MATHS

BOOKS - MODERN PUBLISHERS MATHS (HINGLISH)

MATHEMATICAL INDUCTION

Illustrative Examples

1. If $P(n)$ is the statement $n(n+1)(n+2)$ is divisible is 12 prove that the statements $P(3)$ and $P(4)$ are true, but that $P(5)$ is not true.

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2. If $P(n)$ is the statement $2^n \geq 3n$, and if $P(r)$ is true, prove that $P(r+1)$ is true.

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Questions From Ncert Examplar

1. Prove the following by the principle of mathematical induction: $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n}$ or all natural numbers $n \geq 2$.

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2. Show by the Principle of Mathematical induction that the sum S_n, of the nterms of the series $1^2+2\times 2^2+3^2+2\times 4^2+5^2+2\times 6^2+7^2+\ldots.$ is given by $S_n=\left\{\frac{n(n+1)^2}{2}\right.$, if n is even , then $\frac{n^2(n+1)}{2}$, if n is odd

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Frequently Asked Questions Example

1. Let P(n) be the statement :

"$n^2 + n$ is even"

Prove that P(n) is true for all $n \in N$ by Mathematical Induction

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2. Using the principle of mathematical induction, prove that $n < 2^n$ for all n $\in$ N

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3. By Principle of Mathematical Induction, prove that :

$$1 + 2 + 3 + \ldots. + n = n\frac{n+1}{2}$$

$1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \frac{1}{6}n(n+1)(2n+1)$ for each given $n \in N$

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4. For all $n \geq 1$, prove that

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\vdots+\frac{1}{n(n+1)}=\frac{n}{n+1}$$

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5. Prove the following by using the Principle of mathematical induction

$\forall n \in N$

$$\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\ldots\ldots\left(1-\frac{1}{n+1}\right)=\frac{1}{n+1}$$

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6. For every positive integer n, prove that 7^n-3^n is divisible by 4.

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7. Prove by the principle of mathematical induction that

$n(n+1)(2n+1)$ is divisible by 6 for all n $\in$ N

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8. Prove by induction that the sum of the cubes of three consecutive natural numbers is divisible by 9.

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9. Prove by the principle of mathematical induction that for all $n \in N, 3^{2n}$ when divided by 8 , the remainder is always 1.

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10. Prove the rule of exponents $(ab)^n = a^n b^n$by using principle of mathematical induction for every natural number.

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11. Prove that, $\sin\theta + \sin 2\theta + \sin 3\theta + \ldots \sin n\theta = \dfrac{\frac{\sin n\theta}{2}\sin\frac{n+1}{2}\theta}{\sin\frac{\theta}{2}}$ for all $n \in N$.

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12. Using principle of mathematical induction prove that $\cos\alpha\cos 2\alpha\cos 4\alpha\cos\left(2^{n-1}\alpha\right) = \dfrac{s \in 2^n\alpha}{2^n s \in \alpha} f$ or $all\, nN\cdot$

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13. Prove by the principle of mathematical induction that $\dfrac{n^5}{5} + \dfrac{n^3}{3} + \dfrac{7n}{15}$ is a natural number for all $n \in N\cdot$

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14. For the proposition P(n), given by, $1 + 3 + 5 + \ldots + (2n-1) = n^2 + 2$prove that $P(k)$ is true implies

$P(k+1)$ is true. But, P(n) is not true for all $n \in N$

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Exercise 4 A Short Answer Type Questions

1. If P(n) is the statement $\mathrm{n\hat{}2}> 100$, prove that whenever $P(r)$ is true, $P(r+1)$ is also true.

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2. Let $P(n)$ be the statement: $2^n \geq 3n$. If $P(r)$ is true, show that $P(r+1)$ is true. Do you conclude that $P(n)$ is true for all $n \in N$

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3. If $P(n)$ is the statement $2^{3n}-1$. Is an integral multiple $7,$ and if $P(r)$ is true, prove that $P(r+1)$ is true.

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4. Given an example of a statement $P(n)$ which is true for all $n \geq 4$ but $P(1), \ P(2) and \ P(3)$ are not true. Justify your answer.

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5. If P(n) is the statement : $C_r \leq n!$ for $1 \leq r \leq n$ then:

(i) find $P(n+1)$

(ii) show that $P(3)$ is true

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Exercise 4 B Short Answer Type Questions

1. Give an example of a statement P(n) such that P(3) is true, but P(4) is not true.

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2. Prove that the Principle of Mathematical Induction does not apply to the following:

(i) $P(n): n^3 + n$ is divisible by 3"

(ii) $P(n): n^3 \leq 100$

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Exercise 4 B Long Answer Type Questions I

1. The nth term of an A.P. whose first term is 'a' and common difference d' is $a + (n-1)d$. Prove by PMI.

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2. Prove by PMI that

$$1.2 + 2.3 + 3.4 + \ldots. + n(n+1) = \frac{(n)(n+1)(n+2)}{3}, \forall n \in N$$

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3. $1.2 + 2.2^2 + 3.2^3 \ldots . n.2^n = (n-1)2^{n+1} + 2$

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4. Prove the following by using the principle of mathematical induction for all $n \in N$:$a + ar + ar^2 + \vdots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}$

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5. The statement $x^n - y^n$ is divisible by (x-y) where n is a positive integer is

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6. For every positive integer n, prove that $7^n - 3^n$is divisible by 4.

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7. Prove the following by using the Principle of mathematical induction

$\forall n \in N$

$4^n + 15n - 1$ is divisble by 9.

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8. Using mathematical induction prove that $n^2 - n + 41$ is prime.

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9. Using the principle of mathmatical induction, prove each of the following for all $n \in N$

$3^n \geq 2^n$

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Exercise 4 B Long Answer Type Questions Ii

1. Shwo that $n^3 + (n+1)^3 + (n+2)^3$ is divisible 9 for everynatural number n.

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2. Prove that $(1+x)^n \geq (1+nx)$, for all natural number n, where $x \succ 1$.

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3. If $P(n)$ is the statement $n^2 - n + 41$ is prime. Prove that $P(1),\ P(2) and\ P(3)$ are true. Prove also that $P(41)$ is not true.

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4. If n straight lines be drawn in a plane , no two of them being parallel and no three of them being concurrent , how many points of intersection will be there ?

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Objective Type Questions Questions From Ncert Exemplar

1. Let $P(n): 2^n < (1 \times 2 \times 3 \times \quad \times n)$. Then the smallest positive integer for which $P\ (n)$ is true is 1 b. 2 c. 3 d. 4

A. 1

B. 2

C. 3

D. 4

Answer: D

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2. A student was asked to prove a statement P(n) by induction. He proved that $P(k+1)$ is true whenever P(k) is true for all $k > 5 \in N$ and also that P(n) is true. On the basis of this he could conclude that P(n) is true :

A. for all $n \in N$

B. for all $n > 5$

C. for all $n \geq 5$

D. for all $n < 5$

Answer: C

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3. If $10^n + 3 \times 4^{n+2} + \lambda$ is divisible by 9 or all nN , then the least positive integral value of λ is 5 b. 3 c. 7 d. 1

A. 5

B. 3

C. 7

D. 1

Answer: A

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4. For all $n \in N, 3.5^{2n+1} + 2^{3n+1}$ is divisble by-

A. 19

B. 17

C. 23

D. 25

Answer: B

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5. If $x^n - 1$ is divisible by $x - k$ then the least positive integral value of k is

A. 1

B. 2

C. 3

D. 4

Answer: A

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Objective Type Questions For Board Examinations

1. For each $n \in N, 3^{2n} - 1$ is divisible by

A. 8

B. 16

C. 32

D. None of these.

Answer: A

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2. For each $n \in N, 2^{3n} - 1$ is divisible by

A. 7

B. 8

C. 16

D. 32

Answer: A

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3. For each $n \in N$, $3.\left(5^{2n+1}\right)+2^{3n+1}$ is divisible by

A. 17

B. 19

C. 21

D. 23

Answer: A

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4. For all $n \in N$, $4^n - 3n - 1$ is divisible by

A. 3

B. 8

C. 9

D. 27

Answer: C

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5. For each n in N, $49^n + 16n - 1$ is divisible by :

A. 3

B. 19

C. 29

D. 64

Answer: D

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6. For all $n \in N, n^3 + 2n$ is divisible by

A. 2

B. 3

C. 5

D. 6

Answer: B

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7. For each $n \in N, n(n+1)(2n+1)$ is divisible by

A. 6

B. 7

C. 8

D. 15

Answer: A

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8. For all positive integers $\left\{x\left(x^{n-1}-n.\,a^{n-1}+a^n(n-1)\right\}\right.$ is divisible by

A. $n>1$

B. $n>2$

C. all $n \in N$

D. None of these.

Answer: C

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9. If a, b and n are natural numbers then $a^{2n-1}+b^{2n-1}$ is divisible by

A. a + b

B. $(a+b)^2$

C. a^3+b^3

D. None of these.

Answer: A

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10. For each $n \in N$, the greatest positive integer which divides $(n+1)(n+2)(n+3)$is :

A. 2

B. 6

C. 24

D. 120

Answer: B

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11. For each $n \in N$ the greatest positive integer which divides $(n+1)(n+2)(n+3)\dots(n+r)$ is :

A. r

B. r!

C. $(r+1)!$

D. $n+r$

Answer: B

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12. The inequality $n! > 2^{n-1}$ is true

A. all n

B. no n

C. all $n > 1$

D. all $n > 2$

Answer: C

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13. The inequality $2^n < n!n \in N$is true for :

A. all n

B. all $n > 1$

C. all $n > 2$

D. all $n > 3$

Answer: D

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14. The statement $n! > 2^{n-1}, n \in N$ is true for

A. all $n > 1$

B. all$n \in N$

C. all $n > 2$

D. all$n \in N$

Answer: B

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15. Let $S(k) = 1 + 3 + 5 + ... + (2k - 1) = 3 + k^2$. Then which of the following is true ? (A) S(1) is correct (B) S(k)=S(k+1) (C) $S(k) \neq S(k+1)$ (D) Principal of mathematical induction can be used to prove the formula

A. S(1) is correct

B. $S(k) \Rightarrow S(k-1)$

C. $S(k) \Rightarrow S(k+1)$

D. principle of Mathematical Induction can be used to prove the formula.

Answer: C

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Objective Type Questions Fill In The Blanks

1. Let $P(n)$ be the statement $n(n+1)$ is even, then $P(4) =$ _____

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2. Let $P(n)$ be the statement $3\hat{}n > n$. If $P(n)$ is true, $P(n+1)$ is also true.

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3. Let P (n) be the statement $C_r \leq n!$ for $1 \leq r \leq n$ then $P(n+1)$ = _______

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4. By Principle of Mathematical Induction $1 + 2 + 3$ ____ $+n =$ ____

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5. By Principle of Mathematical Induction :

nth term of A.P. whose first term is 'a' and common difference is 'd' is

______

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Objective Type Questions True False Questions

1. If $P(n)$ is the statement $n(n+1)(n+2)$ is divisible is 12 prove that the statements $P(3)$ and $P(4)$ are true, but that $P(5)$ is not true.

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2. Let P (n) be the statement $2^{3n}-1$ is integral multiple of 7. Then P (3) is true.

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3. If $P(n)$ is the statement $n^3 + n$ is divisible 3 is the statement $P(3)$ true ? Is the statement $P(4)$ true?

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4. Let P (n) be the statement$2^n \geq n$. When P (r) is true, then is it true that $P(r+1)$ is also true ?

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5. Let P (n) be the statement $(n-4)$ is a whole number. Then P (3) true

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Objective Type Questions Very Short Answer Type Questions

1. Let P (n) be the statement. "$P(n) = 16n + 3$ is prime. Is P (3) true ?

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2. If $P(n)$ is the statement $2^n \geq 3n,$ and if $P(r)$ is true, prove that $P(r+1)$ is true.

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3. If $P(n)$ is the statement $n^3 + n$ is divisible 3 is the statement $P(3)$ true ? Is the statement $P(4)$ true?

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4. Let P (n) be the statement." 2^{3n-1} is integral multiple of 7". Then, P (1), P (2) and P (3) are true ?

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5. Prove the following by the principle of mathematical induction: $(ab)^n = a^n b^n$ for all $n \in N$.

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Ncert File Exercise 4 1 Prove The Following By Using The Principle Of Mathematical Induction For All N Inn

1. Prove the following by the principle of mathematical induction: $1 + 3 + 3^2 + \quad + 3^{n-1} = \dfrac{3^n - 1}{2}$

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2. $1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\dfrac{n(n+1)}{2}\right)^2$

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3. $1+\frac{1}{1+2}+\frac{1}{1+2+3}+\frac{1}{1+2+3+n}=\frac{2n}{n+1}$

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4. Prove the following by using the principle of mathematical induction for all $n \in N$:

$1.\ 2.\ 3+2.\ 3.\ 4+\ \text{dot}\ \ \text{dot}\ \ \text{dot}+\text{n}(\text{n}+1)\ \ \ (\text{n}+2)=\frac{n(n+1)(n+2)(}{4}$

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5. Prove the following by using the principle of mathematical induction for all $n \in N$:$1.\ 3+2.\ 3^2+3.\ 3^3+\ \vdots +n.3^n=\frac{(2n-1)3^{n+1}+3}{4}$

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6. Prove the following by using the principle of mathematical induction for all $n \in N$:$1.\ 2+2.\ 3+3.\ 4+\ \vdots +n(n+1)=\left[\frac{n(n+1)(n+2)}{3}\right]$

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7. Prove the following by using the principle of mathematical induction for all $n \in N$:

$$1.\ 3 + 3.\ 5 + 5.\ 7 + \vdots + (2n1)(2n+1) = \frac{n\left(4n^2 + 6n - 1\right)}{3}$$

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8. Prove the following by using the principle of mathematical induction for all $n \in N$:$1.\ 2 + 2.\ 2^2 + 3.\ 2^2 + \vdots + n.2^n = (n-1)2^{n+1} + 2$

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9. Prove the following by using the principle of mathematical induction for all $n \in N$:$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \vdots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$

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10. Prove the following by using the principle of mathematical induction for all $n \in N$:

$$\frac{1}{2.\ 5} + \frac{1}{5.\ 8} + \frac{1}{8.\ 11} + \vdots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{(6n+4)}$$

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11. Prove the following by using the principle of mathematical induction for all $n \in N$:

$$\frac{1}{1.\ 2.\ 3} + \frac{1}{2.\ 3.\ 4} + \frac{1}{3.\ 4.\ 5} + \vdots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

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12. Prove the following by using the principle of mathematical induction for all $n \in N$: $a + ar + ar^2 + \vdots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}$

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13. Prove the following by using the principle of mathematical induction

for all $n \in N$: $\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)1+\vdots\frac{(2n+1)}{n^2}=(n+1)^2$

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14. Prove the following by using the principle of mathematical induction

for all $n \in N$: $\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)1+\vdots\frac{1}{n}=(n+1)$

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15. Prove the following by using the principle of mathematical induction

for all $n \in N$: $1^2+3^2+5^2+\ \vdots+(2n-1)^2=\frac{n(2n-1)(2n+1)}{3}$

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16. Prove the following by using the principle of mathematical induction for all $n \in N$:

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \vdots + \frac{1}{(3n-1)(3n+1)} = \frac{n}{(3n+1)}.$$

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17. Prove the following by using the principle of mathematical induction for all $n \in N$:

$$\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \vdots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}.$$

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18. Prove that $1 + 2 + 3 + 4....... + N < \frac{1}{8}(2n+1)^2$

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19. Prove the following by using the principle of mathematical induction for all $n \in N$: $n(n+1)(n+5)$ is a multiple of 3.

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20. Show that $10^{2n-1}+1$ is divisible by 11 for all natural numbers n.

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21. Prove the following by using the principle of mathematical induction for all $n \in N$: $x^{2n}-y^{2n}$ is divisible by $x+y$.

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22. Prove the following by using the principle of mathematical induction for all $n \in N$: $3^{2n+2}-8n-9$ is divisible by 8.

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23. $41^n - 14^n$ is a multiple of 27

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24. Prove by mathematical induction that, $2n + 7 < (n + 3)^2$ for all $n \in \mathbb{N}$.

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Exercise

1. $4^n - 1$ is divisible by 3, for each natural number n.

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2. Using the principle of mathematical induction, prove that $(2^{3n} - 1)$ is divisible by 7 for all $n \in N$.

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3. $3^{2n} - 1$ is divisible by 8, for all natural numbers n.

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4. $n^3 - n$ is divisible by 6, for each natural numbers $n \geq 2$

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5. $n(n^2 + 5)$ is divisible by 6, for each natural number n.

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6. $n^2 < 2^n$, for all natural number $n \leq 5$.

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7. For all $n \in N$, $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + + \frac{1}{\sqrt{n}}$

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8. $1 + 5 + 9 + \ldots + (4n - 3) = n(2n - 1)$ for all natural numbers n

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9. A sequence $b_0, b_1, b_2, \ldots$ is defined by letting $b_0 = 5$ and $b_k = 4 + b_{k-1}$, for all natural number k. Show that $b_n = 5 + 4n$, for all natural number n using mathematical induction.

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10. Using induction, prove that $\cos\theta\cdot\cos 2\theta\cdot\cos 2^2\theta....\cos 2^{n-1}\theta=\frac{\sin 2^n\theta}{2^n\sin\theta}$

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Revision Exercise

1. Prove, by Induction, that $2^n < n!$for all $n\geq 4$

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2. The number of all possible subsets of a set containing n elements ?

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3. Let P(n) denote the statement:

$2^n\geq n!$.Show that P(1),P(2),P(3) are true butP(4) is not true.

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4. Prove the following by using the principle of mathematical induction for all $n \in N$: $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \vdots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$

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5. Using mathematical induction prove that $x + 4x + 7x + \ldots\ldots + (3n-2)x = \frac{1}{2}n(3n-1)x$

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6. Prove by PMI that $1.2 + 2.3 + 3.4 + \ldots. + n(n+1) = \frac{(n)(n+1)(n+2)}{3}, \forall n \in N$

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7. $1.\ 2.3 + 2.\ 3.4 + \ + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$

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8. Prove the following by the principle of mathematical induction: $7 + 77 + 777 + \ + 777 + \ + \ddot{n} - digits7 = \frac{7}{81}\left(10^{n+1} - 9n - 10\right)$ for all $n \in NB\cdot$

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9. Prove by the principle of mathematical induction, that

$1.1! + 2.2! + 3.3! + \ldots.. + (n.\ n!) - 1 \text{for all natural number } \ n(n \neq 1 \times$

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10. Prove that for $n \in N,\ 10^n + 3.\ 4^{n+2} + 5$ is divisible by 9 .

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11. By Mathematical Induction, prove the following:

(i) $(4^n + 15n - 1)$ is divisible by 9,

(ii) $\left(12^n + 25^{n-1}\right)$ is divisible by 13

(iii) $11^{(n+2)} + 12^{(2n+1)}$ is divisible by 133 for all $n \in N$

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12. For all positive integer n , prove that $\frac{n^7}{7} + \frac{n^5}{5} + \frac{2}{3}n^3 - \frac{n}{105}$ is an integer

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13. Let $U_1 = 1,\ U_2 = 1\ and\ U_{n+2} = U_{n+1} + U_n f$ or $n \geq 1.$ use mathematical induction to show that:

$U_n = \frac{1}{\sqrt{5}}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right\} f$ or $all\ n \geq 1.$

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14. Using the principle of Mathematical Induction, prove that $\forall n \in N, 4^n - 3n - 1$ is divisible by 9.

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Check Your Understanding

1. Does the Principle of Mathematical Induction' apply to :

$P(n): n^3 > 100$?

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2. If $P(n)$ is the statement $n(n+1)(n+2)$ is divisible is 12 prove that the statements $P(3)$ and $P(4)$ are true, but that $P(5)$ is not true.

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3. If $P(n)$ is the statement $n(n+1)(n+2)$ is divisible is 12 prove that the statements $P(3)$ and $P(4)$ are true, but that $P(5)$ is not true.

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4. Let P(n) be the statement : $10n+3$ is prime. Is P(3) true ?

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5. Let P(n) be the statement : $2^n > 1$. Is P(1) true ?

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6. If P(n) is the statement:$n(n+1)$ is even, then what is P(4) ?

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7. If $P(n)$ is the statement $n^3 + n$ is divisible 3 is the statement $P(3)$ true ? Is the statement $P(4)$ true?

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8. Let P(n) be the statement : $n^2 + n$ is even Is P(n) true for all $n \in N$?

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9. Let P(n) be the statement: $C_r \leq n!$ for $1 \leq r \leq n$

Is P(3) true ?

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10. Prove by induction that : $2^n > n$ for all $n \in N$.

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Chapter Test 4

1. For all $n \in N, 3.5^{2n+1} + 2^{3n+1}$ is divisble by-

A. 19

B. 17

C. 23

D. 25

Answer: B

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2. For each $n \in N, n(n+1)(2n+1)$ is divisible by

A. 6

B. 7

C. 8

D. 15

Answer: A

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3. If $P(n)$ is the statement $2^{3n} - 1$. Is an integral multiple $7,$ and if $P(r)$ is true, prove that $P(r+1)$ is true.

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4. Let P(n) be the statement :

$3^n > n$

What is $P(n+1)$?

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5. Given an example of a statement $P(n)$ which is true for all $n \geq 4$ but $P(1),\ P(2) and\ P(3)$ are not true. Justify your answer.

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6. Prove by the principle of mathematical induction that for all $n \in N$:

$$1^2 + 2^2 + 3^2 + \quad + n^2 = \frac{1}{6}n(n+1)(2n+1)$$

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7. Prove the following by using the principle of mathematical induction

for all $n \in N$: $1^2 + 3^2 + 5^2 + \vdots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

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8. Prove the following by using the principle of mathematical induction

for all $n \in N$: $a + ar + ar^2 + \vdots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}$

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9. If n is a positive integer, show that $4^n - 3n - 1$ is divisible by 9

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10. Prove that $2n + 1 < 2^n$ for all natural numbers $n \geq 3$

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11. Prove the following by using the principle of mathematical induction for all $n \in N$:

$$\frac{1}{1.\,2.\,3} + \frac{1}{2.\,3.\,4} + \frac{1}{3.\,4.\,5} + \vdots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

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12. Prove by the principle of mathematical induction that $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$ is a natural number for all $n \in N$.

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