A



Watch Video Solution

3. Show that if $iz^3 + z^2 - z + i = 0$, then $|z| = 1$



Watch Video Solution

4. Evaluate: $\left[i^{18} + \left(\frac{1}{i} \right)^{25} \right]^3$



Watch Video Solution

5. Find the reciprocal of $3 - 4i$.

A. $\frac{3}{25} + \frac{4}{25}i$.

B. $\frac{3}{25} - \frac{4}{25}i$.

C. $\frac{1}{25} + \frac{4}{25}i$.

D. $\frac{4}{25} + \frac{3}{25}i$.

Answer: A

 [Watch Video Solution](#)

6. For any positive integer n , prove that:

$$i^n + i^{n+1} + i^{n+2} + i^{n+3} + i^{n+4} + i^{n+5} + i^{n+6} + i^{n+7} = 0.$$

 [Watch Video Solution](#)

7. If $z_1 = 2 - i$, $+ 2 = - 2 + i$, find : $Re\left(\frac{z_1 z_2}{z_1}\right)$

 [Watch Video Solution](#)

8. If α and β are different complex numbers with $|\beta| = 1$, then find

$$\left| \frac{\beta - \alpha}{1 - \bar{\alpha}\beta} \right|.$$

 [Watch Video Solution](#)

9. If $a^2 + b^2 = 1$, prove that $\frac{1 + b + ia}{1 + b - ia} = b + ia$.

 [Watch Video Solution](#)

10. Solve the equation: $x^2 + \frac{x}{\sqrt{2}} + 1 = 0$

 [Watch Video Solution](#)

11. Given that $(a_1 + ib_1)(a_2 + ib_2) \dots (a_n + ib_n) = c + id$, show that:

$$\tan^{-1}\left(\frac{b_1}{a_1}\right) + \tan^{-1}\left(\frac{b_2}{a_2}\right) + \dots + \tan^{-1}\left(\frac{b_n}{a_n}\right) = m\pi + \tan^{-1}\left(\frac{d}{c}\right)$$

 [Watch Video Solution](#)



Watch Video Solution

12. If $\frac{a + ib}{c + id} = x + iy$, prove that

$$\frac{a - ib}{c - id} = x - iy \text{ and } \frac{a^2 + b^2}{c^2 + d^2} = x^2 + y^2.$$



Watch Video Solution