MATHS

BOOKS - MODERN PUBLISHERS MATHS (HINGLISH)

RELATIONS AND FUNCTIONS

Example

1. If A = {1,2} and B= {a,b,c}, obtain $A \times B$ and represent it by an arrow diagram

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Frequently Asked Questions

1. Find x and y, if $(2x, x + y) = (6, 2)$.

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2. Let $A = \{a, b\}$, $B = \{a, b, c\}$. What is $A \times B$?

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3. If $A \times B = \{(p, q), (p, r), (m, q), (m, r)\}$, find A and B.

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4. Let A and B be two sets such that $n(A) = 5$ and $n(B) = 2$. If $(a_1, 2), (a_2, 3), (a_3, 2), (a_4, 3), (a_5, 2)$ are in $A \times B$ and a_1, a_2, a_3, a_4 and a_5 are distinct, find A and B.

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5. If $P = \{a, b, c\}$ and $Q = \{r\}$, form the sets $P \times Q$ and $Q \times P$.

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6. Express $\{(x, y) : x^2 + y^2 = 25, \text{ where } x, y \in W\}$ as a set of ordered pairs.

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7. Few elements of $A \times B$ are $(1, 5)$, $(1, 10)$ and $(3, 12)$. If $A = \{1, 2, 3\}$, then obtain the remaining elements of $A \times B$ such that $n(A \times B)$ is least.

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8. If $P = \{1, 2\}$form the set $P \times P \times P$.

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9. Let $A = \{1, 2, 3, 4\}$, $B = \{5, 7, 9\}$. Determine :

(i) $A \times B$ and represent it graphically.

(ii) $B \times A$ and represent it graphically.

(iii) Is $A \times B = B \times A$?

(iv) Is n $(A \times B) = n(B \times A)$?

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10. If $A = \{1, 3, 5\}$, $B = \{x, y\}$ represent the following products by arrow diagrams: $A \times B$ (ii) B $\times A$ (iii) $A \times A$ (iv) $B \times B$

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11. If $A = \{1, 2\}$, $B = \{3, 4\}$, $C = \{4, 5\}$, find $A \times (B \cup C)$.

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12. मान लीजिए की $A = \{1, 2, 3\}, B = \{3, 4\}$ और $C = \{4, 5, 6\}$, तो, निम्नलिखित के मान ज्ञात कीजिए:

(i) $A \times (B \cap C)$ (ii) $(A \times B) \cap (A \times C)$

(iii) $A \times (B \cup C)$ (iv) $(A \times B) \cup (A \times C)$

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13. Let $A = \{1, 2, 3\}, B = \{2, 3, 4\}$ and $C = \{4, 5\}$, verify $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

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14. Let $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\} and\ D = \{5, 6, 7, 8\}$.

Verity that: $A \times (B \cap C) = (A \times B) \cap (A \times C)$

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15. Theorem 1(ii) (For any three set $A; B; C$; prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

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16. Theorem 2 (For any three set $A; B; C;$ prove that $A \times (B - C) = (A \times B) - (A \times C)$

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17. For any sets A, B, C, D prove that: $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

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Illustrative Example

1. Let A={1,2} and B={3,4}. Find the number of relations from A to B.

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2. Determine the domain and range of the relation R defined by :

$R = \{(x, x+5) : x \in \{0, 1, 2, 4, 5\}\}$

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3. Let $A = \{1, 2, 3, 4, 6\}$. Let R be the relation on A defined by $\left\{\left(a\dot{b}\right) : a, b \in A, b\text{is exactly divisible by a}\right\}$.(i) Write R in roster form(ii) Find the domain of R(iii) Find the range of R.

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4. If $A = \{4, 9, 16, 25\}$, $B = \{1, 2, 3, 4\}$ and R is the relation "is square of" from A to B, write down the set corresponding to R. Also find the

domain and range of R.

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5. Let R be the relation on the set N of natural numbers defined by $R = \{(a, b) : a + 3b = 12, a \in N, b \in N\}$. Find : (i) R (ii) $Doma \in ofR$ (iii) $Ran \geq ofR$

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6. Mr. Yogender Sharma is husband of Mrs. Leelavati. They have sons and all are married. Five sons Rakesh, Suresh, Munish, Kamlesh and Avnish. Rohini is Munish's wife and Anita is Rakesh's daughter. Neha is a member of Sharma's family. Kamlesh's sister-in-law is Sunita and Aruna's husband. Nisha is Suresh's sister-in-law and lives in the same family. Avnish has no child and so adopts a daughter Ahalya. Express the relation between these members as a function. Find the domain, co-domain and range of the members.

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7. If R is the relation "less than" from $A = \{1, 2, 3, 4, 5\}$ to $B = \{1, 3, 5\}$, write down the Cartesian product corresponding to R. Also find the inverse relation to R.

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8. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and R be the relation on A defined by :

$R = \{(x, y) : x \in A, y \in A \text{ and } x + 2y = 10\}$

Find the domains and ranges of R and R^{-1} after expressing them as sets of ordered pairs.

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9. Let $A = \{1, 2, 3, 4, 5, 6\}$. Define a relation R from A to A by $R = \{(x, y) : y = x + 1\}$(i) Depict this relation using an arrow diagram. (ii) Write down the domain, co-domain and range of R.

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10. Show that the relation '$>$' on the set R of all real numbers is transitive but it is neither reflexive nor symmetric.

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11. Consider that the set $A = \{a, b, c\}$. Give an example of a relation R on A. Which is :

(i)reflexive and symmetric but not transitive

(ii) symmetric and transitive but not reflexive

(iii) reflexive and transitive but not symmetric.

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12. Let R be the relation of the set Z of all integers defined by :

$R = \{(a, b) : a, b \in Z \text{ and } (a - b) \text{ is divisible by } n \in N\}$

Prove that :

(i) $(a, a) \in R$ for all a $\in Z$

(ii) $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in Z$

(iii) $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all$a, b, c \in Z$

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13. Show that the relation is parallel to on the set S of all straight lines in a plane is an equivalence relation.

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14. Prove that the relation R on Z defined by $(a, b) \in R \Leftrightarrow a - b$ is divisible by 5 is an equivalence relation on Z.

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15. Let n be a positive integer. Prove that the relation R on the set Z of all integers numbers defined by $(x, y) \in Rx - y$ is divisible by n, is an

equivalence relation on Z.

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16. Prove that a relation R defined on $N \times N$ where $(a, b)R(c, d) \Leftrightarrow ad = bc$ is an equivalence relation.

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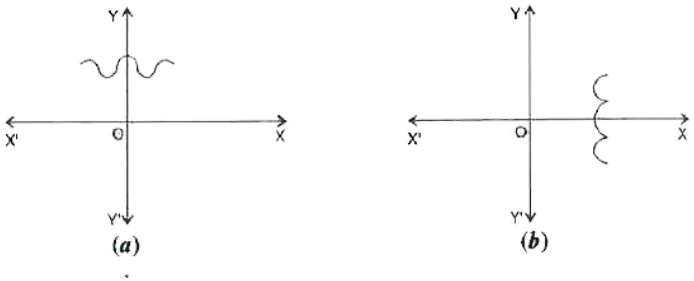
17. Which of the following graphs represent the function of x ? Why



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18. Let N be the set of natural numbers and the relation R be defined on N such that $R = \{(x, y) : y = 2x, x, y \in N\}$.What is the domain, codomain and range of R? Is this relation a function?

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19. Which of the following relations are functions? Give reason. If it is a function, determine its domain and range.

(i) $R = \{(2, 1), (3, 1), (4, 2), (5, 7), (6, 9)\}$

(i) $R = \{(2, 2), (2, 4), (3, 3), (4, 4), (5, 8)\}$

(iii) $R = \{(1, 3), (1, 5), (2, 5), (3, 6), (3, 7)\}$

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20. Let N be the set of natural numbers. Define a real valued function $f : N \to N by \quad f(x) = 2x + 1$. Using this definition, complete the table given below. x 1 2 3 4 5 6 7 y f(1) = . . . f(2) = . . . f(3) = . . . f(4) = . . . f(5)

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21. Let $f = \{(1, 2), (2, 3), (0, 1), (1, 3)\}$be a linear function from Z into Z. Find f(x).

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22. Let A = {1,2,3},B={4,5} and let $f = \{(1, 4), (2, 5), (3, 5)\}$. Show that 'f' is an onto function from A to B

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23. Let $N \to N$ be defined by f(x) = 3x. Show that 'f' is not an onto function.

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24. Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. Then f is .

A. One-one

B. Many-one

C. Not a function

D. None of these

Answer: A

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25. If $f(x) = x^3 - \frac{1}{x^3}$, show that $f(x) + f\left(\frac{1}{x}\right) = 0$.

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26. If f is a real function defined by $f(x) = \frac{x-1}{x+1}$, then prove that $f(2x) = \frac{3f(x)+1}{f(x)+3}$

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27. If $f(x) = \frac{1}{2x+1}, x \neq -\frac{1}{2}$, then show that $f(f(x)) = \frac{2x+1}{2x+3}$, provided that $x \neq -\frac{3}{2}$.

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28. If $f(x) = \log_e\left(\frac{1-x}{1+x}\right)$; prove that $f(a) + f(b) = f\left(\frac{a+b}{1+ab}\right)$

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29. माना $f: R \to R$ एक फलन इस प्रकार परिभाषित है, कि

$$f(x) = \begin{cases} 3x-1 & x > 3 \\ x^2-2 & -2 \leq x \leq 3 \\ 2x+3 & x < -2 \end{cases}$$

तो निम्न का मान ज्ञात कीजिए -

(i) $f(2)$ $(ii) f(-3)$

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30. For the relation $y = +\sqrt{x}$, say whether it is a function or not. If it is a function, find its domain and range.

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31. Find the domain and range of the following functions:

(i) $f(x) = \sqrt{(x-1)(3-x)}$

(ii) $f(x) = 11 - 7\sin x$

(iii) $f(x) = 1 - |x|$

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32. The domain of the function $f(x) = \frac{1}{\sqrt{[x]^2 - 2[x] - 8}}$ is, where [*] denotes greatest integer function

A. $(-2, 5]$

B. $(-2, 5)$

C. $(-\infty, -2) \cup [5, \infty)$

D. None of these

Answer:

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33. Find the domain of the function $f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$

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34. Find the range of the following function: $f(x) = \frac{1}{(2x - 3)(x + 1)}$

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35. Find the domain of the function: $f(x) = \frac{x^2 + 3x + 5}{x^2 - 5x + 4}$

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36. Which of the following functions are odd or even or neither:

(i) $f(x) = \tan x + 3\cos ecx + x$

(ii) $f(x) = |x| + 1$

(iii) $f(x) = |x - 2|$

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37. The period of $x - [x]$ where $[x]$ represents the integral part of x is

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38. Solve: $[2x - 3] = 5$

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39. Let f and g be two functions defined by:

$f(x) = \sqrt{x - 1}$ and $g(x) = \sqrt{4 - x^2}$

Find (i) f+g (ii) g + f (iii) f-g (iv) g-f (v) fg (vi) gf (vii) $\frac{f}{g}$ (viii) $\frac{g}{f}$

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40. The domain of the function $f(x) = \frac{\sin^{-1} x}{[x]}$ is (where is greatest integer function)

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41. Define the function f : R $\rightarrow$ R by y = f(x) = x^2 , $x \in R$. Complete the Table given below by using this definition. What is the domain and range

of this function? Draw the graph of f. x 4 3 2 1 0 1 2 3 4 $y = f(x) = x^2$

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42. Draw the graph of the function: $f: R \to R$ defined by $f(x) = x^3, x \in R$

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43. Let R be the set of real numbers. Define a real function $f: R \to R$ by $f(x) = x + 10$. Sketch the graph of this function.

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44. फलन f, $f(x) = \begin{cases} 1 - x, x < 0 \\ 1, x = 0 \\ x + 1, x > 0 \end{cases}$ द्वारा परिभाषित है। f(x) का आलेख खीचिए।

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45. Draw the graph of the function: $1 - x$

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46. Draw the graph of the function: $f(x) = \begin{cases} x & \text{if } x \leq 0 \\ x^2 & \text{if } 0 < x \leq 2 \\ x & \text{if } x > 2 \end{cases}$

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47. Draw the graph of the function: $f(x) = |1 - x| + |1 + x|, \ -2 \leq x \leq 2$

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48. If $A = \{2, 4, 6, 9\}$ and $B = \{4, 6, 18, 27, 54\}$, $a \in A, b \in B$, find the set of ordered pairs such that 'a' is a factor of 'b' and $a < b$

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49. Find the domain and range of the function f given by

$$f(x) = \frac{x^2}{1 - x^2}$$

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50. Is the following relation a function ? Justify your answer:

(i) $R_1 = \left\{(2, 3), \left(\frac{1}{2}, 0\right), (2, 7), (-4, 6)\right\}$

(ii) $R_2 = \{(x, |x|) : x \text{ is a real number}\}$

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51. For which Domain, the functions $f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ are equal to

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52. Find the domain of the function: $f(x) = [x] + x$

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53. Find the range of the function: $f(x) = \frac{|x-4|}{x-4}$

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54. Find the domain of the function f given by $f(x) = \frac{1}{\sqrt{[x]^2 - [x] - 6}}$

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Exercise 2 A

1. If $A = \{a, b, c\}, B = [p, q], \quad \text{find} \quad B \times A.$

(ii) Let $A = [1, 2, 3, 4, 5, 6], B = \{2, 4, 6, 8\}$. Find $A \times B$

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2. Find x and y if:

(i) $(x+1, y-2) = (3, 1)$

(ii) $(x+2, 4) = (5, 2x+y)$

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3. Let $= \{1, 2, 3, 4\} adn\ R = \{(a, b) : a \in A, b \in A,\ a \text{ divides } b\}$. Write R explicity.

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4. Let A and B be two sets such that $n(A) = 3 and n(B) = 2$. If $(x, 1), (y, 2), (z, 1)$ are in A $\times$ B. find A and B. where x, y and z are distinct elements.

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5. The Cartesian product $A \times A$ has 9 elements among which are found $(1, \quad 0) \quad and \quad (0, \quad 1)$. Find the set A and the remaining elements of $A \times A$.

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6. If R is the set of all real numbers, what do the cartesian products $R \times R$and $R \times R \times R$represent?

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7. If $A = [a, b, c], B = [c, d]$ and $C = [d, e, f]$, Find

(i) $A \times (B \cup C)$ (ii) $(A \cap B) \times C$ (iii) $(A \times B) \cap (B \times C)$

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8. (a) Let A= {1,2,3}, B= [3,4] and C={4,5,6}. Find:

(i) $(A \times B) \cap (A \times C)$

(b) Let A = {2,4,6}, B= {6,8,10} and C= {10,12,14}. Find $B \times (A \cup C)$

(c) If A = [1,2,3], B= [3,4], C = [4,5,6], find $A \times (B \cup C)$

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9. Let $A = \left\{\frac{1}{2}, 2\right\}, B = [2, 3, 5], C = \{-1, -2\}$. Verify that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

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10. Let A = {1,2,3}, B = {2,3,4} and C= {4,5). Verify that:

$A \times (B \cup C) = (A \times B) \cup (A \times C)$

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11. If A = $[1, 2, 3], B = [4], C = [5]$, then verify that:

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

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12. Let $A = [1, 2, 3], B = [-1, 0, 1, 2, 3], C = \{1\}, D = \{-1, 1\}$. Then verify the following:

(i) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

(ii) If $A \subset B$ and $C \subset D$, then $(A \times C) \subset (B \times D)$

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13. Let P = {1,4,9} and Q={2,4,6}. Write the elements of $(P \cap Q) \times (P \cup Q)$. Also, find $(P \times Q) \cap (Q \times P)$

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14. Theorem 3 (If A and B are two non empty set; then prove that $A \times B = B \times A \Leftrightarrow A = B$

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15. Let A be a non-empty set such that $A \times B = A \times C$. Show that $B = C$.

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16. If $A \subseteq B$ and $C \subseteq D$, then prove that:

$A \times C \subseteq B \times D$

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17. (i) If $A \subseteq B$, prove that $A \times A \subseteq (A \times B) \cap (B \times A)$

(ii) If $A \subseteq B$, prove that $A \times C \subseteq B \times C$ for any set C.

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18. Theorem 1(i) (For any three set $A; B; C;$ prove that $A \times (B \cup C) = (A \times B) \cup (A \times C))$

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19. For any three sets A, B and C, prove that:

$(A - B) \times C = (A \times C) - (B \times C)$

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20. For any A sets A, B, C and D, prove that:

$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

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21. For any three sets A, B, C prove that:

$A \times (B' \cup C')' = (A \times B) \cap (A \times C)$

$A \times (B' \cap C')' = (A \times B) \cup (A \times C)$

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22. If A= {1,2,3} and B={1,2}, then find:

(i) $A \times B$ (ii) $B \times A$

(iii) Is $A \times B = B \times A$?

(iv) Represent $A \times B$ graphically and by arrow diagram.

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23. Let $X = (-2, 0, 1), Y = \{2, 3\}$. Reprsent $X \times Y$ and $Y \times X$ graphically.

Also find $n(X \times Y)$ and $n(Y \times X)$

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24. Let $A = \{2, 3, 5, 7\}, B = \{1, 12, 13, 15\}$. How many elements are there in $A \times B$? In $B \times A$? Is $A \times B = B \times A$? Is $n(A \times B) = n(B \times A)$?

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25. Theorem 9(A and B are two non empty set having n element in common ; then prove that $A \times B$ and $B \times A$ have n^2 element in common.)

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Exercise 2 B

1. (a) Let R be the relation on Z defined by aRb if and only if $a - b$ is an integer. Find: (i) R (ii) domain of R (iii) range of R.

(b) Let R be the relation on Z defined by:

$R = \{(a, b) : a \in Z, b \in Z, a^2 = b^2\}$

Find (i) R (ii) domain of R (iii) range of R.

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2. Let A= {1,2,3,4,5} and B= {2,4,6,8,10}. Let $R = \{(a, b) : a \in A, b \in B, \text{ a divides } b\}$ be a relation from A into B. Find R. Show that domain of R is A and range of R is B.

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3. Determine the domain and range of the relation R defined by :

(i) $R = \{(x + 1, x + 5), x \in \{0, 1, 2, 3, 4, 5\}\}$

(ii) R= $\{(x, x^3) : x$ is prime number less than10}

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4. Determine the domain and range of the following relations:

(i) $\{(1, 2), (1, 4), (1, 6), (1, 8)\}$

(ii) $\{(x, y), x \in N, y \in N \text{ and } x + y = 10\}$

(iii) $\{(x, y) : x \in N, x < 5, y = 3\}$

(iv) $\{(x, y) : y = |x - 1|, x \in Z \text{ and } |x| \leq 3\}$

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5. Let $A = \{1, 2, 3, 4\}$ and $B = \{x, y, z\}$. Let $R : A \to B$ is defined as $R = \{(1, x), (1, z), (3, x), (4, y)\}$ then find domain and range of R.

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6. Let L be the set of all lines in a plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1(\text{ is perpendicular to L})_2\}$. Show that R is symmetric but neither reflexive nor transitive.

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7. Show that the relation geq on the set R of all real numbers is reflexive and transitive but not symmetric.

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8. The relation "is a factor of" on the set N of all natural number is not

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9. Let A={1,2,3,4,6}. Let R be the relation on A defined by $\{\{a, b): a, b \in A,$ b is exactly divisible by a}.

(i) Write R in roster form, (ii) Find the domain of R, (iii) Find the range of R.

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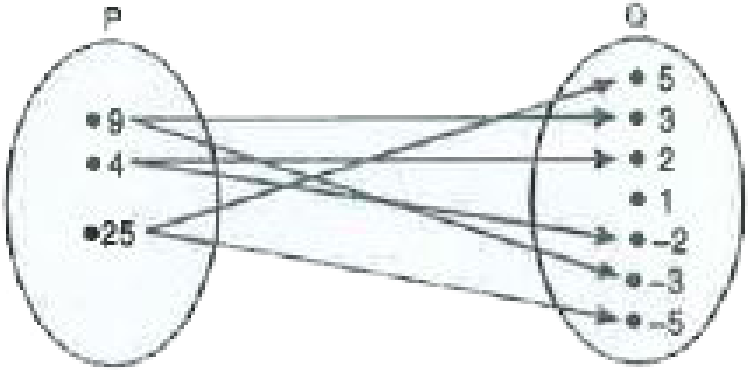
10. The following figure shows a relation between P and Q. Write the relation in: (i) set builder form (ii) roster form

What is its domain and range?



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11. For the given relation R on a set S, determine which are equivalence relations:

(a) (i) S is the set of all rational number a R b iff a= b

(ii) S is the set of all real numbers iff:

(I) $|a| = |b|$ (II) $a \geq b$

(b) (i) S is the set of all traingles in a plane a R b iff a is congrunet to b.

(ii) S is the set of all triangles in a place a R b iff a and b have equal perimeters.

(iii) S is the set of all people in India today, a R b iff a and b have same mother-tongue.

(iv) S is the of all people in the world today aRb iff a and b have same mother-tongue.

(v) S is the set of all people in the world today a R b iff a lives within 75 kilometres of b.

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12. Statement-1: The relation R on the set $N \times N$ defined by (a, b) R (c, d) $\Leftrightarrow$ a+d = b+c for all a, b, c, d $\in$ N is an equivalence relation.

Statement-2: The union of two equivalence relations is an equivalence relation.

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13. Is inclusion of a subset in another, in the context of a universal set, an equivalence relation in the class of subsets of the sets? Justify your answer.

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14. Given the relation R= {(1,2), (2,3)} on the set of natural numbers, add a minimum of ordered pairs so that the enlarged relation is symmetric, transitive and reflexive.

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Exercise 2 C

1. Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range:

(i) {(2,1), (3,1),(4,2)}

(ii) {(2,2),(2,4),(3,3),(4,4)}

(iii) {(2,1),(5,1),(8,1),(11,2),(14,2),(17,2)}

(iv) {(1,2),(2,3),(3,4),(4,5),(5,6),(6,7)}

(v){(2,1),(4,2),(6,3),(8,4),(10,5)}

(vi) {(1,2),(2,3),(3,4),(3,5),(3,7),(4,8)}

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2. Determine if each function given below is one-to-one

(i) To each state of India assign its capital.

(ii) To each person on earth assign the number, which corresponds to his height.

(iii) To each country in the world assign the latitude and longitude of its capital.

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3. If $f: A \to B$ is an injection such that range of $f = \{a\}$. Determine the number of elements in A .

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4. (i) If $f(x) = 3x^4 - 5x^2 + 7$, find $f(x-1)$

(ii) If $f(x) = x^2 - 3x + 4$, then find the values of x satisfying $f(x) = f(2x+1)$

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5. (i) If f(x) $= x + \frac{1}{x}$ prove that:

$[f(x)]^3 = f(x^3) + 3f\left(\frac{1}{x}\right)$

(ii) If $f(x) = x^3 - \frac{1}{x^3}$, prove that $f(x) + f\left(\frac{1}{x}\right) = 0$

(iii) If $f(x) = \frac{1-x^2}{1+x^2}$, prove that $f(\tan\theta) = \cos 2\theta$

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6. (i) If $y = f(x) = \frac{3x-1}{5x-3}$, prove that f(y) = x

(ii) If $y = f(x) = \frac{ax-b}{bx-a}$, prove that f(y) = x

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7. If f(x) = $\log_e\left(\frac{1-x}{1+x}\right)$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to :

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8. (a) What are the real numbers x such that [x] =2?

(b) What are the value taken by the function |x|?

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9. What values does the function $x \to 2x^2 - 1$ associate with the number 7 in the range?

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10. Given $f(x) = \begin{cases} 3x - 8 & \text{for} \quad x \leq 5 \\ 7 & \text{for} \quad x > 5 \end{cases}$

What is the value of the function:

(i) at x = 3 and (ii) at x = 7?

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11. What is the domain of the function $\frac{x}{x^2 - 3x + 2}$?

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12. What is the range of the constant function 1 ?

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13. For what values of x are the following functions not defined ? (i) $\frac{3x}{4x - 3}$ (ii) $\sqrt{x - 2}$ (iii) $\frac{1}{\sqrt{x - 3}}$ (iv) $\frac{\sin x}{x}$ (v) $\sin\frac{1}{x}$ (vi) $\sqrt{(x + 2)(x - 3)}$

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14. Find the periods of the following functions, if periodic : (i) $|\cos x|$ (ii) $2\cos\frac{1}{3}(x-\pi)$

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15. Show that $f: N \to N$ defined by $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$ is many-one onto function.

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16. If $f(x) = \cos(\log x)$, then $f(x)f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy)\right]$=

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17. (1) If $f(x) = \sqrt{x}$, prove that :

$\frac{f(x+h) - f(x)}{h} = \frac{1}{\sqrt{x+h} + \sqrt{x}}$.

(2) If $f(x) = x^2$, find :

$$\frac{f(1.1) - f(1)}{1.1 - 1}.$$

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18. Find domain(i) $f(x) = \frac{1}{x-5}$ (ii) $f(x) = \frac{3-x}{x-3}$ (iii) $f(x) = \frac{x^2-1}{x-1}$ (iv) $f(x) = \frac{|x-3|}{x-3}$ (v) $f(x) = \frac{1}{2-\sin 3x}$

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19. (i) $f(x) = \sqrt{3-2x}$

(ii) $f(x) = \frac{1}{\sqrt{x+2}}$

(iii) $f(x) = 1 + x - [x-2]$.

Find the domain of the following (23 - 24) :

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20. Find the domain of $f(x) = \sqrt{\log\left(\frac{5x - x^2}{6}\right)}$

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21. State, given justification for your answer, which of the following pairs are equal :

(i) $f(x) = \frac{x}{x^2}, g(x) = 1$

(ii) $f(x) = \sqrt{x^2}, g(x) = |x|.$

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22. Are the following functions invertible in their respective domains ? If so, find the inverse in each case :

(i) $f(x) = -\frac{1}{3}x + 4$ (ii) $f(x) = \frac{x - 1}{x + 1}$ (iii) $f(x) = \sqrt{1 - x^2}, 0 \leq x \leq 1.$

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23. Let $f: R \to R$ be defined by $f(x) = 3x - 7$. Show that f is invertible and hence find f^{-1}.

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Exercise 2 D

1. Let $f(x) = x^2$ and $g(x) = 2x + 1$ be two real functions. Find : $(f + g)(x), (f - g)(x), (fg)(x)$ and $\left(\frac{f}{g}\right)(x)$.

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2. Let $f(x) = \sqrt{x}$ and $g(x) = x$ be two functions defined over the set of nonnegative real numbers. Find $(f + g)(x)$, $(fg)(x)$, $(fg)(x)$ and $\left(\frac{f}{g}\right)(x)$.

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3. If f and g are functions defined by :

$f(x) = \sqrt{x-1}, g(x) = \frac{1}{x}$, then describe the following : (i) $f+g$ (ii) $f-g$ (iii) fg (iv) $\frac{f}{g}$.

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Exercise 2 E

1. Define the real valued function $f : R \quad \{0\} \to R$ defined by $f(x) = \frac{1}{x}$, $x \in R \quad \{0\}$. Complete the Table given below using this definition. What is the domain and range of this function? x 2 1.5 1 0.5 0.25 1 1.5 2 $y = \frac{1}{x}$

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2. Sketch the graphs of the following functions : (a) (i) $f(x) = 2x + 3$ (ii) $f(x) = 4 - 2x$

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3. Draw the graph of $f(x) = sgn(x - 2)$.

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4. Draw the graph of the function :

$$f(x) = \begin{cases} 0 \text{ if x is an even integer} \\ 1 \text{ if x is an odd integer} \end{cases}$$

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Objective Type Questions

1. Let n(A) = m and n(B) = n. The total number of non-empty relations that can defined from A to B is

A. m^n

B. $n^m - 1$

C. $mn - 1$

D. $2^{mn} - 1$

Answer:

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2. Domain of $\sqrt{a^2 - x^2}(a > 0)$ is

A. $(-a, a)$

B. $[-a, a]$

C. $[0, a]$

D. $(-a, 0]$.

Answer: B

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3. Range of $f(x) = \dfrac{1}{1 - 2\cos x} is:$

A. $\left[\dfrac{1}{3}, 1\right]$

B. $\left[-1, \dfrac{1}{3}\right]$

C. $(-\infty, -1) \cup \left[\dfrac{1}{3}, \infty\right)$

D. $\left[-\dfrac{1}{3}, 1\right].$

Answer:

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4. The domain of the function f given by $f(x) = \dfrac{x^2 + 2x + 1}{x^2 - x - 6}$

A. $R - \{3, -2\}$

B. $R - \{-3, 2\}$

C. $R - [3, -2]$

D. $R - (3, -2).$

Answer: B

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5. The correct match by the following is :

Column I		Column II	
(*i*)	Polynomial Function	(*a*)	$F(x) = \|x\|$
(*ii*)	Constant Function	(*b*)	$F(x) = x^2 + 2x + 3$
(*iii*)	Rational Function	(*c*)	$F(x) = \frac{f(x)}{g(x)}$
(*iv*)	Modulus Function	(*d*)	$F(x) = 2, x \in \mathbf{R}$

A. (i) -(b), (ii) -(c), (iii)- (d), (iv)- (a)

B. (i)- (b), (ii) - (d), (iii)- (c), (iv) - (a)

C. (i) - (c), (ii) - (b), (iii) - (d), (iv) - (a)

D. (i) - (a), (ii) - (b), (iii) - (c), (iv) - (d)

Answer: B

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6. If R is a relation from a finite set A having m elements to a finite set B having n elements then the number of relations from A to B is

A. 2^m

B. mn

C. $m + n$

D. 2^{mn}

Answer: D

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7. (i) If $A = [-1, 1]$, find $A \times A$

(ii) If P = {1,2}, find $P \times P$

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8. If G={7,8} and H={5,4,2}, find $G \times H$ and $H \times G$.

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9. If P ={ a, b, c} and Q = {r} , find $P \times Q$.

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10. If $n(A) = 3, n(B) = 4,$ then write $n(A \times A \times B)$.

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11. Let A and B be two sets such that n (A) = 3 and n(B) =2. If (x,1) (y,2) (z,1) are in $A \times B$, find A and B, where x,y,z are distinct elements.

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12. Let $A = \{1, 2, 3, 4, 6\}$. Let R be the relation on A defined by $\left\{\left(a\dot{b}\right) : a, b \in A, b$is exactly divisible by a}.(i) Write R in roster form(ii)

Find the domain of R(iii) Find the range of R.

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13. Let R be the relation "is divisor of" from the set A{1,2,3} to B= [4,10,15].

Write down the set of ordered pairs corresponding to R.

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14. If $A = \{1, 3, 5\} and\ B = \{2, 4\}$ list the elements of R, if $R = \{(x, y) : x, y \in A \times B\ and\ x > y\}$.

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15. Let $R = \{(x, y) : x, y \in Z, y = 2x - 4]$. If $(p, -2)$ and $[4, q^2) \in R$, then find the value of p and q

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16. If $R = \{(x, y) : x, y \in Z,\ x^2 + y^2 \leq 4\}$ is a relation defined on the set Z of integers, then write domain of R.

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17. A function f is defined by $f(x) = 2x \quad 5$. Write down the values of (i) $f(0)$, (ii) $f(7)$, (iii) $f(3)$.

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18. Write the range of the function $f(x) = \cos[x]$, where $-\frac{\pi}{2} < x < \frac{\pi}{2}$

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19. Find the domain and range of each of the following real valued function: $f(x) = \frac{x-2}{2-x}$.

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20. Write of the domain and range of $f(x) = \sqrt{x - [x]}$.

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21. Let f and g be real functions, defined by $f(x) = \sqrt{x-1}$ and $g(x) = \sqrt{x+1}$. Find

(i) (f+g)(x) (ii) (f-g)(x) (iii) (fg)(x) (iv) $\left(\frac{f}{g}\right)(x)$.

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22. Let fg: $R \to R$ be defined respectively by $f(x) = x + 1, g(x) = 2x - 3$. Find f+g, f-g and $\frac{f}{g}$.

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23. Find the domain of $f(x) = \frac{1}{\sqrt{x - |x|}}$

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24. Find the range of $f(x) = e^{x - [x]}, x \in R$

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25. If f and g are real fucntions defined by $f(x) = x^2 + 7$ and $g(x)3x + 5$ Then , find each of the following .

$f(3) + g(-5)$ (ii) $f\left(\frac{1}{2}\right) \times g(14)$

(iii) f(-2)+g(-1) (iv) f(t)-f(-2)

(v) $\frac{f(t) - f(5)}{t - 5}$ if $t \neq 5$

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26. Find the period of $\tan 4x$

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Ncert File Question For Ncert Book Exercise 2 1

1. If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$, find the values of x and y.

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2. If the set A has 3 elements and the set $B=\{3,4,5\}$, then find the number of elements in $(A \times B)$.

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3. If $G=\{7,8\}$and $H=\{5,4,2\}$, find $G \times HandH \times G$.

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4. Stat whether each of the following statements are true or false. If the statement is false re-write the given statement correctly: If

$p = \{m, n\} and\ Q = \{n, m\},\ then\ P \times Q = \{(m, n),\ (n, m)\}$ If A and B are non empty sets then $A \times B$ is a non empty set of ordered pairs (x, y) such that $x \in B\ and\ y \in A$. If $A = \{1, 2\},\ B = \{3, 4\},\ then\ A \times (B \cap \varphi) = \varphi$.

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5. If $A = \{-1, 1\}$, find $A \times A \times A$.

Watch Video Solution

6. If $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$. Find A and B.

Watch Video Solution

7. Let A={1,2},B={1,2,3,4},C={5,6}and D={5,6,7,8}. Verify that:

(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

(ii) $A \times C$ is a subset of $B \times D$.

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8. Let $A = \{1, 2\}, B = \{3, 4\}$. Write $A \times B$. How many subsets will $A \times B$ have ?

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9. Let A and B be two sets such that $n(A) = 3 and n(B) = 2$. If $(x, 1), (y, 2), (z, 1)$ are in A $\times$ B. find A and B. where x, y and z are distinct elements.

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10. The Cartesian product $A \times A$ has 9 elements among which are found $(1, \quad 0) \quad and \quad (0, \quad 1)$. Find the set A and the remaining elements of $A \times A$.

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Ncert File Question For Ncert Book Exercise 2 2

1. Let $A = \{1, 2, 3;\cdot 14\}$. Define a relation on a set A by $R = \{(x, y) : 3x - y = 0. \, where \, x, y \in A\}$. Depict this relationship using an arrow diagram. Write down its domain, co-domain and range.

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2. Define a relation R on the set N of natural numbers by $R = \{(x, y) : y = x + 5, x$ is a natural number less than $4, x, y \in N\}$. Depict this relationship using i. roster form ii. an arrow diagram. Write down the domain and range or R.

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3. If A = {1, 2, 3, 5} and B = {4, 6, 9} . Define a relation R from A to B by R = {(x, y) : the difference between x and y is odd , x in A, y in B} Write R in roaster form .

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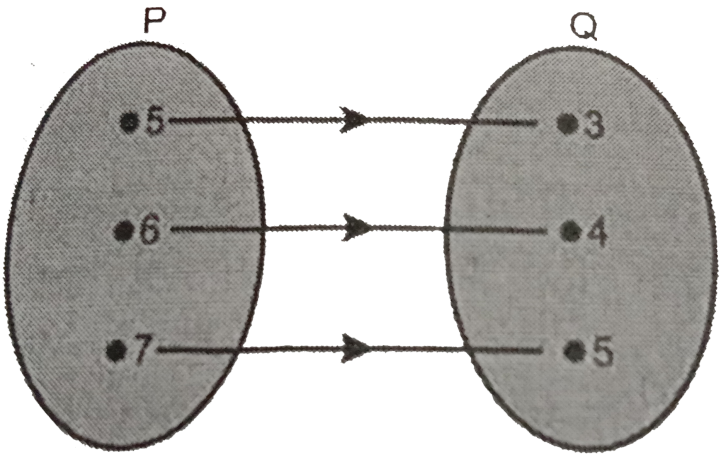
4. The figure shows a relationship between the sets P and Q. Write this relation :

(i) in set-builder from

(ii) Roaster from. What is its domain and range?



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5. Write the relation $R = \{(x, x^3) : x$ is a prime number less than 15$\}$ in roster form.

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6. Let $A = \{x, y, z\} and B = \{1, 2\}$. Find the number of relations from A to B.

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7. Let R be the relation on Z defined by $R = \{(a, b) : a, b \in Z, ab$ is an integer$\}$.Find the domain and range of R.

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Ncert File Question For Ncert Book Exercise 2 3

1. Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

(i) {(2,1),(5,1),(8,1),(11,1),(14,1),(17,1)}

(ii) {(2,1),(4,2),(6,3),(8,4),(10,5),(12,6),(14,7)}

(iii) {(1,3),(1,5),(2,5)}

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2. Find the domain and range of the following real functions:(i) $f(x) = -|x|$ (ii) $f(x) = \sqrt{9 - x^2}$

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3. A function f is defined by $f(x) = 2x \quad 5$. Write down the values of (i) $f(0)$, (ii) $f(7)$, (iii) $f(3)$.

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4. The function t which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$. Find (i) t (0) (ii) t(28) (iii) $t(\quad 10)$ (iv) The value of C, when $t(C) = 212$.

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5. Find the rage of each of the following functions. (i) $f(x) = 2 \quad 3x, \quad x \in R, x > 0$ (ii) $f(x) = x^2 + 2$, x is a real number. (iii) $f(x) = x$, x is a real number.

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Miscellaneous Exercise

1. The relation 'f' is defined by: $f(x) = \begin{cases} x^2, 0 \leq x \leq 3 \\ 3x \quad , 3 \leq x \leq 10 \end{cases}$. The relation g is defined by $g(x) = \begin{cases} x^2, 0 \leq x \leq 2 \\ 3x, 2 \leq x \leq 10 \end{cases}$

Show that f is a function and g is not a function.

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2. $f(x) = x^2$, $find \dfrac{f(1.1) - f(1)}{1.1 - 1}$

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3. Find the domain of the function $f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$

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4. Find the domain and the range of the real function defined by $f(x) = \sqrt{x - 1}$

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5. Find the domain and the range of the real function/defined by $f(x) = |x - 1|$

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6. Let $f = \left\{\left(x, \frac{x^2}{1+x^2}\right) : x \in R\right\}$ be a function from R into R. Determine the rage of f.

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7. Let f, g : R $\rightarrow$ R be defined, respectively by $f(x) = x + 1, g(x) = 2x3$. Find $f + g$, fg and $\frac{f}{g}$.

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8. Let f={(1,1),(2,3), (0,-1), (-1, -3)} be a function from Z to Z defined by f(x) = ax +b, for some integers a, b. Determine a, b.

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9. Let A={1,2,3,4}, B={1,5,9,11,15,16} and f={(1,5),(2,9),(3,1),(4,5),(2,11)}. Are the following true?

(i) f is a relation from A to B.

(ii) f is a function from A to B. Justify your answer in each case.

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10. Let f be the subset of $Z \times Z$defined by $f = \{(ab, a + b) : a, b \in Z\}$. Is f a function from Z to Z? Justify your answer.

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11. Let $A = \{9, 10, 11, 12, 13\}$and let $f : A \to N$be defined by f(n) = the highest prime factor of n. Find the range of f.

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Exercise

1. If $A = \{x : x \in W, x < 2\}, B = \{x : x \in N, 1 < x < 5\}, C = \{3, 5\}$, find: (i) $A \times (B \cap C)$ (ii) $A \times (B \cup C)$

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2. $A = \{1, 2, 3, 4, 5\}, S = \{(x, y) : x \in A, y, \in A\}$, then find the ordered which satisfy the conditions given below.

(i) x+y=5 (ii) $x + y < 5$

(iii) $x + y = \ > 8$

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3. If $R_1 = \{(x, y) \mid y = 2x + 7,$ where $x \in R$ and $-5 \leq x \leq 5\}$ is a relation. Then find the domain and Range of R_1.

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4. If $R_2 = \{(x, y) \mid x \text{ and } y \text{ are integers and } x^2 + y^2 = 64\}$ is a relation.

Then find R_2.

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5. If the given relation a fucntion? Give reason for your answer.

(i) $h = \{(4, 6), (3, 9), (-11, 6), (3, 11)\}$

(ii) $f = \{(x, x) \mid x \text{ is a real number }\}$

(iii) $g = \left\{\left(x, \frac{1}{x}\right) x \text{ is a positive integer}\right\}$

(iv) $s = \left\{\left(x, x^2\right) \mid x \text{ is positive integer}\right\}$

(v) t={(x,3)|x is a real number}

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6. If f and g are real fucntions defined by $f(x) = x^2 + 7 \text{ and } g(x)3x + 5$

Then , find each of the following .

$f(3) + g(-5)$ (ii) $f\left(\frac{1}{2}\right) \times g(14)$

(iii) f(-2)+g(-1) (iv) f(t)-f(-2)

(v) $\frac{f(t)-f(5)}{t-5}$ if $t \neq 5$

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7. If f and g are two real valued functions defined as f(x) =2x +1 and $g(x)=x^2+1$ then find

(i) f+g (ii) f-g (iii) fg (iv) $\frac{f}{g}$

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8. Find the domain of the function

$$f(x)=\frac{1}{\sqrt{1-\cos x}}$$

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9. If $f(x)=\frac{x-1}{x+1}$, then show that $f\left(\frac{1}{x}\right)=-f(x)$ (ii) $f\left(-\frac{1}{x}\right)=\frac{1}{f(x)}$

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10. If $y = f(x) = \frac{ax - b}{bx - a}$, the prove that : $x = f(y)$

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Revision Exercise

1. Let R be a relation from set Q to Q defined as:

$R = \{(a, b) : a, b \in Q \text{ and } a - b \in Z\}$

Prove that

(i) For each $a \in Q$, $(a, a) \in R$

$(ii)(a, b) \in R \Rightarrow (b, c) \in R$ where $a, b \in Q$

(iii) $(a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R$, where $a, b, c \in Q$

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2. Let R be a relation from N to N defined by $R = \left\{(a, b) : a\dot{b} \in N \text{and } a = b^2\right)$. Are the following true?(i) $(a, a) \in R, f \text{ or } alla \in N$(ii) $(a, b) \in R, implies(b, a) \in R$(iii) `(a ,

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3. If $f(x) = \frac{1+x}{1-x}$, show that $f(f(\tan\theta)) = -\cot\theta.$

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4. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ show that $f\left(\frac{2x}{1+x^2}\right) = 2(f(x))$

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5. If $f(x) = \frac{2x}{1+x^2}$, show that $f(\tan\theta) = \sin 2\theta)$

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6. यदि $f(x) = \log_e x$, तो सिद्ध कीजिए की f(x y z) = f(x) + f(y) + f(z)

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7. If for non-zero x, if $f(x) + mf\left(\frac{1}{x}\right) = \frac{1}{x} - 5$, where $1 \neq m$, then obtain f(x)

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8. Let $f(x)$ be defined on $[-2, 2]$ and be given by

$$f(x) = \begin{cases} -1, & -2 \leq x \leq 0 \\ x - 1, & 1 < x \leq 2 \end{cases} \text{ and } g(x) = f(|x|) + |f(x)|.$$

Then find $g(x)$.

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9. Period of $f(x) = \sin^4 x + \cos^4 x$

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10. Find the domain and range of the function:

$$f(x) = \begin{cases} x^2 & \text{when } x < 0 \\ x & \text{when } 0 \leq x \leq 1 \\ \frac{1}{x} & \text{when } x > 1 \end{cases}$$

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11. Find the domain of the following:

(i) $f(x) = \dfrac{1}{\log_{10}(1-x)} + \sqrt{x+2}$

(ii) $f(x) = \sqrt{1-2x} + 3\sin^{-1}\left(\dfrac{3x-1}{2}\right)$

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12. Find the domain of $F(x) = \dfrac{1}{x} + 2^{\sin^{-1}x} + \dfrac{1}{\sqrt{x-2}}$

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13. Find the domain of the following functions:

(i) $f(x) = \dfrac{1}{\sqrt{|x| - x}}$

(ii) $f(x) = \sqrt{\cos(\sin x)} + \sin^{-1}\left(\dfrac{1 + x^2}{2x}\right)$

(iii) $\dfrac{1}{\log_{10}(1 - x)} + \sqrt{x + 2}$

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14. Find the domain and range of the following functions:

(i) $f(x) = \dfrac{1}{\sqrt{x - [x]}}$

(ii) $f(x) = \sin\left(\log\left(\dfrac{\sqrt{4 - x^2}}{1 - x}\right)\right)$

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15. Is the function: $f(x) = \dfrac{x^2 - 8x + 18}{x^2 + 4x + 30}$ one-one?

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16. Prove that $f:(-1,1) \to R$ defined by:

$$f(x) = \begin{cases} \frac{x}{1+x} & -1 < x < 0 \\ 0 & x = 0 \\ \frac{x}{1-x} & 0 < x < 1 \end{cases}$$ is bijective. Prove that f is one -one onto

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17. Let $f: N \to N$ be defined by: $f(n) = \{n+1, \quad \text{if} \quad n\, is\, odd n-1, \quad \text{if} \quad n\, is\, even$ Show that f is a bijection.

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18. Let $f: N \cup \{0\} \to N \cup \{0\}$ be defined by $f(n) = \{n+1, \quad \text{if} \quad n\, is\, eve \cap -1, \quad \text{if} \quad n\, is\, odd$ Show that f is invertible and $f = f^{-1}$.

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Check Your Understanding

1. If $A \times B = \phi$, then is it necessary that both A and B are empty?

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2. If $A = \{-1, 1\}$, find $A \times A \times A$.

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3. Let $A = \{1, 2\}$and $B = \{3, 4\}$. Find the number of relations from A to B.

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4. Write the relation R = $\{(x, x^3) : x$ is a prime number less than 20$\}$ in roster from.

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5. Find the domain and Range of Absolute-Value function:

$$f(x) = |x| = \begin{cases} x & \text{if} x > 0 \\ 0 & x = 0 \\ -x & \text{if } x < 0 \end{cases}$$

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6. Which of the following functions are odd or even or neither?

(i) $f(x) = \cot x + 4\cos ecx + x$

(ii) $f(x) = \sec x + 4\cos x + 3x^2$

(iii) $f(x) = \sin x + \cos x$

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7. Find the periods of the following : (i) $|\cos x|$ (ii) $\tan 4x$

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8. The domain of $f(x) = \frac{1}{[x] - x}$ is

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9. Let $f(x) = x^2$ and $g(x) = 2x + 1$ be two real functions. Find $(f + g)(x), (f \quad g)(x), (fg)(x), \left(\frac{f}{g}\right)(x)$.

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Competition File

1. Let $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ be a relation the set $A = \{1, 2, 3, 4\}$. The relation R is

A. a function

B. transitive

C. not symmetric

D. reflexive

Answer: C

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2. Let $R = \{(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9(, (3, 12), (3, 6)\}$ be relation on the set $A = \{3, 6, 9, 12\}$. The relation is-

A. reflexive only

B. reflexive and transitive only

C. reflexive and symmetric only

D. an equivalence relation.

Answer: B

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3. Let w denote the words in the english dictionary. Define the relation R by: R = $\{(x, y) \in W \times W \mid$ words x and y have at least one letter in common$\}$. Then R is: (1) reflexive, symmetric and not transitive (2) reflexive, symmetric and transitive (3) reflexive, not symmetric and transitive (4) not reflexive, symmetric and transitive

A. not reflexive, symmetric and transitive

B. reflexive, symmetric and not transitive

C. reflexive, symmetric and transitive

D. reflexive, not symmetric and transitive

Answer: B

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4. Let R be the real line. Consider the following subsets of the plane $R \times R$. $S = \{(x, y) : y = x + 1 and 0 < x < 2\}, T = \{(x, y) : x - y$ is an integer $\}$. Which one of the following is true? (1) neither S nor T is an

equivalence relation on R (2) both S and T are equivalence relations on R (3) S is an equivalence relation on R but T is not (4) T is an equivalence relation on R but S is not

A. T is an equivalence relation on R but S is not

B. Neither S nor T is an equivalence relation on R

C. Both S and T are equivalence relations on R

D. S is an equivalence relation on R but T is not

Answer: A

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5. Consider the following relations: R = {(x, y) | x, y are real numbers and x = wy for some rational number w}; $S = \left\{ \left(\frac{m}{n}, \frac{p}{q} \right) \mathrm{m}, \mathrm{n}, \mathrm{pandqa\ r\ ei\ n\ t\ e\ g\ e\ r\ ss\ u\ c\ ht\ h\ a\ tn}, \mathrm{q} \neq 0 \mathrm{andq\ m} \right.$. Then (1) neither R nor S is an equivalence relation (2) S is an equivalence relation but R is not an equivalence relation (3) R and S both are

equivalence relations (4) R is an equivalence relation but S is not an equivalence relation

A. R is an equivalence relation but S is not an equivalence relation

B. neither R nor S is an equivalence relation

C. S is an equivalence relation but R is not an equivalence relation

D. R and S both are equivalence relations.

Answer: C

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6. The largest interval lying in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ for which the function $\left[f(x) = 4^{-x} \hat{} 2 + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)\right]$ is defined, is

A. $-\frac{\pi}{2}, \frac{\pi}{2}$

B. $\left(-\frac{\pi}{4}, \frac{\pi}{2}\right)$

C. $\left(0, \frac{\pi}{2}\right)$

D. $[0, \pi]$

Answer: C

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7. Let $f: N \overrightarrow{Y}$ be a function defined as $f(x) = 4x + 3$, where $Y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$. Show that f is invertible and its inverse is (1) $g(y) = \frac{3y + 4}{3}$ (2) $g(y) = 4 + \frac{y + 3}{4}$ (3) $g(y) = \frac{y + 3}{4}$ (4) $g(y) = \frac{y - 3}{4}$

A. $g(y) = \frac{y - 3}{4}$

B. $g(y) = \frac{3y + 4}{3}$

C. $g(y) + 4 + \frac{y + 3}{4}$

D. $g(y) = \frac{y + 3}{4}$

Answer: A

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8. The domain of the function

$f(x) = \frac{1}{\sqrt{|x| - x}}$, is

A. $(-\infty, \infty)$

B. $(0, \infty)$

C. $(-\infty, 0)$

D. $(-\infty, \infty) - \{0\}$

Answer: C

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9. Let $f(x) = x^2 and g(x) = \sin x f$ or $all x \in R$. Then the set of all x satisfying $(fogogof)(x) = (gogof)(x), where (fog)(x) = f(g(x)),$ is

$\pm\sqrt{n\pi}, n \in \{0, 1, 2, .\}$ $\pm\sqrt{n\pi}, n \in \{1, 2, .\}$

$\frac{\pi}{2} + 2n\pi, n \in \{, -2, -1, 0, 1, 2\}$ $2n\pi, n \in \{, -2, -1, 0, 1, 2, \}$

A. $\pm\sqrt{n\pi}, n \in \{0, 1, 2. \ldots . \}$

B. $+\sqrt{n\pi}, n \in \{0, 1, 2. \ldots .\}$

C. $\frac{\pi}{2} + 2n\pi, n \in \{. \ldots . , -2, -1, 0, 1, 2, \ldots .\}$

D. $2n\pi, n \in \{. \ldots . -2, -1, 0, 1, 2, \ldots\}$

Answer: A

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10. The function $f: [0, 3] \to [1, 29]$, defined by

$f(x) = 2x^3 - 15x^2 + 36x + 1$, is

A. one-one and onto

B. onto but not one-one

C. one-one but not onto

D. neither one-one nor onto.

Answer: B

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11. If $a \in R$ and the equation $-3(x-[x])^2+2(x-[x])+a^2=0$ (where [x] denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval: (1) (-2,-1) (2) $(\infty, -2) \cup (2, \infty)$ (3) $(-1, 0) \cup (0, 1)$ (4) (1,2)

A. (1,2)

B. $(-2, -1)$

C. $(-\infty, -2) \cup (2, \infty)$

D. $(-1, 0) \cup (0, 1)$

Answer: D

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12. The function $f: R \to \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x)=\frac{x}{1+x^2}$ is

A. Surjective but not injective

B. Neither injective nor surjective

C. Invertible

D. Injective but not surjective.

Answer: A

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Chapter Test

1. Range of $f(x) = \frac{1}{1 - 2\cos x} is:$

A. $\left[\frac{1}{3}, 1\right]$

B. $\left[-1, \frac{1}{3}\right]$

C. $(-\infty, -1] \cup \left[\frac{1}{3}, \infty\right)$

D. $\left[-\frac{1}{3}, 1\right]$

Answer: C

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2. If R is a relation from a finite set A having m elements to a finite set B having n elements then the number of relations from A to B is

A. 2^m

B. mn

C. $m + n$

D. 2^{mn}

Answer: B

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3. $\left(\frac{x}{3} + 1, y - \frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$ then $x =$, $y =$

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4. $f(x) = x^2$, $find \frac{f(1.1) - f(1)}{1.1 - 1}$

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5. Find the domain of $f(x) = \frac{x^2 + 3x + 5}{x^2 - 5x + 4}$

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6. Theorem 1(i) (For any three set $A; B; C;$ prove that $A \times (B \cup C) = (A \times B) \cup (A \times C))$

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7. Let $A = \{1, 2, 3;, 14\}$. Define a relation on a set A by $R = \{(x, y) : 3x - y = 0. where\ x, y \in A\}$. Depict this relationship using an arrow diagram. Write down its domain, co-domain and range.

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8. Let $f(x) = \ln\left(\frac{1-x}{1+x}\right)$. Find x,y for which $f(x) + f(y) = f\left(\frac{x+y}{1+xy}\right)$

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9. Find the domain of $f(x) = \sqrt{\log\left(\frac{5x - x^2}{6}\right)}$

A. $(2, 3)$

B. $[2, 3)$

C. $(2, 3]$

D. $[2, 3]$

Answer: D

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10. The period of $\sin^4 x + \cos^4 x$ is

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11. The functin 'f' defined by: $f(x) = \begin{cases} x^2 & ,0 \leq x \leq 3 \\ 3x, & 3 \leq x \leq 10 \end{cases}$

The relation 'g' is defined by $g(x) = \begin{cases} x^2 ,0 \leq x \leq 2 \\ 3x & ,2 \leq x \leq 10 \end{cases}$

Show that 'f' is a function and 'g' is not a function

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